CIRCUITS, NODES, AND BRANCHES

Background:

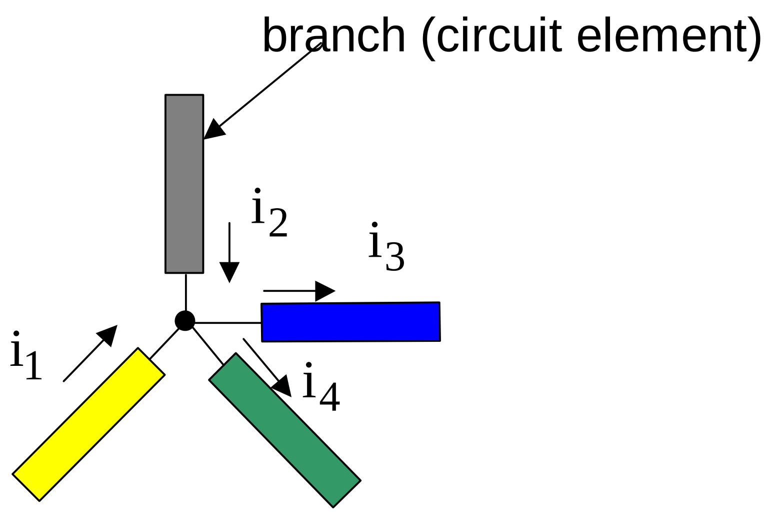
- We know the properties of a few ideal circuit elements (voltage source, current source, resistor)
- Ideal wires hook these elements up with no loss of voltage or loss of power (wires are kind of like resistors in the limit R approaches 0)
- We saw intuitively that conservation of charge leads to a conservation of current when wires intersect, and conservation of energy means we can add voltages as we move around in a circuit.
- Now we will formalize the latter observations

Today:

- We begin with elementary circuit analysis - methods for rigorous approach to determining circuit response
- Circuit theorems for analysis of branch currents and voltages
  - Kirchhoff's current law and voltage law

BRANCHES AND NODES

Circuit with several branches connected at a node:



KIRCHOFF's CURRENT LAW "KCL":
(see Text 1.2 and 1.3)

(Sum of currents <u>entering</u> node) – (Sum of currents <u>leaving</u> node) = 0

q = charge stored at node is zero. If charge *is* stored, for example in a capacitor, then the capacitor is a branch and the charge is stored there NOT at the node.

WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is 1μA = 1 μC/s (net current entering node)

Assuming that q = 0 at t = 0, the charge increase is 10–6 C each second

or $10^{-6}/1.6\times10^{-19} = 6\times10^{12}$ charge carriers each second

But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite (Q = CV).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields … there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn't pile up at the nodes!

SIGN CONVENTIONS FOR SUMMING CURRENTS

Kirchhoff's Current Law (KCL)

Sum of currents *entering* node = sum of currents *leaving* node

Use reference directions to determine "entering" and "leaving" currents … **no concern** about actual polarities

☞ KCL yields one equation per node

Alternative statements of KCL

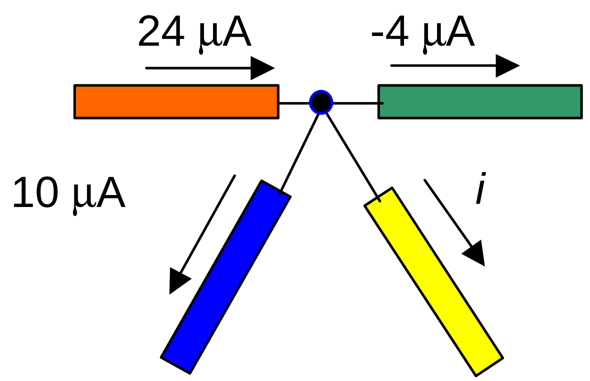
1 "Algebraic sum" of currents *entering* node = 0

