

NODAL ANALYSIS CONTINUED

Lecture 9 review:

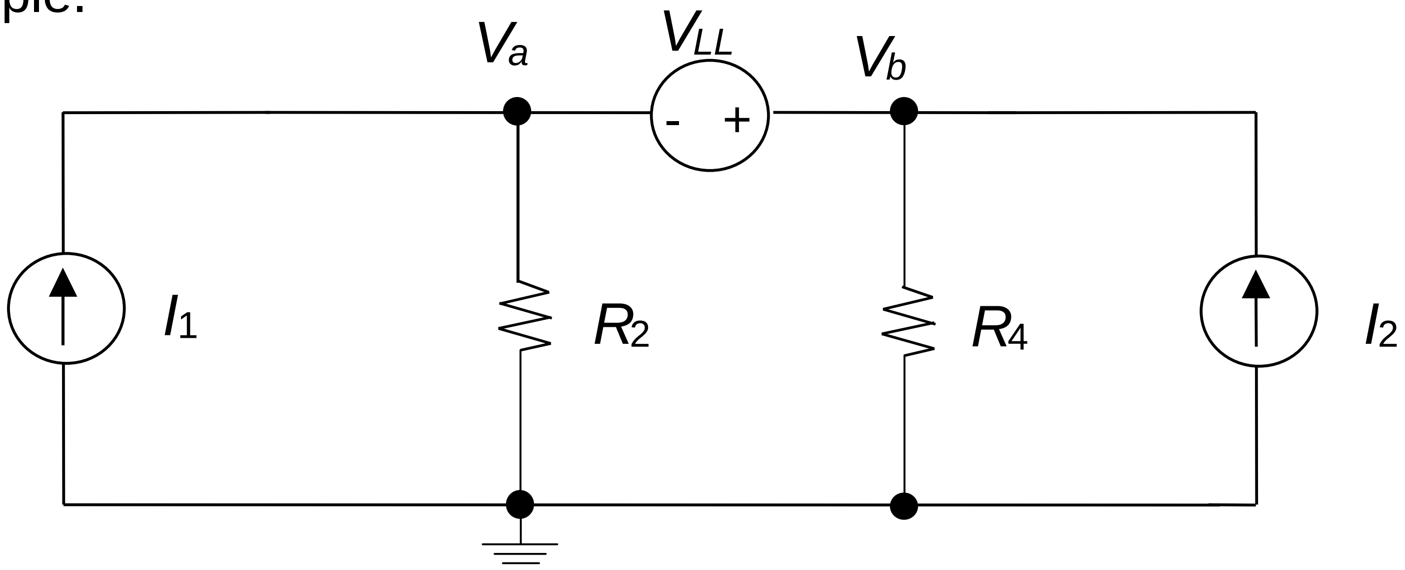
- Formal nodal analysis
- Voltage divider example

Today:

- Nodal analysis with floating voltage sources
- Real voltmeters
- Real ammeters
- Parallel resistors
- Current dividers
- Series and parallel capacitors

NODAL ANALYSIS WITH "FLOATING" VOLTAGE SOURCES

A "floating" voltage source is a voltage source for which neither side is connected to the reference node. V_{LL} in the circuit below is an example.

