

## EECS 42 Introduction to Electronics for Computer Science Andrew R. Neureuther

### Lecture #3

- Kirchhoff's Laws
- Ideal independent sources
- Resistors

<http://inst.EECS.Berkeley.EDU/~ee42/>

## Game Plan 01/22/03

Monday 01/27/03

- Electrical Quantities  
Schwarz and Oldham: 1.3-1.4

Today 01/29/03

- Kirchhoff Laws  
Schwarz and Oldham: 2.1-2.2

Next (3<sup>rd</sup>) Week

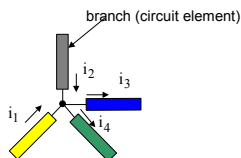
- Capacitors, inductors, I vs. V  
Schwarz and Oldham: 5.1, 2.2, 3.1
- Power and Energy  
Schwarz and Oldham: 5.1, 2.2, 3.1

Problem Set #2 – Out 1/27/03 - Due 2/5/03 2:30 in box in 240 Cory

2.1 Flow; 2.2 KCL; 2.3 KVL; 2.4 resistor circuit; 2.5 Power

### BRANCHES AND NODES

Circuit with several branches connected at a node:

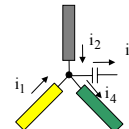


KIRCHHOFF's CURRENT LAW "KCL":  
(see Text 1.2 and 1.3)

(Sum of currents entering node) – (Sum of currents leaving node) = 0  
q = charge stored at node is zero. If charge *is* stored, for example in a capacitor, then the capacitor is a branch and the charge is stored there NOT at the node.

### Capacitor at a Node

Circuit with several branches, including a capacitor



(Sum of currents entering node) – (Sum of currents leaving node) = 0  
q = charge stored at node is zero. If charge *is* stored, for example in the capacitor shown as branch 3, the charge is accounted for as the time-integral of  $i_3$ . Thus the charge is not over at the node; it is on the capacitor.

### WHAT IF THE NET CURRENT WERE NOT ZERO?

Suppose imbalance in currents is  $1\mu\text{A} = 1\mu\text{C/s}$  (net current entering node)  
Assuming that  $q = 0$  at  $t = 0$ , the charge increase is  $10^{-6}$  C each second  
or  $10^{-6}/1.6 \times 10^{-19} = 6 \times 10^{12}$  charge carriers each second

But by definition, the capacitance of a node to ground is ZERO because we show any capacitance as an explicit circuit element (branch). Thus, the voltage would be infinite ( $Q = CV$ ).

Something has to give! In the limit of zero capacitance the accumulation of charge would result in infinite electric fields ... there would be a spark as the air around the node broke down.

Charge is transported around the circuit branches (even stored in some branches), but it doesn't pile up at the nodes!

### SIGN CONVENTIONS FOR SUMMING CURRENTS

Kirchhoff's Current Law (KCL)

Sum of currents entering node = sum of currents leaving node  
Use reference directions to determine "entering" and "leaving" currents ... no concern about actual polarities

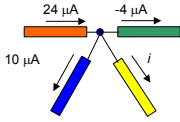
☞ KCL yields one equation per node

Alternative statements of KCL

1 "Algebraic sum" of currents entering node = 0  
where "algebraic sum" means currents leaving are included with a minus sign

2 "Algebraic sum" of currents leaving node = 0  
where currents entering are included with a minus sign

### KIRCHHOFF'S CURRENT LAW EXAMPLE



Currents entering the node:  $24 \mu\text{A}$   
 Currents leaving the node:  $-4 \mu\text{A} + 10 \mu\text{A} + i$

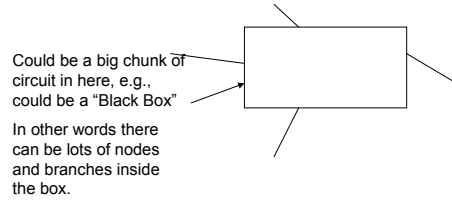
$$\left. \begin{aligned} 24 &= 10 + (-4) + i \\ i &= 18 \mu\text{A} \end{aligned} \right\}$$

Three statements of KCL

$$\left. \begin{aligned} \sum_{\text{IN}} i_{\text{in}} &= \sum_{\text{OUT}} i_{\text{out}} & 24 &= -4 + 10 + i & \Rightarrow & i = 18 \mu\text{A} \\ \sum_{\text{ALL}} i_{\text{in}} &= 0 & 24 &- (-4) - 10 - i &= 0 & \Rightarrow & i = 18 \mu\text{A} \\ \sum_{\text{ALL}} i_{\text{out}} &= 0 & -24 &- 4 + 10 + i &= 0 & \Rightarrow & i = 18 \mu\text{A} \end{aligned} \right\} \text{EQUIVALENT}$$

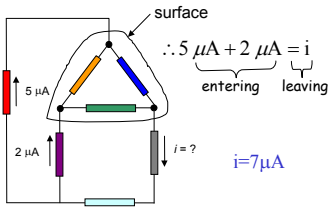
### GENERALIZATION OF KCL TO SURFACES

Sum of currents entering and leaving any "black box" is zero

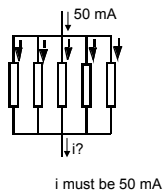


### KIRCHHOFF'S CURRENT LAW USING SURFACES

Example



Another example

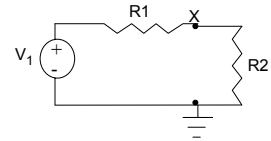


### Example of the use of KCL

At node X:

