Problem 1: Boolean Algebra

(a) Simplify the following expression: \((A + B) + A\)

Simplified Expression: \(A + B\)

(b) Simplify the following expression: \((A + BC)(\overline{AC} + B)\)

Simplified Expression: \(AB + BC\)
(c) Simplify the following expression: \( A(ABC + BCA) + \overline{CA} + B \)

Simplified Expression: \( \overline{A} + B + \overline{C} \)

Problem 2: Boolean Logic Gates

For the following circuits built with logic gates, determine the equivalent and simplified boolean expressions.

(a) Circuit:

Simplified Expression: \( C = AB \)
Problem 3: Basic Verilog

For each numbered blank, fill in the corresponding box below in the correct snippet of Verilog so that it matches the output of the 4-input 3-output Boolean function calculator. The inputs are $a$, $b$, $c$, and $d$, and the outputs are $e$, $f$ and $g$, which have the following boolean expression.

\[
e = \overline{a}bc + \overline{a}bc \\
f = \overline{a}bc + ab + \overline{a}bcd \\
g = b\overline{c} + ad
\]

```verilog
module boolean_calculator(
    ___(1)___ a, b, c, d,
    ___(2)___ wire e, g,
);```

Simplified Expression: \[ C = A \oplus B, \overline{A} \oplus \overline{B} \]

Simplified Expression: \[ D = \overline{ABC}, \overline{A} + \overline{B} + \overline{C} \]
module boolean_calculator (  
    input a, b, c, d,  
    output wire e, g,  
    output reg f  
);  
  assign e = (a ^ b) && c;  
  always @(*) begin  
    case ({a, b})  
      2'b00: f = 1'b0;  
      2'b01: f = !c && d;  
      2'b10: f = c;  
      2'b11: f = 1'b1;  
  endcase  
endmodule

Full solution:

module boolean_calculator (  
    input a, b, c, d,  
    output wire e, g,  
    output reg f  
);
  assign e = (a ^ b) && c;
  always @(*) begin
    case ({a, b})
      2'b00: f = 1'b0;
      2'b01: f = !c && d;
      2'b10: f = c;
      2'b11: f = 1'b1;
    endcase
end
wire g0, g1;
and gate0 (g0, b, !c);
and gate1 (g1, a, d);
or gate2 (g, g0, g1);
endmodule