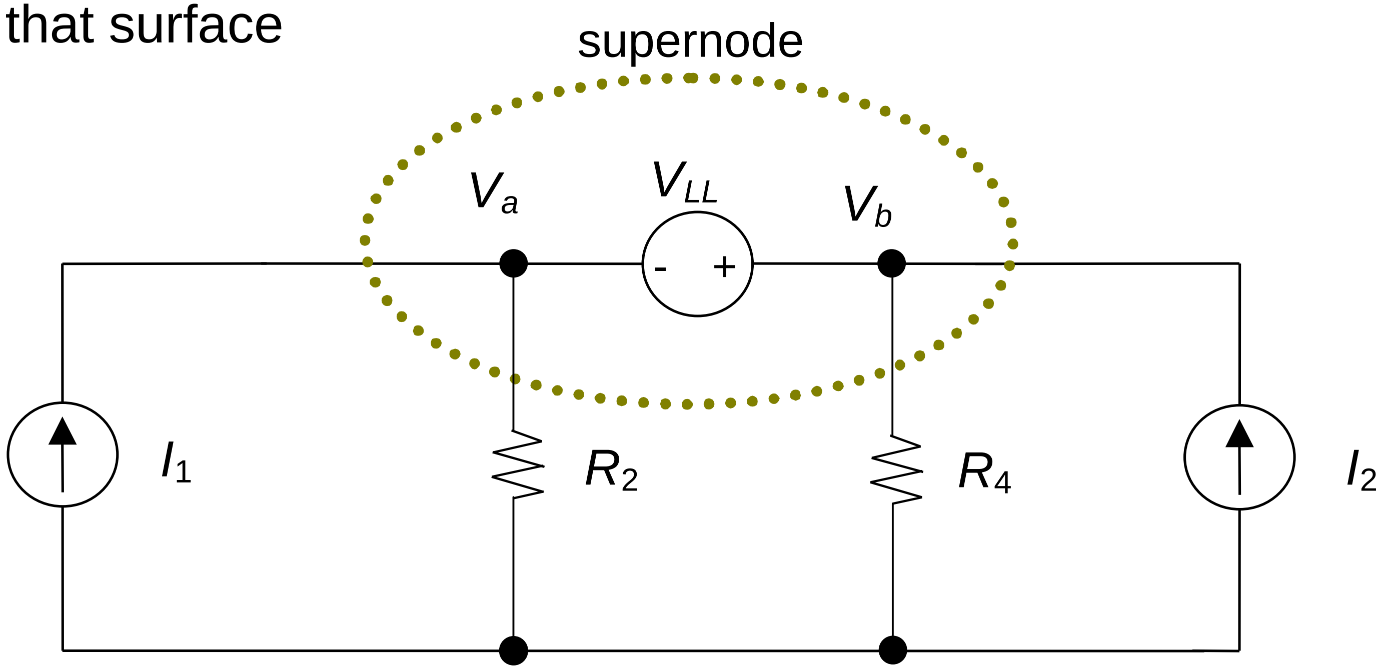


What is the problem? $\rightarrow$ We cannot write KCL at node a or b because there is no way to express the current through the voltage source in terms of $V_a - V_b$.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a <u>and</u> b. Express KCL at this supernode.

FLOATING VOLTAGE SOURCES (cont.)

Use a Gaussian surface to enclose the floating voltage source; write KCL for that surface



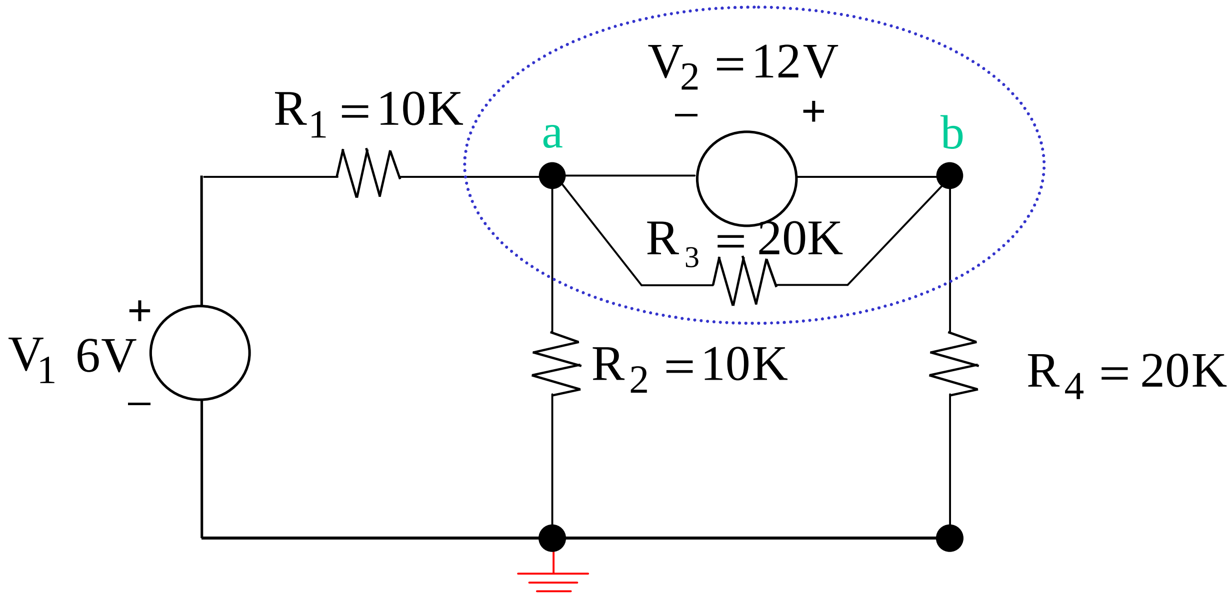
We have two unknowns: V_a and V_b.