Current into X from the left:  
 $(V_1 - v_x)/R_1$

Current out of X to the right:  
 $v_x/R_2$



**KCL:**  $(V_1 - v_x)/R_1 = v_x/R_2$

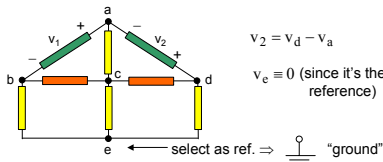
Given  $V_1$ , This equation can be solved for  $v_x$

$v_x = V_1 R_2 / (R_1 + R_2)$

Of course we just get the same result as we obtained from our series resistor formulation. (Find the current and multiply by  $R_2$ )

### BRANCH AND NODE VOLTAGES

The voltage across a circuit element is defined as the difference between the node voltages at its terminals



Specifying node voltages: Use one node as the implicit reference (the "common" node ... attach special symbol to label it)

Now single subscripts can label voltages:

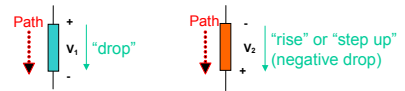
e.g.,  $v_b$  means  $v_b - v_e$ ,  $v_a$  means  $v_a - v_e$ , etc.

### KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of the "voltage drops" around any "closed loop" is zero.

Why? We must return to the same potential (conservation of energy).

Voltage drop  $\rightarrow$  defined as the branch voltage if the + sign is encountered first; it is (-) the branch voltage if the - sign is encountered first ... important bookkeeping



Closed loop: Path beginning and ending on the same node

**KVL EXAMPLE**

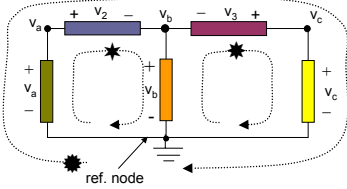
Examples of Three closed paths:



Note that:

$$V_2 = V_a - V_b$$

$$V_3 = V_c - V_b$$



Path 1:

$$-V_a + V_2 + V_b = 0$$

↑

$$V_a - V_b$$

YEP!

Path 2:

$$-V_b - V_3 + V_c = 0$$

Path 3:

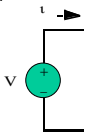
$$-V_a + V_2 - V_3 + V_c = 0$$

**BASIC CIRCUIT ELEMENTS**

- Voltage Source (always supplies some constant given voltage - like ideal battery)
- Current Source (always supplies some constant given current)
- Resistor (Ohm's law)
- Wire ("short" – no voltage drop)
- Capacitor (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1)
- Inductor (inductor law – based on energy storage in magnetic field in space S&O 5.1)

**DEFINITION OF IDEAL VOLTAGE SOURCE**

Symbol



Note: The current and voltage are unassociated here.

Examples:

$$1) V = 3V$$

$$2) v = v(t) = 160 \cos 377t$$

Special cases:

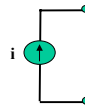
upper case V → constant voltage ... called "DC"

lower case v → general voltage, may vary with time

Current through voltage source can take on any value (positive or negative) *but not infinite*

**IDEAL CURRENT SOURCE**

"Complement" or "dual" of the voltage source: Current through branch is fixed and independent of the voltage across the branch



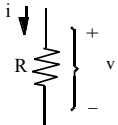
note unassociated direction

Actual current source examples – hard to find except in electronics (transistors, etc.), as we will see

upper-case I → DC (constant) value

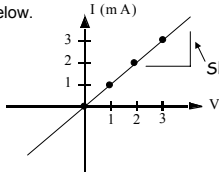
lower-case implies current could be time-varying i(t)

**RESISTOR**



We use associated current and voltage (i.e., i is defined as into + terminal), then  $v = iR$  (Ohm's law).

Question: What is the I-V characteristic for a 1kΩ resistor? Draw on axis below.



Answer:  $V = 0 \Rightarrow I = 0$

$$V = 1V \Rightarrow I = 1 \text{ mA}$$

$$V = 2V \Rightarrow I = 2 \text{ mA}$$

etc

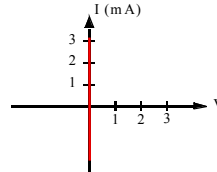
Slope =  $1/R$

**IDEAL WIRE**



Think of a resistor with zero resistance. Clearly V is identically zero, for any current.

Question: What is the I-V characteristic?



Answer:  $V = 0$  for all I  
see red line

Note that all real wires and circuit connections have resistance, but we will most often approximate it to be zero, that is assume an ideal wire.

CURRENT-VOLTAGE CHARACTERISTICS OF VOLTAGE & CURRENT SOURCES

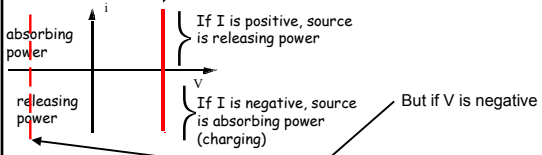
Describe a two-terminal circuit element by plotting current vs. voltage

**Ideal voltage source**

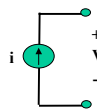


Assume unassociated signs

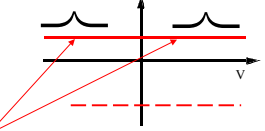
If  $V$  is positive



CURRENT-VOLTAGE CHARACTERISTICS OF VOLTAGE & CURRENT SOURCES (con't)



absorbing power releasing power



If  $i$  is positive then we are confined to quadrants 4 and 1:

Remember the voltage across the current source can be *any* finite value (not just zero)

And do not forget  $i$  can be positive or negative. Thus we can be in any quadrant.