EECS 42 Introduction to Electronics for Computer Science

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Lecture #4

- **Capacitors and Inductors**
- **Energy Stored in C and L**
- **Equivalent Circuits**
  - **Thevenin**
  - **Norton**

http://inst.EECS.Berkeley.EDU/~ee42/

BASIC CIRCUIT ELEMENTS

- Voltage Source (always supplies some constant given voltage - like ideal battery)
- Current Source (always supplies some constant given current)
- Resistor (Ohm's law)
- Wire ("short" – no voltage drop)
- Capacitor (capacitor law – based on energy storage in electric field of a dielectric S&O 5.1)
- Inductor (inductor law – based on energy storage in magnetic field in space S&O 5.1)

Game Plan 02/03/03

Monday 02/03/03

- **Capacitors and Inductors; Equivalent Sources**
  Schwarz and Oldham: 5.1-5.2, 3.1

Wednesday 02/05/03

- **N-L Elements; Graphical Solutions; Power**
  Schwarz and Oldham: 3.2-3.4

Next (4th) Week

- **RC Transient**
  Schwarz and Oldham: 8.1 plus Handouts

Problem Set #2 – Out 1/27/03 - Due 2/5/03 2:30 in box near 275 Cory

2.1 Flow; 2.2 KCL; 2.3 KVL; 2.4 resistor circuit; 2.5 Power

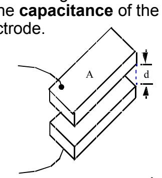
Problem Set #3 – Out 2/2/03 - Due 2/12/03 2:30 in box near 275 Cory

3.1 and 3.2 charging capacitors; 3.3 –3.5; Equivalent Circuits;

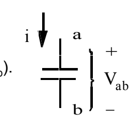
CAPACITOR

Any two conductors a and b separated by an insulator with a difference in voltage V_{ab} will have an equal and opposite charge on their surfaces whose value is given by $Q = CV_{ab}$, where C is the **capacitance** of the structure, and the + charge is on the more positive electrode.

A simple *parallel-plate capacitor* is shown. If the area of the plate is A, the separation d, and the *dielectric constant* of the insulator is ε, the capacitance equals $C = A\,\varepsilon/d$.



Symbol or



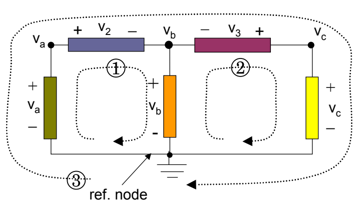
Constitutive relationship: $Q = C (V_a – V_b)$.

(Q is positive on plate a if $V_a > V_b$)

But $i = \frac{dQ_a}{dt}$ so $i = C\frac{dv}{dt}$ equivalent to $Q = C v$

where we use the *associated reference directions*.

What Goes In the Circuit Element Boxes?



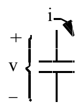
ENERGY STORED IN A CAPACITOR

You might think the energy (in Joules) is QV, which has the dimension of joules. But during charging the average voltage was only half the final value of V.

Thus, energy is $\frac{1}{2}QV = \frac{1}{2}CV^2$.

ENERGY STORED IN A CAPACITOR (cont.)

More rigorous derivation: During charging, the power flow is v · i into the capacitor, where i is into + terminal. We integrate the power from t = 0 (v = 0) to t = end (v = V). The integrated power is the energy



$$E = \int_{t = t_{Initial}}^{t = t_{Final}} v \cdot i\, dt = \int_{v = V_{Initial}}^{v = V_{Final}} v \frac{dq}{dt} dt = \int_{v = V_{Initial}}^{v = V_{Final}} v\, dq$$

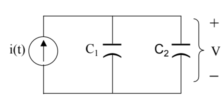
but dq = C dv. (We are using small q instead of Q to remind us that it is time varying . Most texts use Q.)

$$E = \int_{v = V_{Initial}}^{v = V_{Final}} Cv\, dv = \frac{1}{2} CV_{Final}{}^2 - \frac{1}{2} CV_{Initial}{}^2$$

