Problem Set 3 Solutions
Last edited by: Isaac Seetho, Sept 15, 2003
Problem 1: Thévenin and Norton Equivalents:

a) Determine V_{OC} and I_{SC}

In determining V_{OC}, notice the current source controls current throughout the entire loop.

-notice current goes into the resistor from the <u>right</u> so you get a voltage gain as you travel from the (+) on the voltage source to the (+) on V_{OUT}

-current sources can take ANY VOLTAGE—therefore, you can't use the right branch.

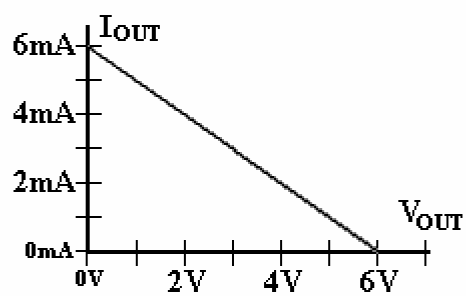
Using the left branch (voltage source + resistor),

$$\boxed{V_{OUT} = 5V + 1mA(1kO) = 6V}$$

In determining I_{SC}, since you have two sources that cause current (a current source that controls current and a voltage source that drives current) you cannot simply use the current on the current source (seeing as the right side is a wire and the left side has a resistance). Instead, you can use KCL & KVL: note the left branch is in parallel with a wire (which has 0 voltage). Therefore, the current through the left branch must be 5mA, moving upward. The middle branch has a forced 1mA current upward, so I_{OUT} is the sum of these:

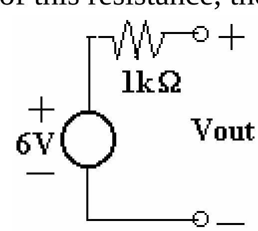
$$\boxed{I_{SC} = -I_{OUT} = -(5mA + 1mA) = -6mA}$$

b)

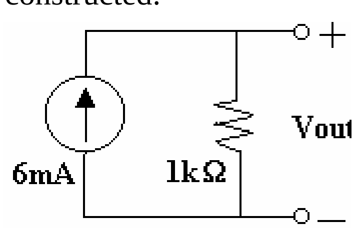


c,d) The Thévenin/Norton resistance can be found taking $R_{TH} = -V_{OC}/I_{SC} = 6V/-6mA = 1kO$
Making use of this resistance, the following circuits can be constructed:

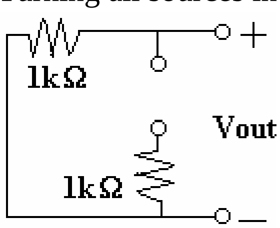
Thévenin:



Norton:



e) Turning all sources in the original circuit off, the following circuit results:



The resistor at the bottom sits on a path that ends in an open circuit, so no current can flow—it is as if that resistor is not there. Therefore, it can be dropped, resulting in a single-resistor circuit, with "equivalent resistance" of $\boxed{R_{TH} = 1kO}$.

Problem 2: Working with Loads

a) Looking at the circuit, it seems that the easiest way to calculate V_{TH} and R_{TH} is to calculate V_{OC} and R_{TH} directly, and not deal with current.

Calculating V_{OC}, the positive terminal of V_{OUT} is at +5V, but the negative terminal voltage is unknown. This can be found using voltage divider, but the lower two resistors must be combined first. As these are in parallel, the equivalent is:

$$R_{eq} = \left(\frac{1}{4k\Omega} + \frac{1}{4k\Omega}\right)^{-1} = 2k\Omega$$

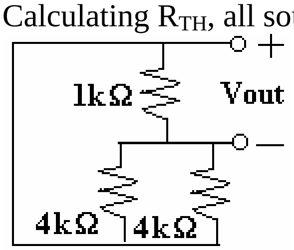
With this value, voltage divider can be used.

$$V_{NEG} = 5V \times \frac{2k\Omega}{2k\Omega + 4k\Omega} = \frac{10}{3}V$$

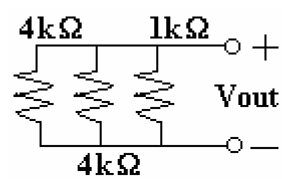
V_{OUT} and V_{OC} are the difference in voltage across the two terminals (+ and -). Therefore,

$$\boxed{V_{OC} = V_{OUT} = 5V - \frac{10}{3}V = \frac{5}{3}V}$$

Calculating R_{TH}, all sources can be turned off. The result is the following circuit:



This circuit can be redrawn to look like this:

