EECS 151/251A SP2022 Discussion 9

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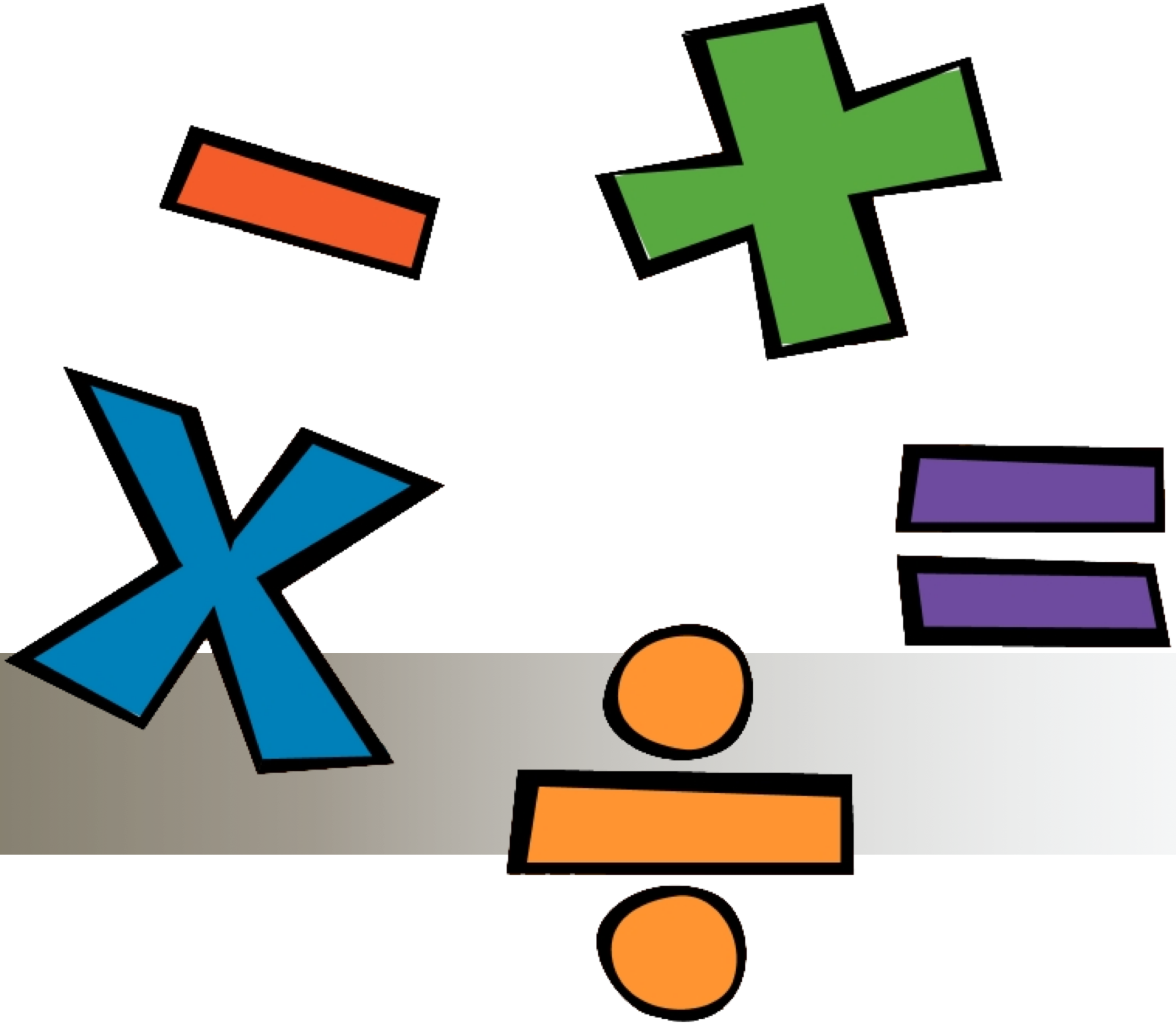
Slides modified from Zhenghan Lin’s and Harrison Liew’s slides