We obtain one equation from KCL at supernode: $I_1 - \frac{V_a}{R_2} - \frac{V_b}{R_4} + I_2 = 0$

We obtain a second "auxiliary" equation from the property of the voltage source: $V_{LL} = V_b - V_a$ (often called the "constraint")

$\Rightarrow$ 2 Equations & 2 Unknowns

ANOTHER EXAMPLE



1 Choose reference node (can it be chosen to avoid floating voltage source?)

2 Label unknowns V_a and V_b

3 Equation at supernode: $$\frac{V_1 - V_a}{R_1} = \frac{V_b}{R_4} + \frac{V_a}{R_2} \rightarrow V_a\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_b}{R_4} = \frac{V_1}{R_1}$$

4 Auxiliary equation: $$V_b - V_a = V_2 \longrightarrow V_a - V_b = -V_2$$

Solve: $$V_a\left(\frac{R_4}{R_1} + \frac{R_4}{R_2} + 1\right) = V_1\frac{R_4}{R_1} - V_2$$

$$V_b = V_a + V_2$$

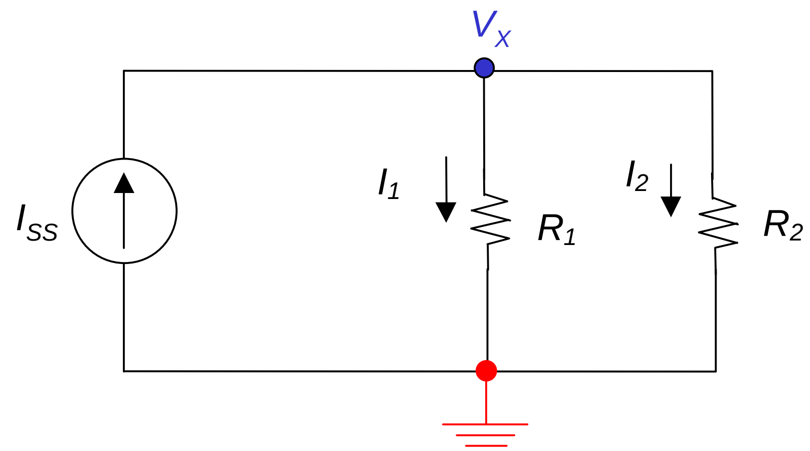
SOLUTION: $V_a = 0$

$V_b = 12$

RESISTORS IN PARALLEL

1 Select Reference Node

2 Define unknown node voltages



Note: $I_{ss} = I_1 + I_2$, i.e.,

$$I_{SS} = \frac{V_X}{R_1} + \frac{V_X}{R_2} \quad \Rightarrow V_X = I_{SS} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \quad = I_{SS} \cdot \frac{R_1 R_2}{R_1 + R_2}$$

