

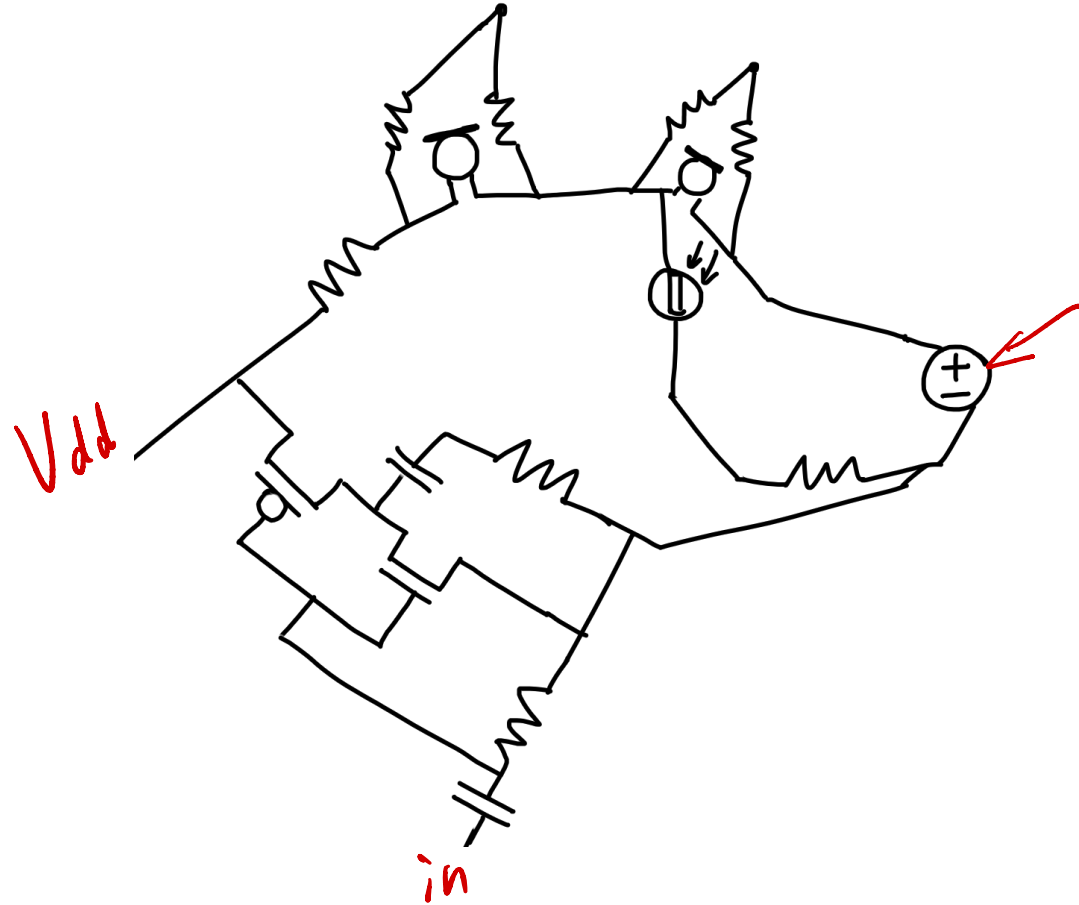
DOGGO 151/251A SP2022 Discussion 8

GSI: DIMA NIKIFOROV, YIKUAN CHEN



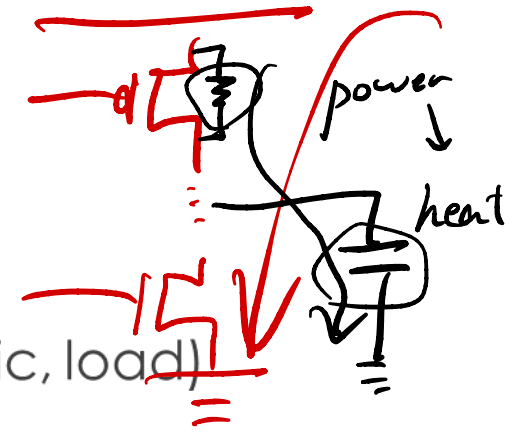
Agenda

- Power/Energy
- Adders



Energy

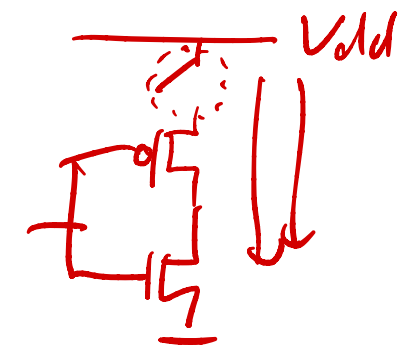
Power/Energy in Digital Circuits



- Fundamentally, charging/discharging capacitors (gate, parasitic, load) through resistances (PMOS, NMOS, wires)