Agenda

- Multiplier
- Latch

Multiplier



Unsigned Multiplication Example

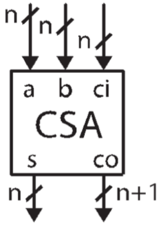
```
4'b0011 (3)
× 4'b0110 (6)
```

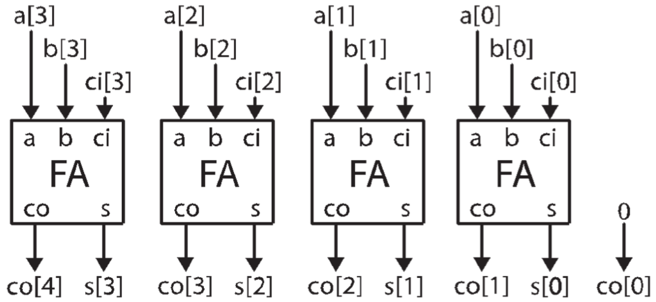
- Partial Products can be generated in parallel
- Challenge: improve the **addition** of partial products

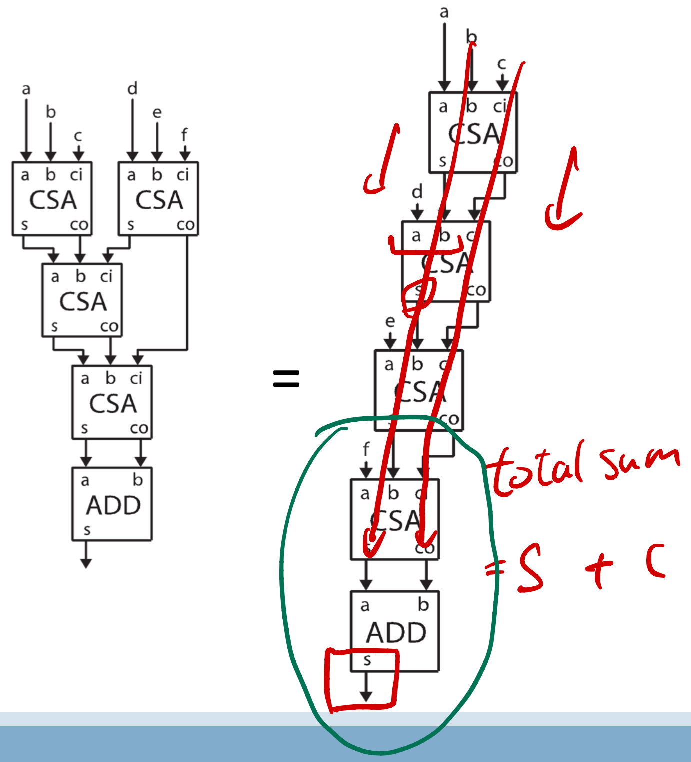
```
            4
  4'b0011 (3)
× 4'b0110 (6)
  --------
      0000
     0011
    0011      4 } Partial Products
+  0000
---------
 00010010 (18)
    16  + 2
```