RESULT 1 EQUIVALENT RESISTANCE: $R_{\parallel} \equiv R_1 \parallel R_2 = \dfrac{R_1 R_2}{R_1 + R_2}$

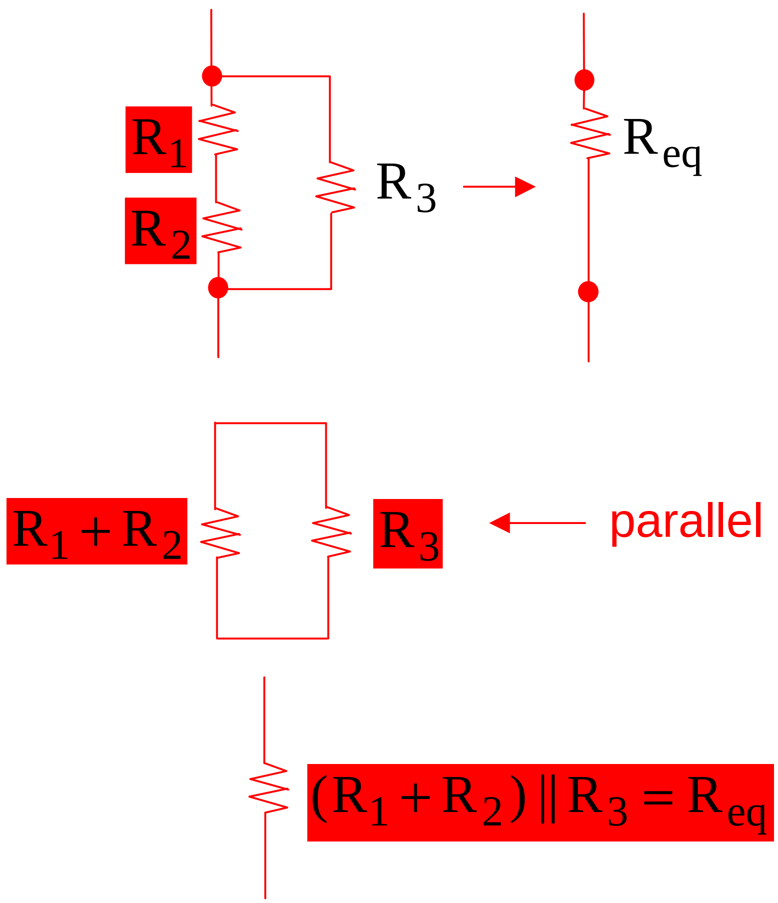
RESULT 2 CURRENT DIVIDER:

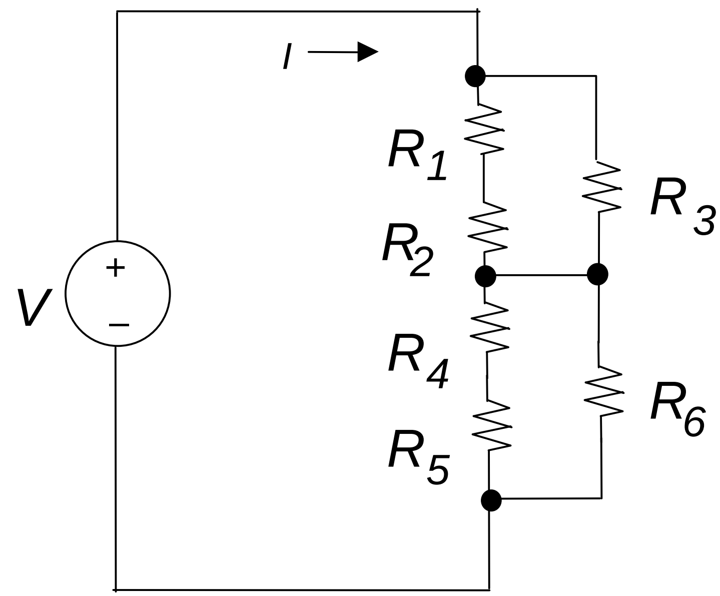
$$I_1 = \frac{V_X}{R_1} = I_{SS} \times \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V_X}{R_2} \quad = I_{SS} \times \frac{R_1}{R_1 + R_2}$$