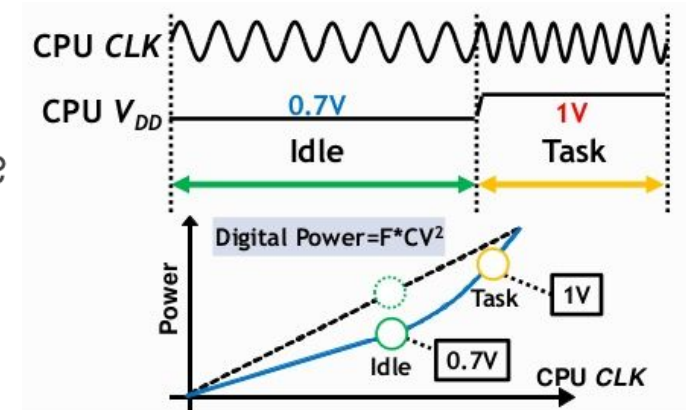
- Capacitors draw CV^2 joules from supply over 1 charge/discharge cycle

- $\frac{1}{2}CV^2$ dissipated in PMOS as heat when charging
 - $\frac{1}{2}CV^2$ stored on capacitor, then dissipated in NMOS when discharging



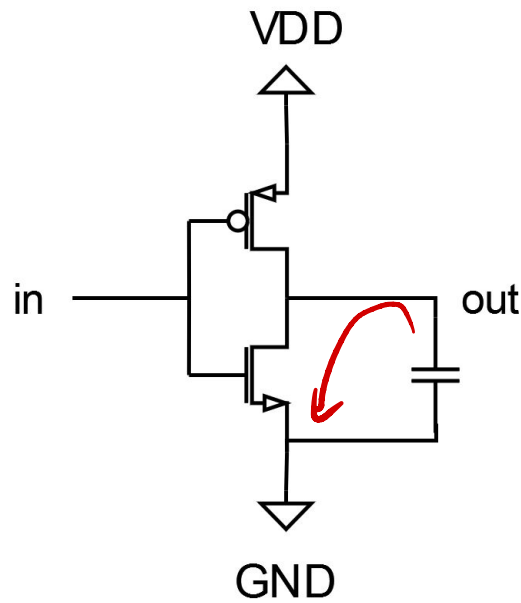
- Dynamic power = $P_{\text{switching}} = \alpha CV^2f$
 - How to minimize each term?
 - α → $\in [0, 1]$
 - f → \uparrow freq → $V \downarrow$ → slower clk
 - Minimizing which terms reduces total energy consumed?
- Static power = leakage → wasted energy!

Dynamic Voltage and Frequency Scaling (DVFS)



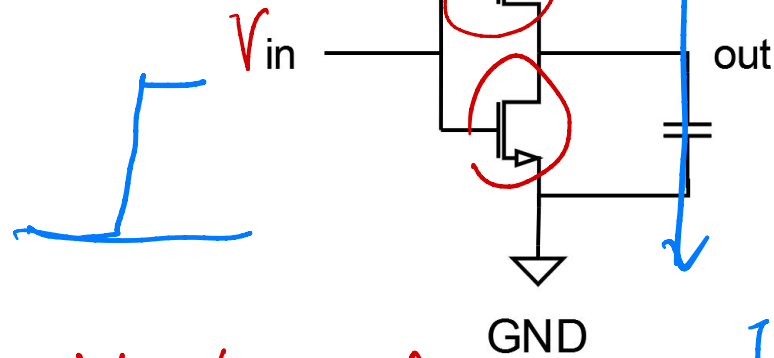
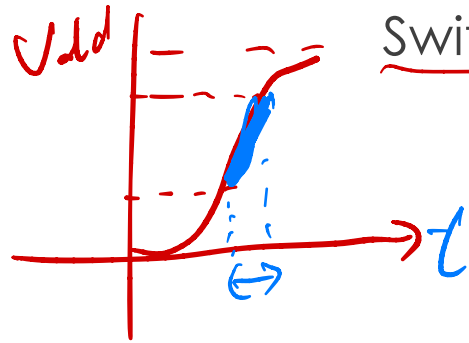
Power/Energy in Digital Circuits Causes