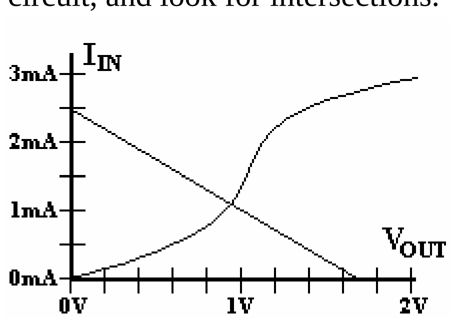


These resistors can be combined into an equivalent resistance as parallel resistances.

$$\boxed{R_{TH} = \left(\frac{1}{1k\Omega} + \frac{1}{4k\Omega} + \frac{1}{4k\Omega}\right)^{-1} = \frac{2}{3}k\Omega}$$

The values for V_{TH} and R_{TH} can then be plugged into a Thévenin equivalent template.

b) Taking the IV graph for the load, you can superimpose the I_{OUT} vs. V_{OUT} graph for the circuit, and look for intersections. These intersections are allowed modes of operation.



In this case, V_{OC} is 5/3 V and I_{SC} = -5/2 mA. The following graph results, where the straight line is the I_{OUT} vs. V_{OUT} graph for your circuit, and the curve represents the load:

Estimating off of the graph, the current and voltage of operation are approximately $\boxed{\text{-1mA, and 1V.}}$

c) Given that 1mA of current goes into the load, and the voltage difference across the connection is 1V, the power calculation is easy. $P = I \times V = 1mA \times 1V = 1mW$. Since current goes into the positive terminal, this power is positive. The load absorbs energy.

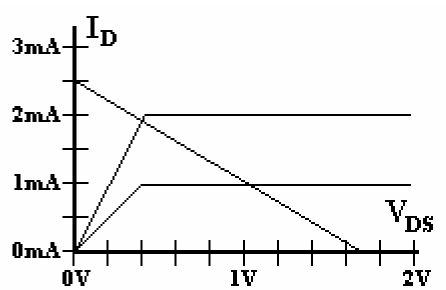
d) To find power out of the voltage source, one must first find out how much current leaves the voltage source. This can be done by determining current through the 1kO resistor and adding that to the current into the load. This current must cause a voltage drop of 1V across the resistor because it is in parallel with the load. Therefore,

$I_{Source} = 1mA + \frac{1V}{1k\Omega} = 1mA + 1mA = 2mA$, and the power is:

$\boxed{P_{Source} = -(2mA \times 5V) = -10mW}$

Problem 3: More Load-Line Analysis

The process in this problem is identical to the process in the last one, except this time we have a graph that has multiple possible IV curves. This means there are 2 modes of operation possible, given that V_{GS} can only hold values of 1V and 2V. Superimposing a graph of I_{OUT} vs. V_{OUT} for your circuit onto the IV graph of the load, the following graph results:



Estimating off of the graph for I_{IN} vs. V_{OUT} combinations, the following are obtained:

I_{IN} ~ -2mA, V_{OUT} ~ 0.2 V
I_{IN} ~ -1mA, V_{OUT} ~ 1.0 V

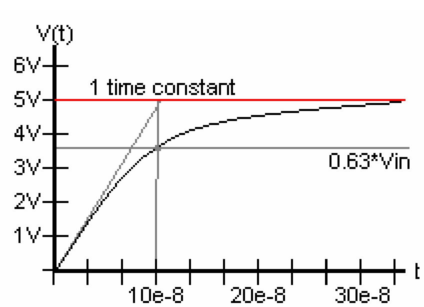
Problem 4: Transient Analysis

a) Finding an expression for $V_c(t)$: Given that the capacitor is completely discharged when the timer starts running, you can use the general formula from the RC Handout. Simply put, if an uncharged capacitor is hooked up to a driving voltage V, then $V_c(t) = V - Ve^{-t/RC}$, where R and C are equivalent resistance and capacitance.

In this case, an uncharged capacitor in a single resistor, single capacitor series circuit is loaded onto a driving voltage of 5V, R = 10kO and C = 1pF, so:

$\boxed{V_c(t) = 5V - 5e^{-t/(10^{-8}s)}}$

b) When making a plot of the capacitor voltage over time, you can use the "EE42 Easy Method." Using this method, all you need are: the RC time constant, the initial slope and the final voltage. The following graph results:



c) To determine the time when the capacitor is halfway charged, something that must be satisfied is that $e^{-t/(10^{-8}s)} = 0.5$. This occurs when $-t/(10^{-8}s) = \ln(0.5)$. Solving for t:

$\boxed{t = \ln(0.5) \times -(10^{-8}s) = 6.93ns}$

d) If you extend a line from (t, V) = (0s, 0V) with slope = $dV_c(0)/dt$, the result you should get is that it intersects the 5V line at exactly t = RC. This time is exactly 1 time constant.