IDENTIFYING SERIES AND PARALLEL COMBINATIONS

Use series/parallel equivalents to simplify a circuit before starting KVL/KCL



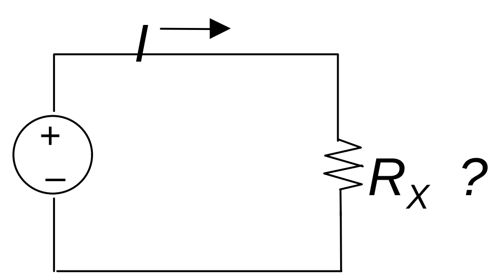


$R_1 = R_2 = 10\ \text{K}$

$R_3 = 20\ \text{K}$

$R_4 = 5 = 5\ \text{K}$

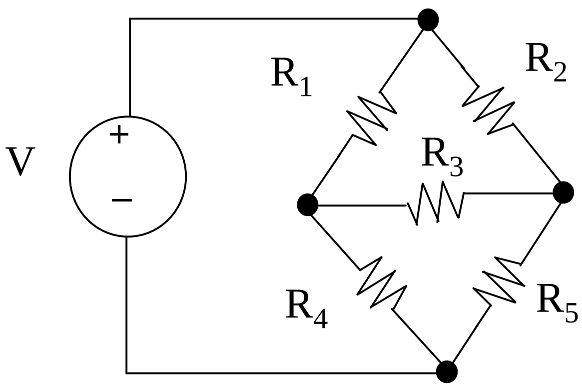
$R_6 = 10\ \text{K}$



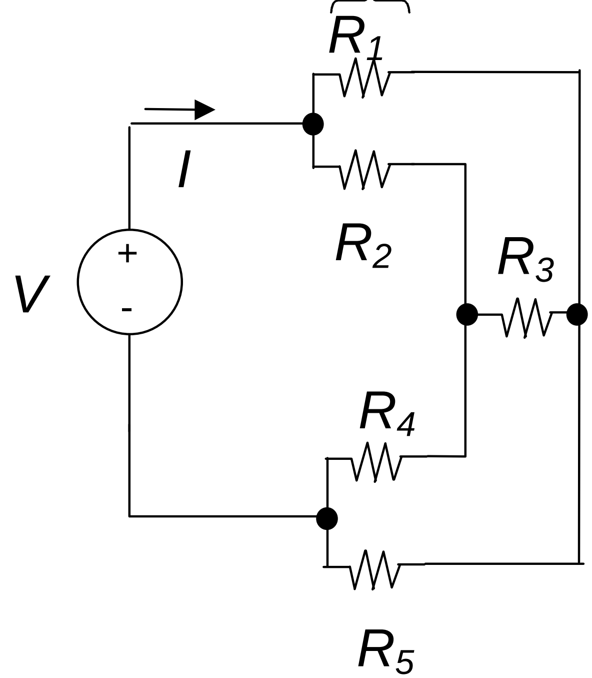
$$R_X = (R_1 + R_2) \| R_3 + (R_4 + R_5) \| R_6$$
$$= 15\ \text{K}$$

IDENTIFYING SERIES AND PARALLEL COMBINATIONS (cont.)