Carry-Save Addition

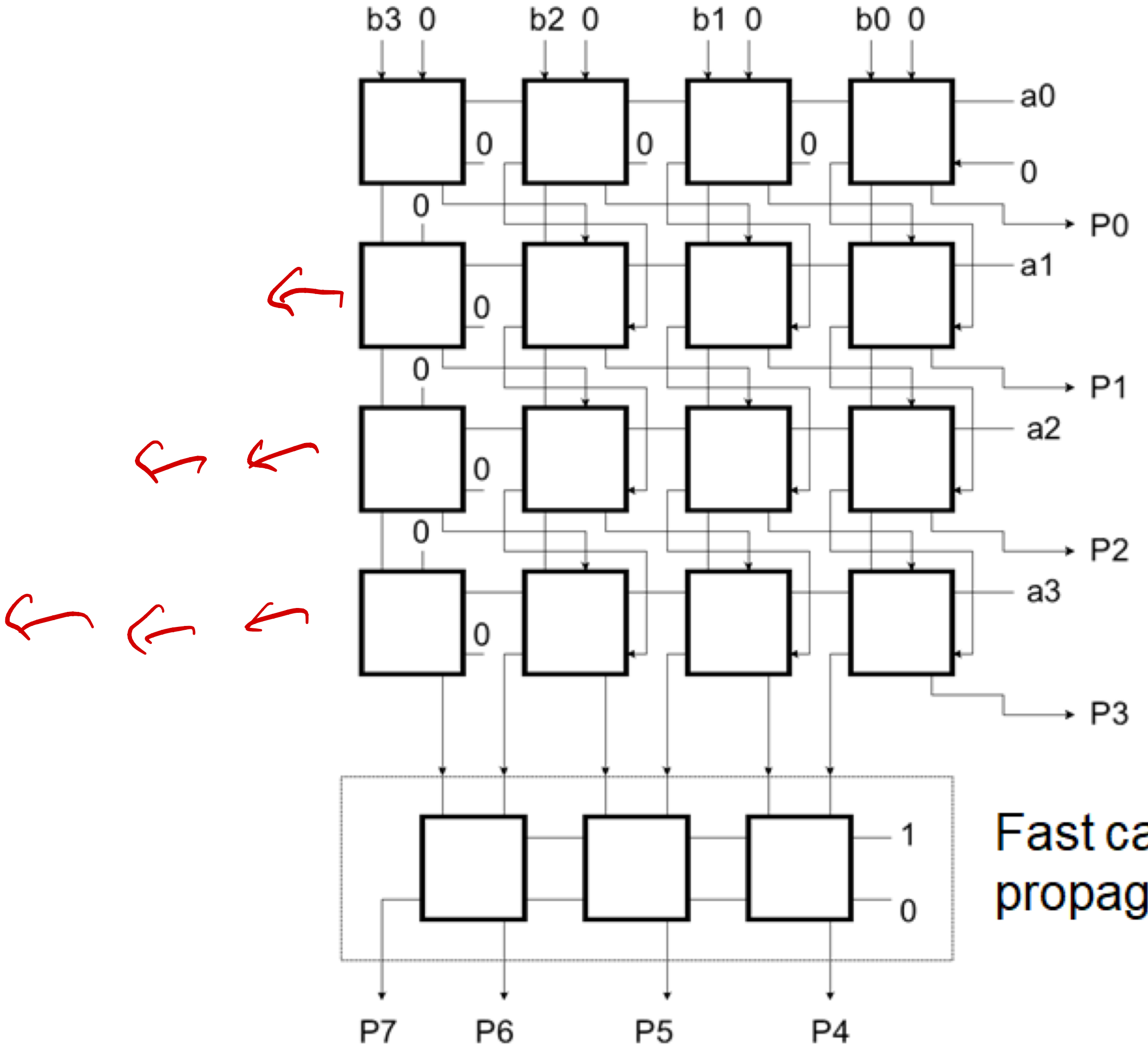
- When you want to add **more than 2 numbers**
- When we generate a carry in a given column, add it to the 2 values in the next column.
  - In turn, may generate its own new carry
- Delay adding carry bits until the end
- Basis of Unit Carry-Save Adder:
  - Takes in a, b, c_{in} (all are multi-bit)
  - Produces a sum and c_{out} (all are multi-bit)
- Benefits:
  - CSAs have no carry ripple => small & fast!
  - Only 1 standard CLA/Parallel Prefix Adder at end
  - Addition is associative => trees!