Dynamic Power



$$\frac{1}{2} C V^2$$

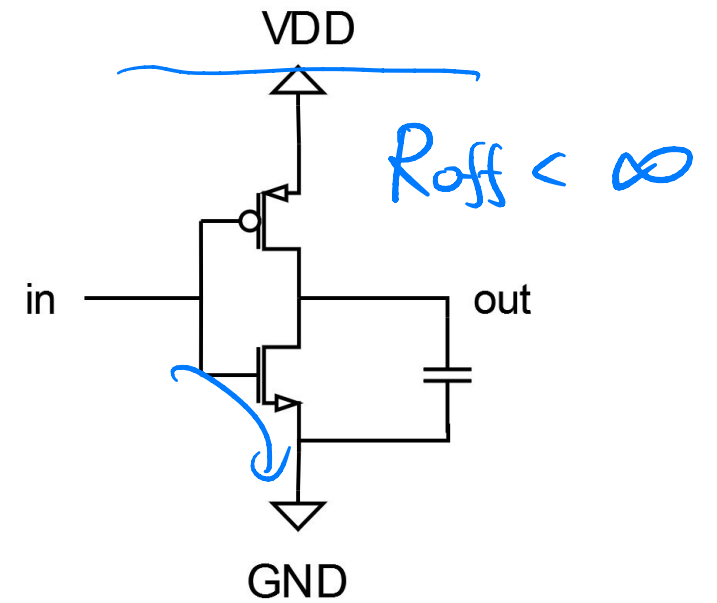
Switching Power



$$V_{DD} - V_{thp} > V > V_{thn}$$

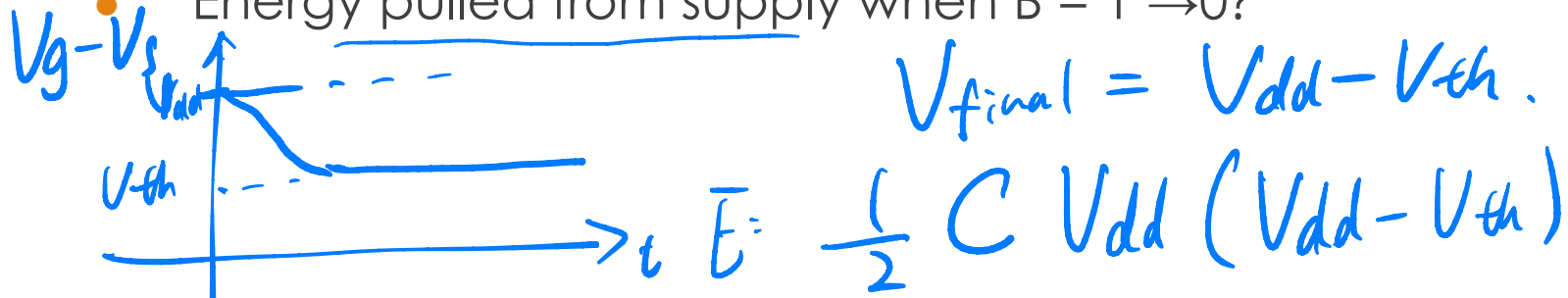
$$= \frac{IV}{R} = \frac{V^2}{R}$$

Leakage Power



Energy Example

- Initially: $A = 1$, $C = 1$, $Out = 0$
- Energy pulled from supply when $B = 1 \rightarrow 0$?

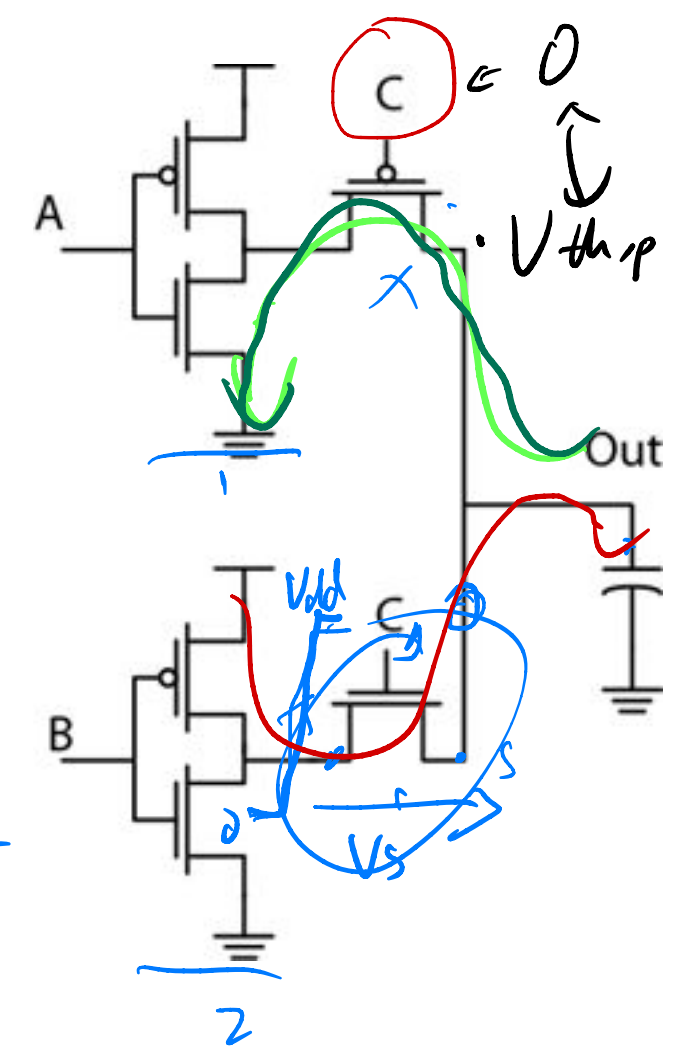


$$\bar{E} = \frac{1}{2} C V_{dd} (V_{dd} - V_{th})$$

- Then, how much energy dissipated when $C = 1 \rightarrow 0$?