Some circuits *must* be analyzed (not amenable to simple inspection)



R_1 and R_2 are **not** in ||



R_1 and R_5 are **not** in series

Special cases:

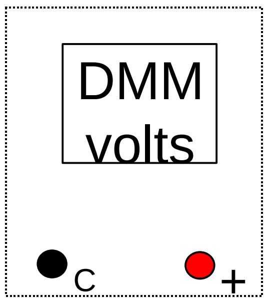
$R_3 = 0$ **OR** $R_3 = \infty$

Example: $R_3 = 0 \Rightarrow R_1 \,||\, R_2$; $R_4 \,||\, R_5$ in series;

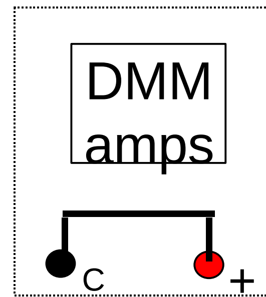
$R_{eq} = R_1 \,||\, R_2 + R_4 \,||\, R_5$

OR IF $R_3 = \infty \Rightarrow (R_1 + R_5) \,||\, (R_2 + R_4)$

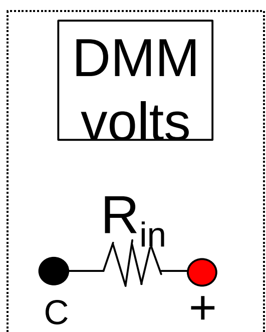
IDEAL AND NON-IDEAL METERS



IDEAL



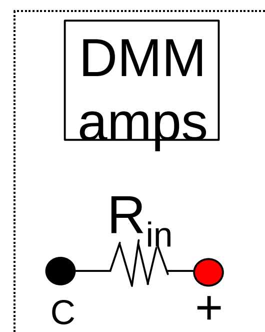
IDEAL



MODEL OF REAL DIGITAL VOLTMETER

Note: R_{in} may depend on range

R_{in} typically $\gtrsim$ 10 MΩ



MODEL OF REAL DIGITAL AMMETER

Note: R_{in} usually depends on current range

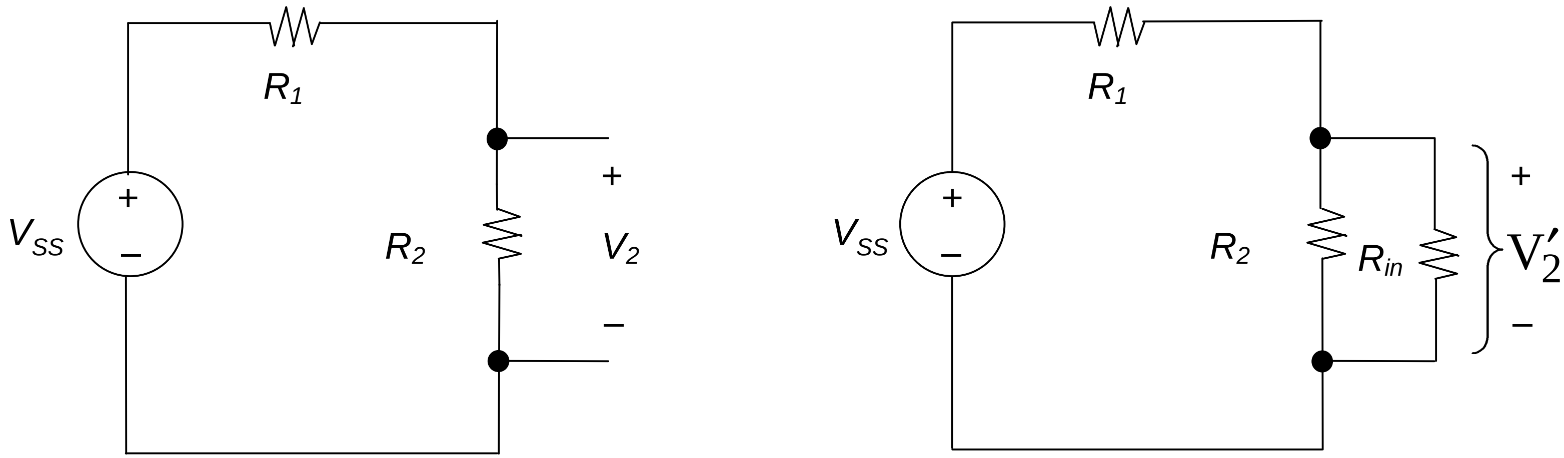
R_{in} typically $\lesssim$ 1 Ω

REAL VOLTMETERS

Concept of “Loading” as Application of Parallel Resistors

How is voltage measured? Modern answer: Digital multimeter (DMM)