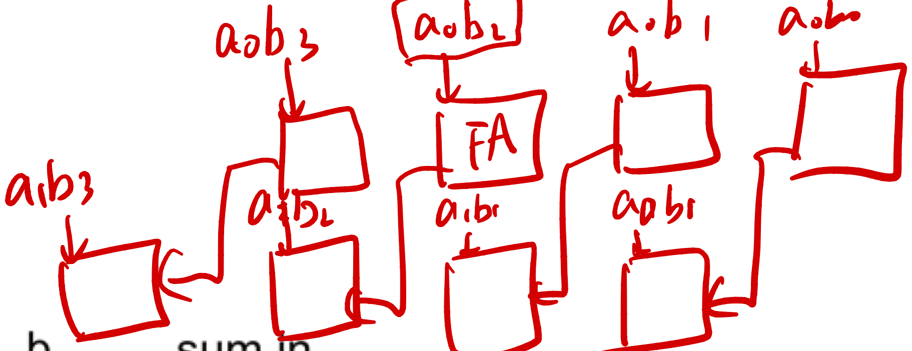


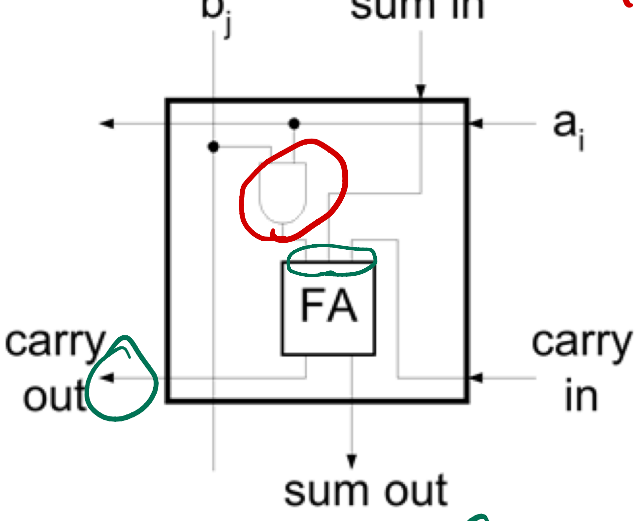


Array Multiplier w/ CSA



Fast carry-propagate adder





A = 101000 (40)

B = 011001 (25)

C_{in} = 010100 (20)

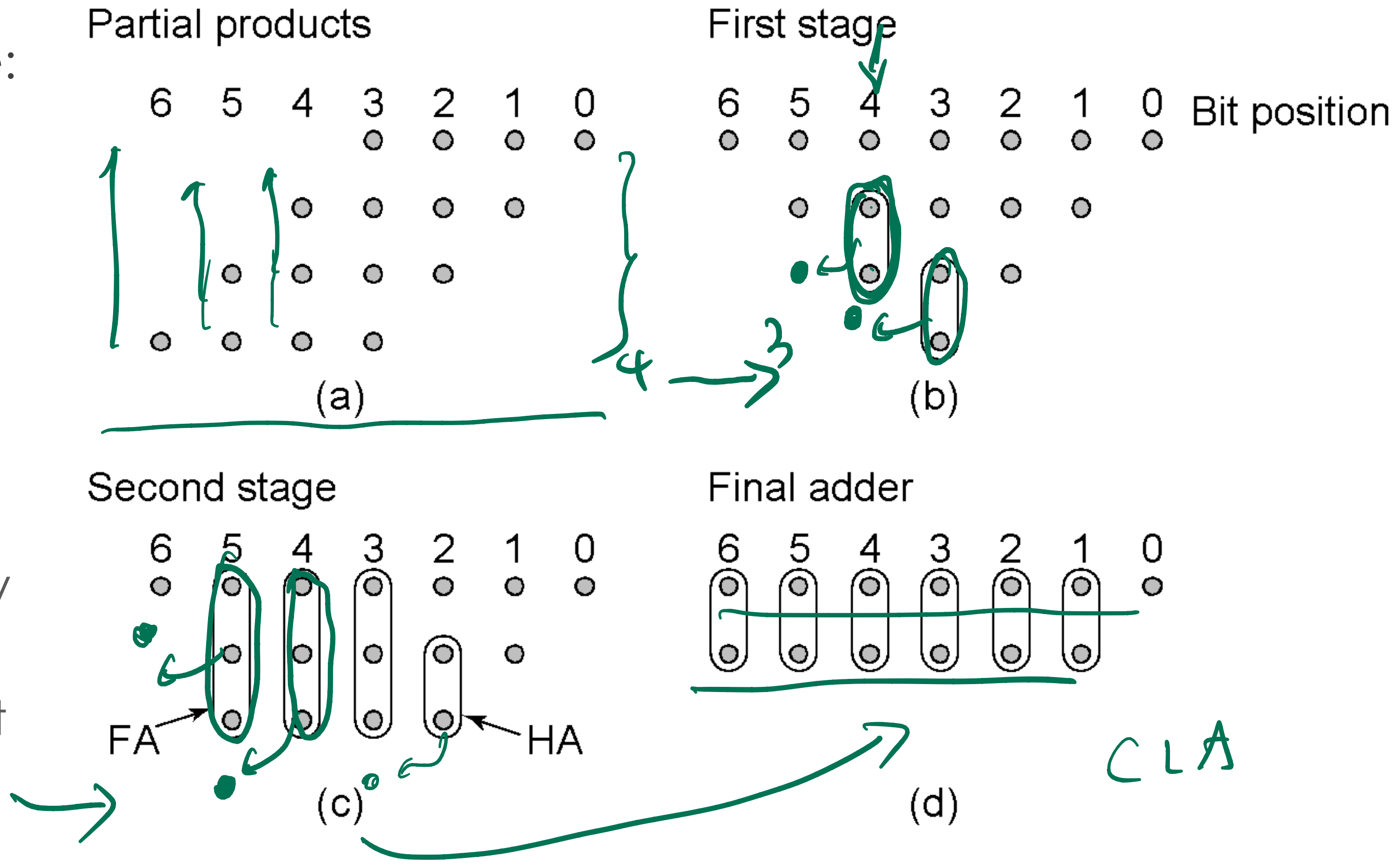
A ⊕ B ⊕ C

```
    101000  A
    011001  B
  + 010100  Cin
"Sum"   100101     O(1)
"Cout" 011000      O(1)
      1010101  ←  O(log N)
```