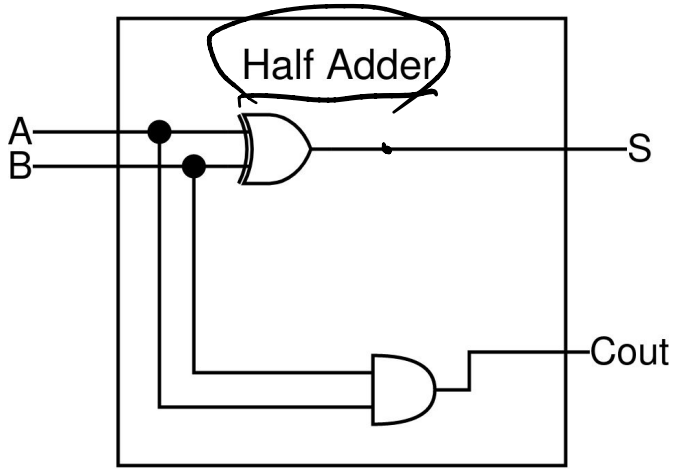
$$V_{cap} : V_{dd} - V_{th,n} \rightarrow V_{th,p}$$

$$\bar{E} = \frac{1}{2} C (V_{dd} - V_{th,n})^2 - \frac{1}{2} C V_{th,p}^2$$



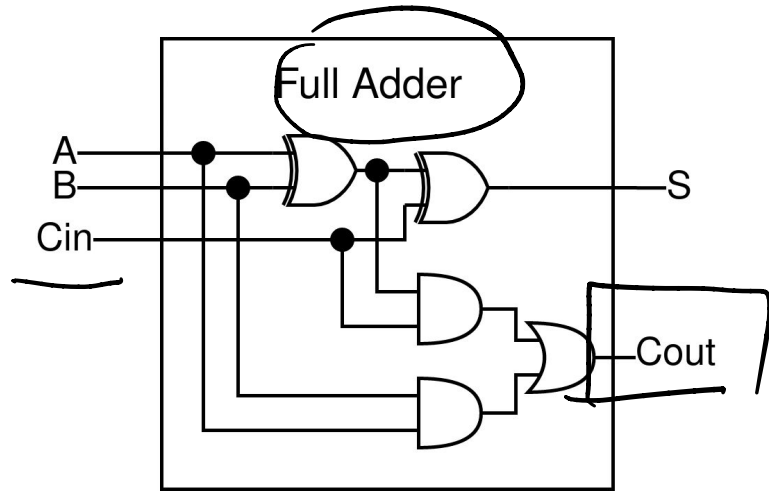
Adders

$2 + 3 = 5$
 Adder Components
 $2 + 3 = 5$



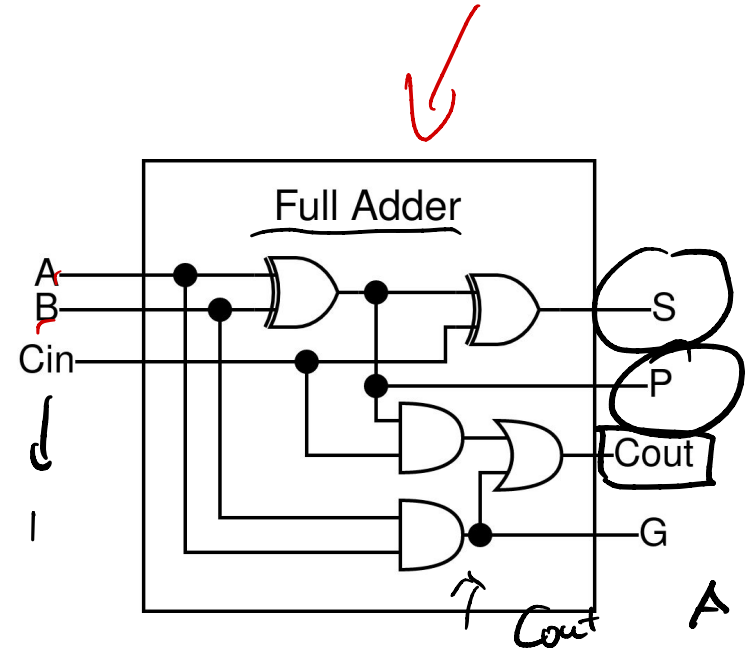
$$S = A \oplus B$$