where "algebraic sum" means currents leaving are included with a minus sign

2 "Algebraic sum" of currents leaving node = 0

where currents entering are included with a minus sign

KIRCHHOFF'S CURRENT LAW EXAMPLE



Currents entering the node: 24 µA

Currents leaving the node: –4 µA + 10 µA + i

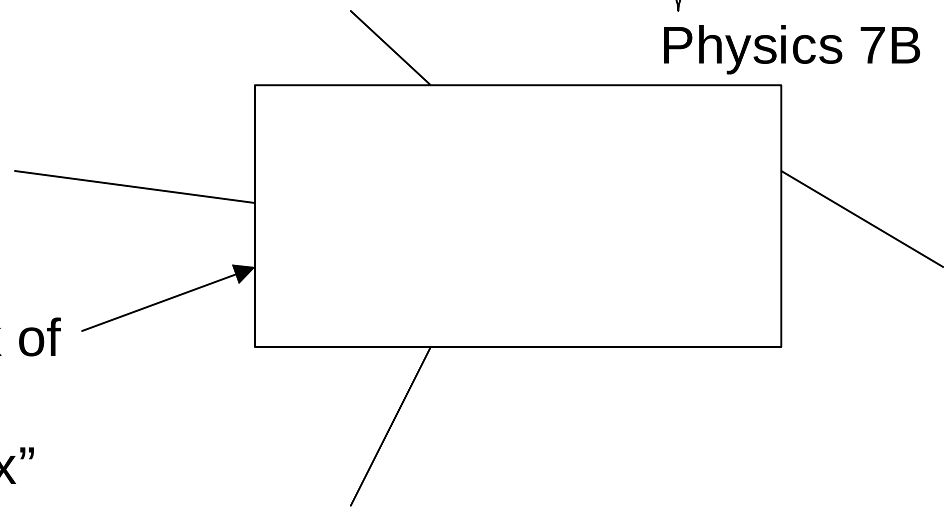
$$\boxed{\begin{gathered}24 = 10 + (-4) + i \\ i = 18\ \mu A\end{gathered}}$$

Three statements of KCL

$$\left.\begin{array}{lll}
\sum\limits_{IN} i_{in} = \sum\limits_{OUT} i_{out} & 24 = -4 + 10 + i & \Rightarrow \quad i = 18\, mA \\
\sum\limits_{ALL} i_{in} = 0 & 24 - (-4) - 10 - i = 0 & \Rightarrow \quad i = 18\, mA \\
\sum\limits_{ALL} i_{out} = 0 & -24 - 4 + 10 + i = 0 & \Rightarrow \quad i = 18\, mA
\end{array}\right\} \text{EQUIVALENT}$$

GENERALIZATION OF KCL

Sum of currents entering and leaving $\underbrace{\text{a closed surface}}_{\text{Physics 7B}}$ is zero



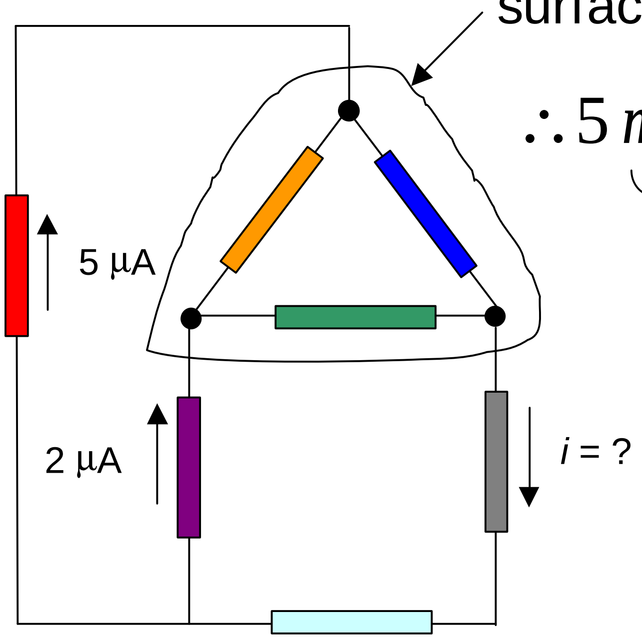
Could be a big chunk of circuit in here, e.g., could be a “Black Box”

Note that *<u>circuit branches</u>* could be inside the surface.