Wallace Tree Multiplier – Reduce the rows!

Method to construct Wallace Tree:

1. Draw a dot diagram where each column has as many dots as number of partial products
2. Group dots in the same column by 2 (~~half adder~~) or 3 (full adder)
   2-input add
   3-input add
3. Propagate carries and sum by adding one dot in the grouped column and one dot in the next column



Radix-4 Multiplication

- Binary multiplication -> N partial products! Can we reduce this?
  - Yes! Let's use a larger radix (think: base)
- E.g. 2 bits at a time (radix 4) -> halve number of partial products

001	1
010	2
011	3
100	10
101	11
⋮	

Radix-4

b Digit	Partial Product	Partial Product (Rewritten)
00	0*A	0
01	1*A	A
10	2*A	4*A - 2*A
11	3*A	4*A - A

1101 * 0110 → 1101* [01][10]

0

② 1 2

1×4^1 2×4^0

4 -2

Booth Recoding –

- 4*A = A << 2
- 2*A = A << 1
- Recall: radix 4 multiplication => shift left by 2 positions for next partial product
- Therefore, any 4*A term can be handled in the next partial product!
    - Multiplier looks a 3 (rather than just 2) bits
    - Extra bit is MSB of the previous

B Digit	Partial Product	Partial Product (Rewritten)
00	0*A	0
01	1*A	A
10	2*A	4*A - 2*A
11	3*A	4*A - A

$B = 0 1 1 0 \leftarrow 4A - 2A$

Booth Recoding

B_{i+1}	B_i	B_{i-1}	Action	Comment
0	0	0	Add 0	
0	0	1	Add A	Includes +4*A from previous radix 4 digit = +A in this position due to left shift by 2
0	1	0	Add A	
0	1	1	Add 2*A	Includes +4*A from previous round (+A in this position). *2 is implemented as a left shift by 1
1	0	0	Sub 2*A	4*A will be added in when handling next radix 4 digit. *2 is implemented as a left shift by 1
1	0	1	Sub A	4*A will be added in when handling next radix 4 digit. Includes +4*A from previous radix 4 digit (+A in this position)
1	1	0	Sub A	4*A will be added in when handling next radix 4 digit.
1	1	1	Add 0	4*A will be added in when handling next radix 4 digit. Includes +4*A from previous radix 4 digit (+A in this position)

Booth Recoding Example (Unsigned)