$$C_o = AB$$



$$S = A \oplus B \oplus C_i$$

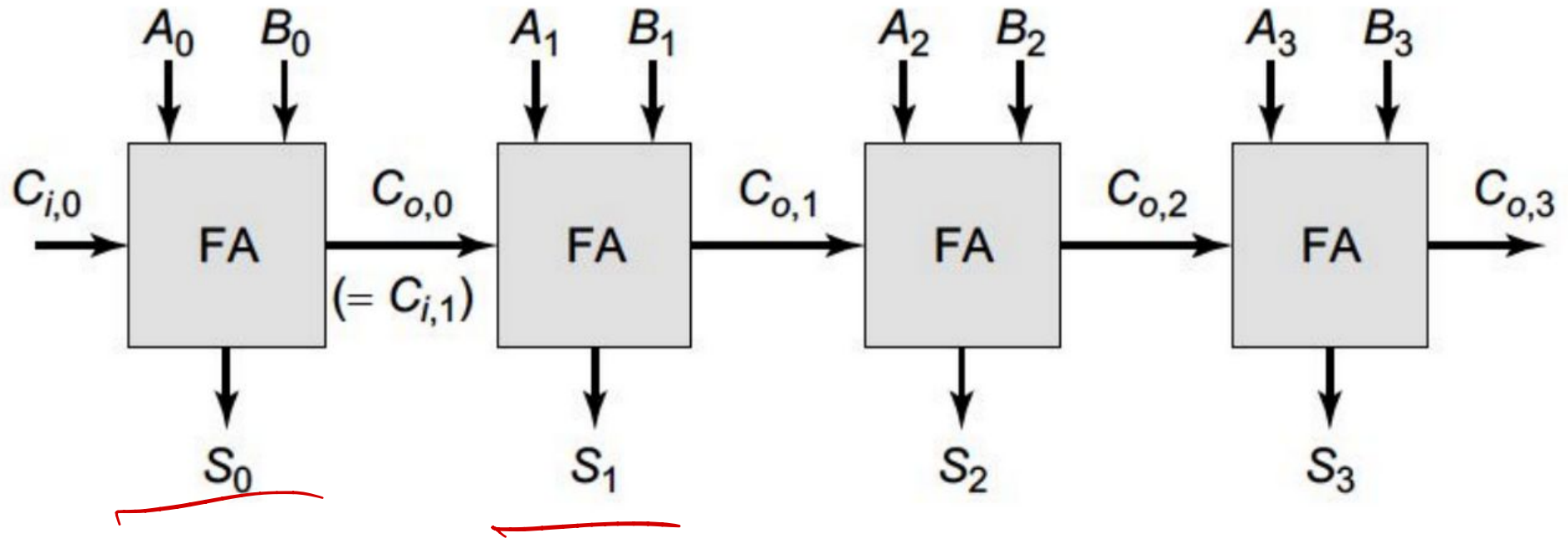
$$C_o = AB + B C_i + A C_i$$



propagate 1
 $P = A \oplus B$
generate
 $G = AB$

| A | B | Cin |
|---|---|-----|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

Ripple-Carry Adder



- Time Complexity?
- Area Complexity?