The surface can enclose more than one node

KIRCHHOFF'S CURRENT LAW USING SURFACES

Example

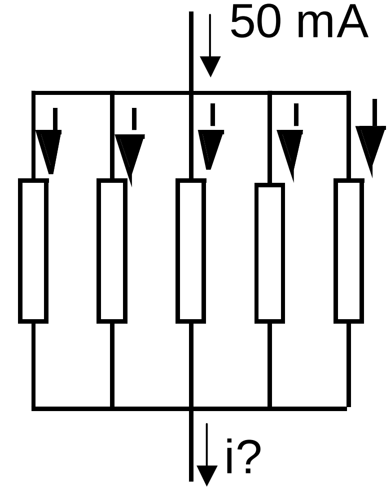


$$\therefore 5\,m\mathrm{A} + 2\,m\mathrm{A} = \mathrm{i}$$

entering leaving

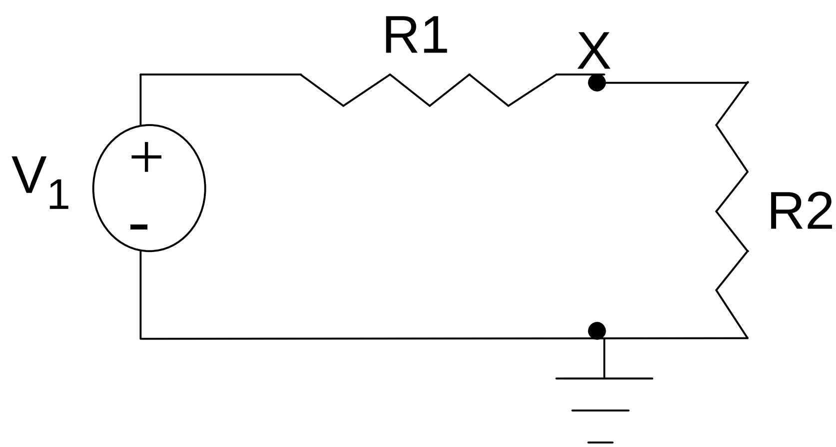
i=7µA

Another example



i must be 50 mA

Example of the use of KCL



At node X:

Current into X from the left:

$(V_1 - v_X)/R1$

Current out of X to the right:

$v_X/R2$

KCL: $(V_1 - v_X)/R1 = v_X/R2$

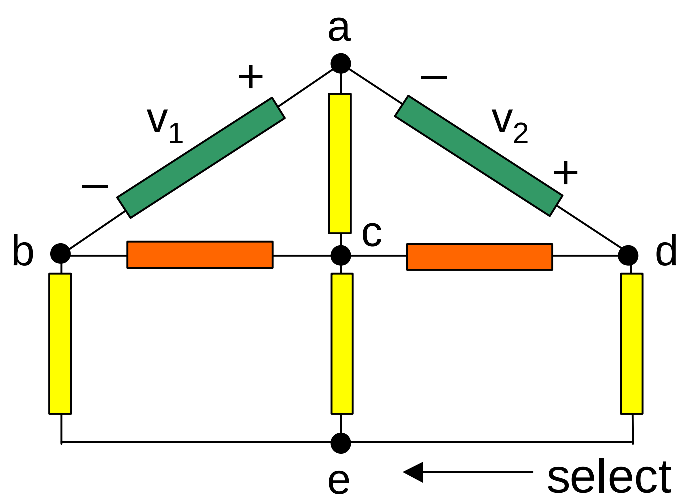
Given V_1, This equation can be solved for v_X

$v_X = V_1\ R2\ /(R1 + R2)$

Of course we just get the same result as we obtained from our series resistor formulation. (Find the current and multiply by R2)

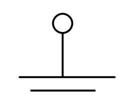
BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals



$$v_2 = v_d - v_a$$

$v_e \equiv 0$ (since it's the reference)

select as ref. ⇒  "ground"

Specifying node voltages: Use one node as the implicit reference (the "common" node … attach special symbol to label it)

Now single subscripts can label voltages:

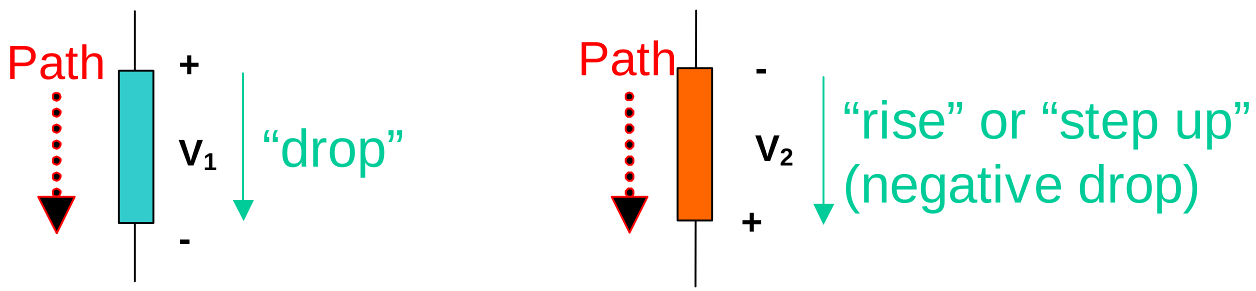
$\boxed{\text{e.g., } v_b \text{ means } v_b - v_e,\ v_a \text{ means } v_a - v_e, \text{ etc.}}$

KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of the "voltage drops" around any "closed loop" is zero.