Capacitance and Inductance

- Capacitors: two plate example; Store energy in the electric field Q = CV, I = C dV/dt and V = (1/C) integral of voltage
- Computer example 1 mA current charging 1 pF
  V(t) = (I/C)t = (10-3 A/10^{-12} F) t = 10^9 V/s t
- At D.C. time derivatives are zero => C is open circuit
- C in parallel add; series 1/C = sum (1/C_i); short together (infinite current but conserve charge)
- Inductors: coil example; Store energy in the magnetic field; Flux = LI, V = L dI/dt and I = (1/L) (integral of voltage)
- At D.C. time derivatives are zero => L is short circuit
- L in parallel 1/L = sum (1/L_i); series add; connect in series when have different currents => $L_1I_1+L_2I_2 = (L_1+L_2)I_{NEW}$

CAPACITORS IN PARALLEL

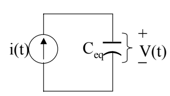


Add Currents

$i(t) = C_1 \, dV + C_2 \, dV$

Equivalent capacitance defined by

$i = C_{eq}\, dV$



Clearly, C C1 + C2 CAPACITORS IN PARALLEL

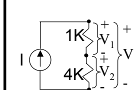
Example of I-V Graphs

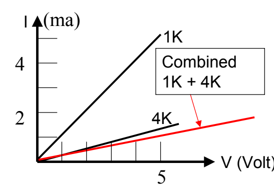
Resistors in Series

If two resistors are in series the current is the same; clearly the total voltage will be the sum of the two IR values i.e. $I(R_1+R_2)$.

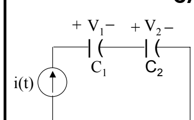
Thus the equivalent resistance is R_1+R_2 and the I-V graph of the series pair is the same as that of the equivalent resistance.

Of course we can also find the I-V graph of the combination by adding the voltages directly on the I-V axes. Lets do an example for 1K + 4K resistors

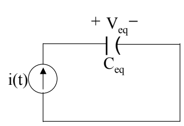




CAPACITORS IN SERIES



Equivalent to



Add Voltages

Equivalent capacitance defined by

$V_{eq} = V_1 + V_2$ and $i = C_{eq} \frac{dV_{eq}}{} = C_{eq} \frac{d(V_1 + V_2)}{}$

$i = C_1 \frac{dV_1}{} = C_2 \frac{dV_2}{}$

So $\frac{dV_1}{} = \frac{i}{C_1}$, $\frac{dV_2}{} = \frac{i}{C_2}$, so $\frac{dV_{eq}}{dt} = i(\frac{1}{C_1} + \frac{1}{C_2}) \equiv \frac{i}{C}$

Clearly, $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$ CAPACITORS IN SERIES

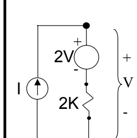
Example of I-V Graphs

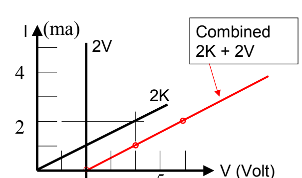
Simple Circuit, e.g. voltage source + resistor.

If two circuit elements are in series the current is the same; clearly the total voltage will be the sum of the voltages i.e. V_S + IR.

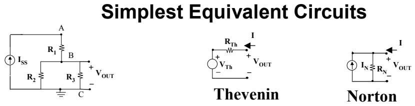
We can graph this on the I-V plane. We find the I-V graph of the combination by adding the voltages V_S and I R at each current I.

Lets do an example for =2V, R=2K

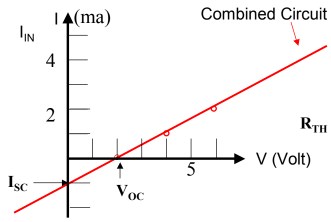




Simplest Equivalent Circuits



An adequately equivalent circuit is one that has an I vs. V graph that is identical to that of the original circuit.



$V_{TH} = V_{OC}$

$I_N = -I_{SC}$

$R_{TH} = R_N = (-I_{SC}/V_{OC})$

I vs. V and Equivalent Circuits

- I vs. V for ideal voltage source is a vertical line at $V = V_{SV}$
- I vs. V for ideal current source is a horizontal line at $I = I_{SC}$
- I vs. V for a circuit made up of ideal independent sources and resistors is a straight line.
- The simplest circuit for a straight line is an ideal voltage source and a resistor (Thevenin) or a current source and a parallel resistor (Norton)
- The easiest way to find the I vs. V line is to find the intercepts where I = 0 (open circuit voltage V_T) and where V = 0 (Short circuit current I_N)
- The short-cut for finding the $(\text{slope})^{-1} = R_T = R_N$ is to turn off all of the dependent sources to zero and find the remaining equivalent resistance between the terminals of the elements.