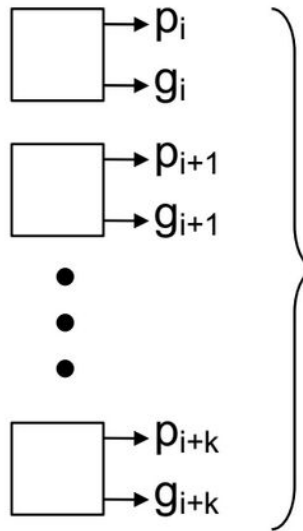
$O(N)$

$O(N)$

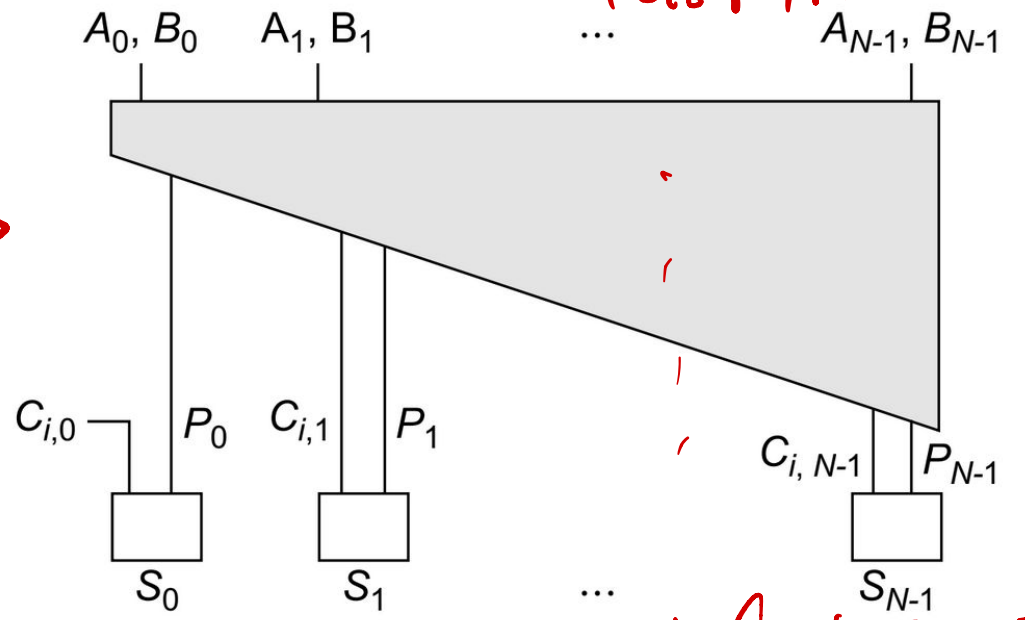
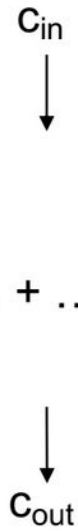
Carry-Lookahead Adder

Handwritten notes:
 - $D_2 = 10$ bit adder
 - $\log_2(N)$ (with arrow pointing to the right)

Handwritten equation:
 $C_{10} = G_{10} + G_8 P_9 P_{10}$



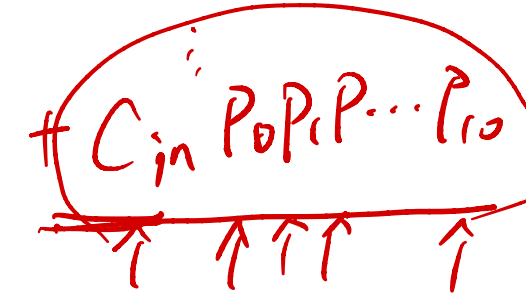
$P = p_i p_{i+1} \dots p_{i+k}$
 $G = g_{i+k} + p_{i+k} g_{i+k-1} + \dots + (p_{i+1} p_{i+2} \dots p_{i+k}) g_i$



$C_{o,k} = f(A_k, B_k, C_{o,k-1}) = G_k + P_k C_{o,k-1}$
Handwritten expansion: $+ G_1 P_2 P_3 \dots P_{10}$

- Time Complexity?
- Area Complexity?

Handwritten notes:
 - $O(1) \leftarrow N$ is big, not possible.
 - $O(\log N)$
 - $O(N^2)$ (circled in green)
 - $\propto N^2$



Carry-Lookahead Tree Adder

Group Gen ; Group Prop. $P_{10:0} = \prod P_i$

$$C_{out6} = G_{6:0} + P_{6:0} C_{0,in}$$

$$P_{1:0} = P_1 \cdot P_0, G_{1:0} = G_1 + P_1 \cdot G_0, \rightarrow C_{out1} = G_{1:0} + P_{1:0} \cdot C_{0,in}$$

- Time Complexity?