Why? We must return to the same potential (conservation of energy).

Voltage drop $\rightarrow$ defined as the branch voltage if the + sign is encountered first; it is (-) the branch voltage if the – sign is encountered first … important bookkeeping

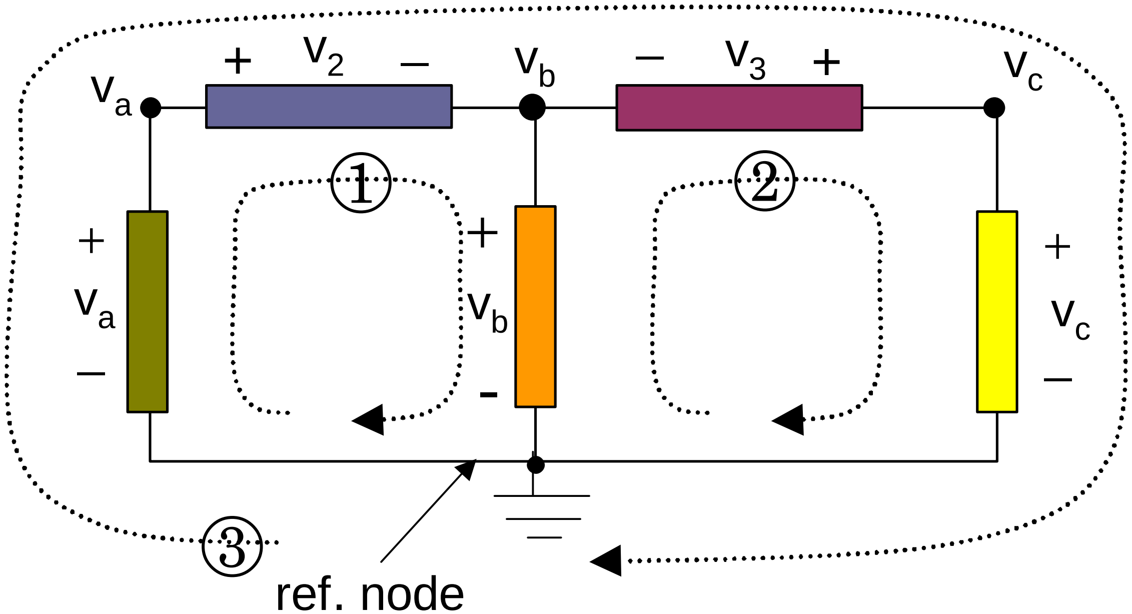


Closed loop: Path beginning and ending on the same node

KVL EXAMPLE

Examples of
Three closed paths:

①, ②, ③



Note that:

$$v_2 = v_a - v_b$$

$$v_3 = v_c - v_b$$

Path 1:

$$-v_a + v_2 + v_b = 0$$

↑

$v_a - v_b$

YEP!

Path 2:

$$-v_b - v_3 + v_c = 0$$

Path 3:

$$-v_a + v_2 - v_3 + v_c = 0$$

ALTERNATIVE STATEMENTS OF KIRCHHOFF'S VOLTAGE LAW

1 For any node sequence A, B, C, D, …, M around a closed path, the voltage drop from A to M is given by

$$v_{AM} = v_{AB} + v_{BC} + v_{CD} + \cdots + v_{LM}$$

2 For all pairs of nodes i and j, the voltage drop from i to j is

$$v_{ij} = v_i - v_j$$

where the node voltages are measured with respect to the common node.

FORMAL CIRCUIT ANALYSIS

Systematic approaches to writing down KCL and KVL: Text 2.3

Nodal Analysis: Node voltages are the unknowns
Text section 2.3

Mesh Analysis: Branch currents are the unknowns

Use one **or** the other for circuit analysis

☞ We will do only nodal analysis – (because voltages make more convenient variables than currents) So ignore Text 2.4