Problem: Connecting leads from voltmeter across two nodes changes the circuit. The voltmeter is characterized by how much current it draws at a given voltage → “voltmeter input resistance,” R_{in}. Typical value: 10 MΩ. Lets do an example; measure V in voltage divider:



$$V_2 = V_{SS}\left[\frac{R_2}{R_1 + R_2}\right] \qquad V_2' = V_{SS}\left[\frac{R_2 \parallel R_{in}}{R_2 \parallel R_{in} + R_1}\right]$$

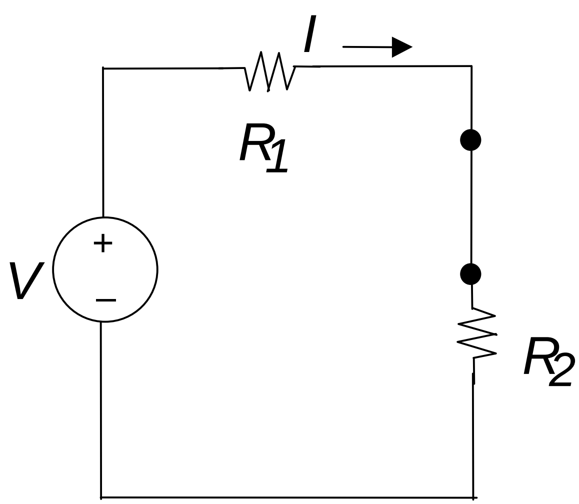
Example: $V_{SS} = 10V, R_2 = 100K, R_1 = 900K \Rightarrow V_2 = 1V$

But if $R_{in} = 10M, V_2' = 0.991V$, a 1% error

MEASURING CURRENT

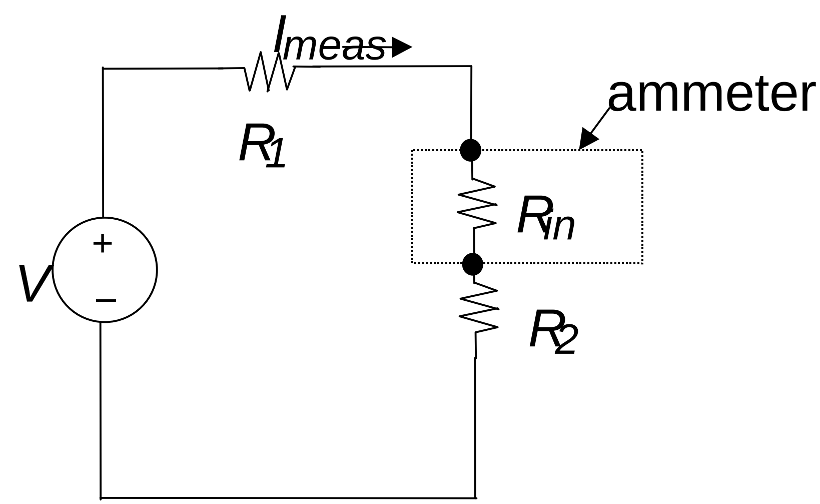
Insert DMM (in current measurement mode) into circuit. But ammeters disturb the circuit. (Note: Ammeters are characterized by their “ammeter input resistance,” R_{in}. Ideally this should be very low. Typical value (in mA range) 1Ω.)