$O(\log_2 N)$

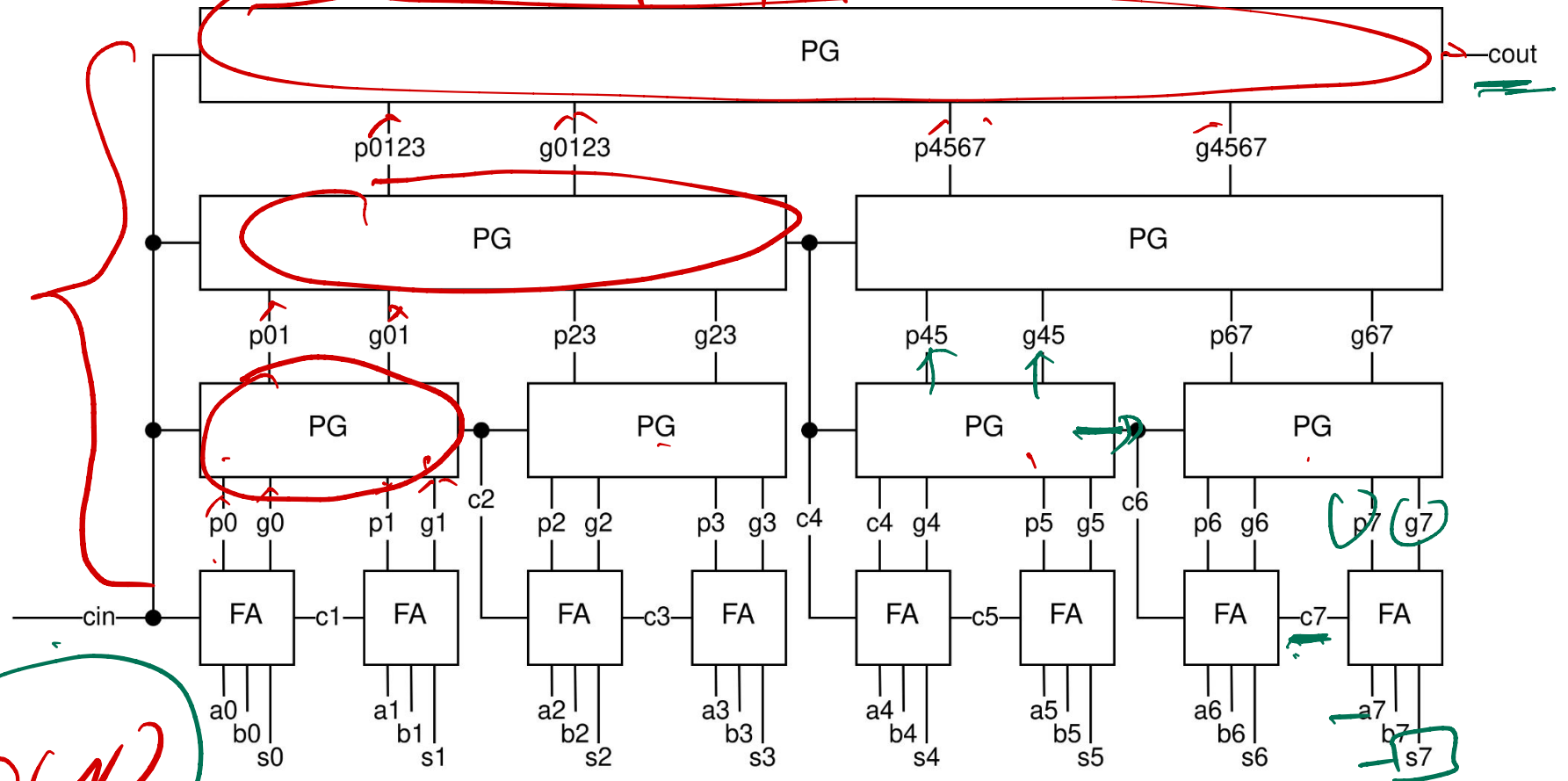
- Area Complexity?

$4 + 2 + 1$

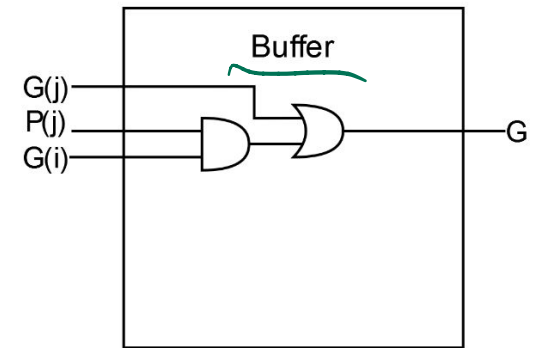
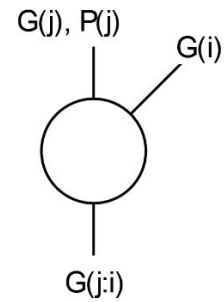
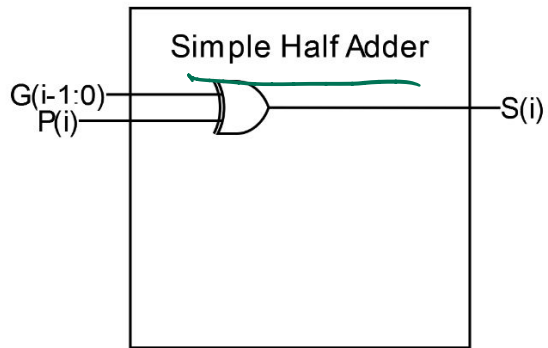
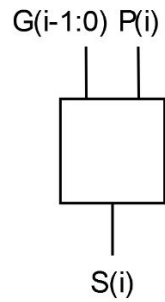
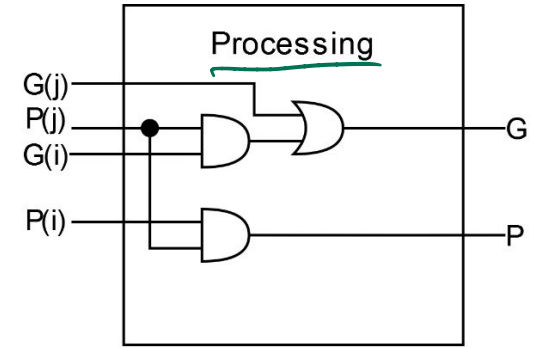
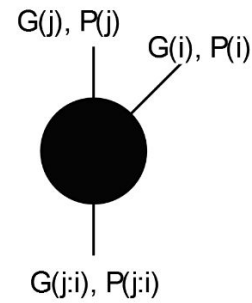
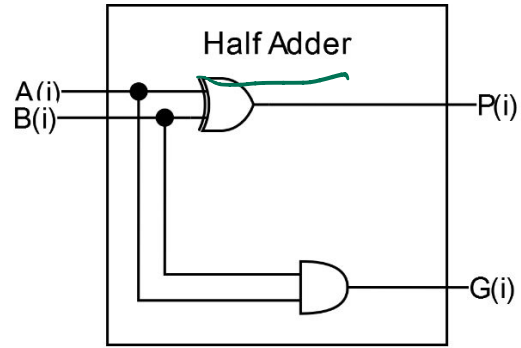
$= 2^3 - 1$

$= \frac{2^{\log_2 8} - 1}{8 - 1} \Rightarrow$

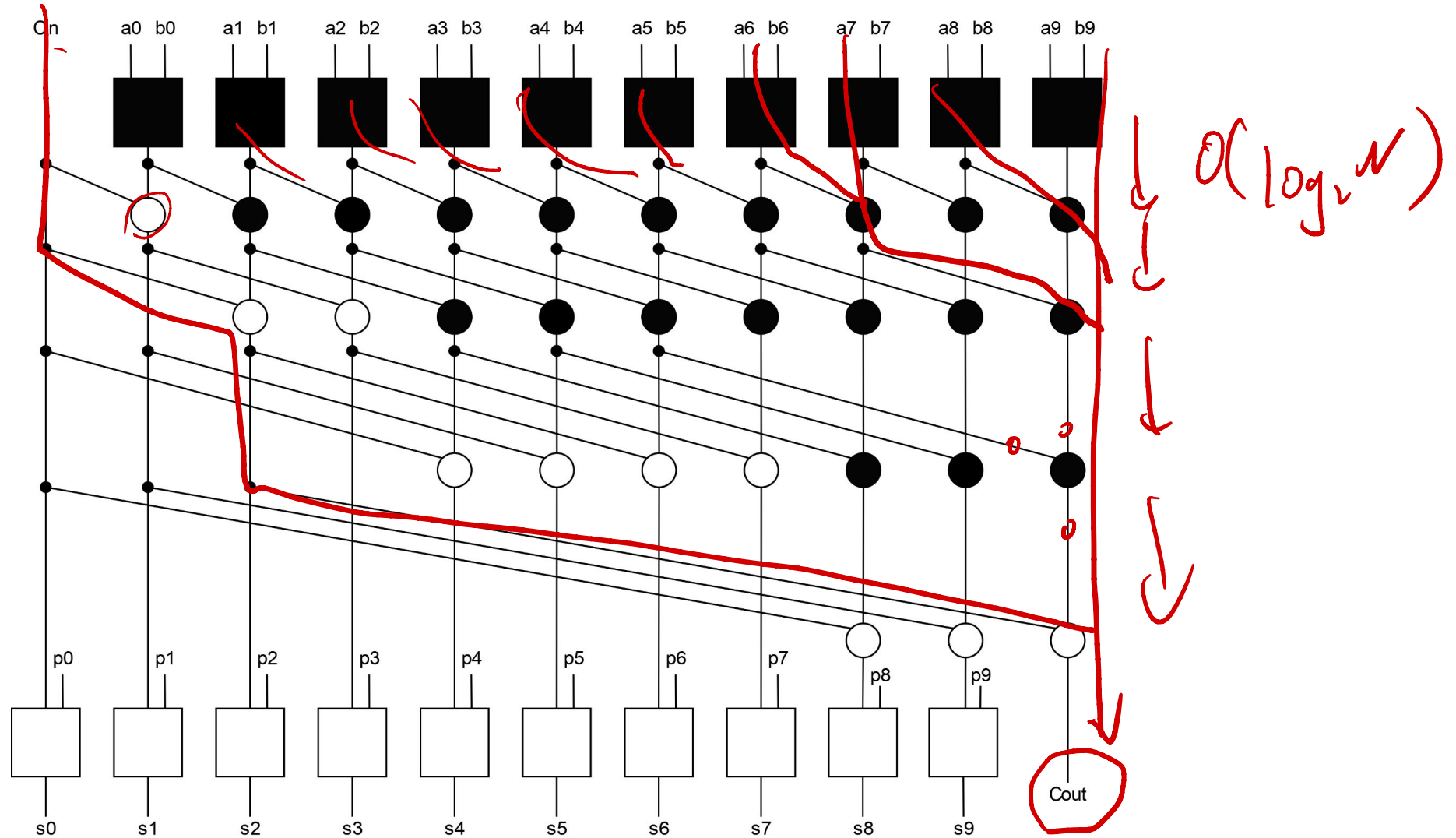
$O(N)$



Kogge-Stone Tree Adder (Components)



Kogge-Stone Tree Adder



Doggo Adder



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Questions?