Potential measurement error due to non-zero input resistance:



undisturbed circuit

$$I = \frac{V_1}{R_1 + R_2}$$



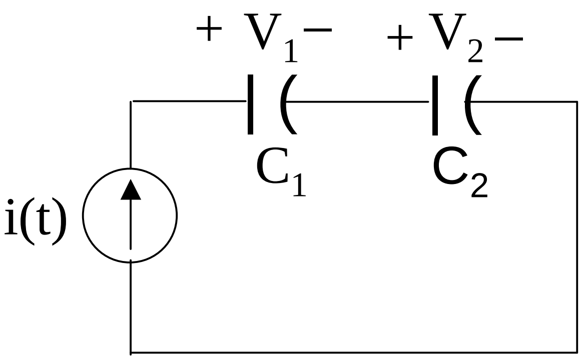
with ammeter

$$I_{meas} = \frac{V_1}{R_1 + R_2 + R_{in}}$$

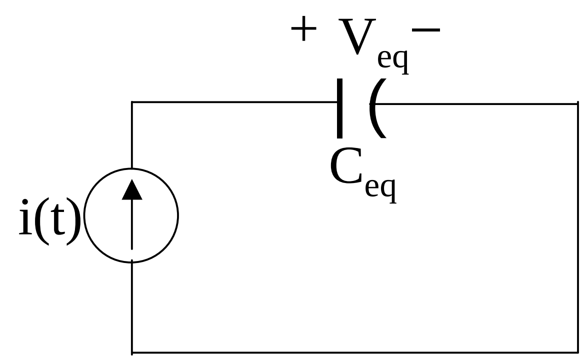
Example V = 1 V: R1 + R2 = 1 KΩ , Rin = 1Ω

$$I = 1\text{mA} \quad , \quad I_{meas} = \frac{1}{1\text{K} + 1\Omega} \cong 0.999\ \text{mA} \qquad (0.1\%\ \text{error})$$

CAPACITORS IN SERIES



Equivalent to →



Equivalent capacitance defined by

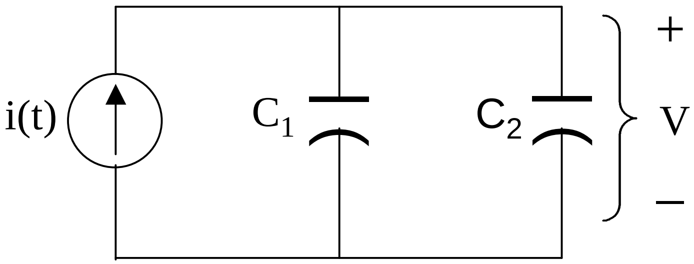
$$V_{eq} = V_1 + V_2 \ \text{ and } \ i = C_{eq}\frac{dV_{eq}}{dt} = C_{eq}\frac{d(V_1+V_2)}{dt}$$

$$i = C_1\frac{dV_1}{dt} = C_2\frac{dV_2}{dt}$$

$$\text{So } \frac{dV_1}{dt} = \frac{i}{C_1}, \quad \frac{dV_2}{dt} = \frac{i}{C_2}, \qquad \text{so } \frac{dV_{eq}}{dt} = i\left(\frac{1}{C_1}+\frac{1}{C_2}\right) \equiv \frac{i}{C_{eq}}$$

Clearly, $\boxed{C_{eq} = \dfrac{1}{\dfrac{1}{C_1}+\dfrac{1}{C_2}} = \dfrac{C_1C_2}{C_1+C_2} \qquad \text{CAPACITORS IN SERIES}}$

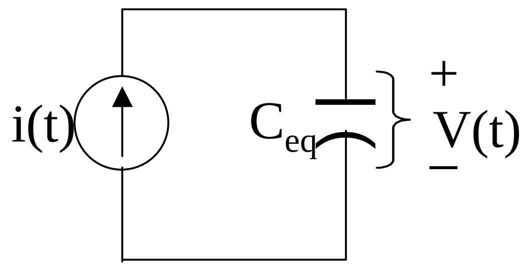
CAPACITORS IN PARALLEL



$$i(t) = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt}$$

Equivalent capacitance defined by

$$i = C_{eq} \frac{dV}{dt}$$



Clearly, $\boxed{C_{eq} = C_1 + C_2 \qquad \text{CAPACITORS IN PARALLEL}}$