A for 01

A for previous stage's 4A

- 6 * 7
- $B_{-1} = 0$

```
     4'b0110   (6)
   * 4'b0111   (7)
     --------
-       0110 ( Sub A)
+    01100   (Add 2A)
+   0000     ( Add 0)
    ---------
+   11111010 ( Sub A)
+    01100   (Add 2A)
+   0000     ( Add 0)
    ---------
~~(1)~~00101010 (42)
```

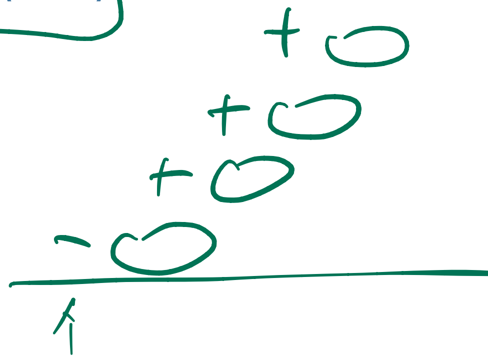
B_{i+1}	B_i	B_{i-1}	Action
0	0	0	Add 0
0	0	1	Add A
0	1	0	Add A
0	1	1	Add 2*A
1	0	0	Sub 2*A
1	0	1	Sub A
1	1	0	Sub A
1	1	1	Add 0

Signed Multiplication: Baugh-Wooley

- Recall: 2’s complement MSB has negative weight, meaning:

  If a[N-1:0] has its MSB=1, then rather than meaning 2^(N-1), it means -2^(N-1)

- Nominally:
  1. Subtract last partial product
  2. Sign-extend the rest of the partial products
- Recall 2’s complement negation: subtract A = add (~(A) + 1)
  - Result: basically same hardware as unsigned mult.
  - Implementation: invert some bits, insert a 1 left of the first & last partial products



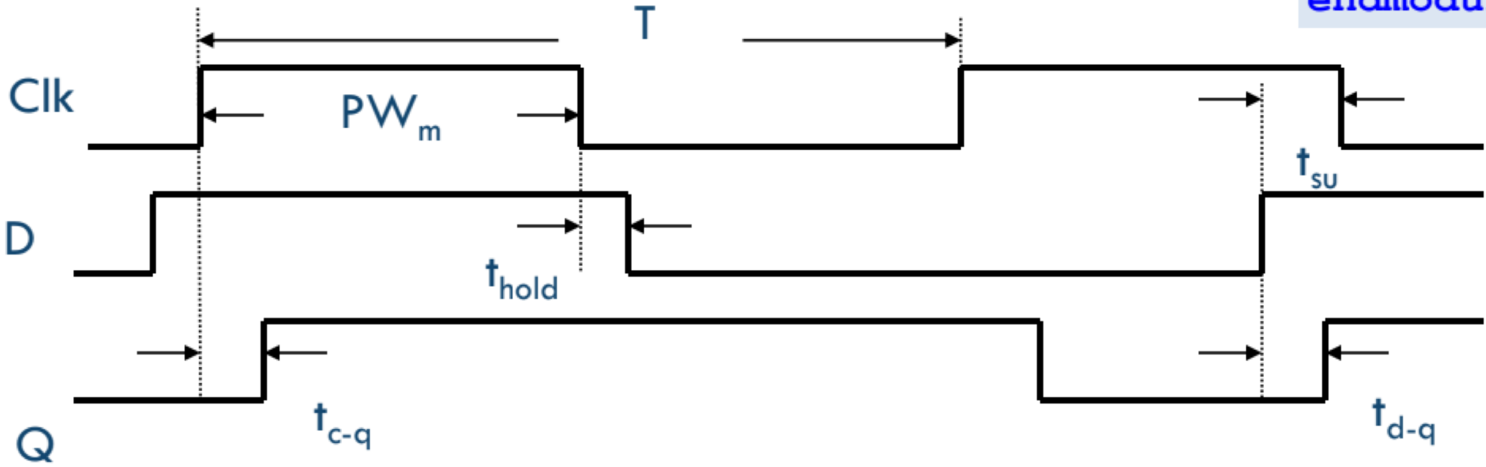
```
                          1    p0[3]  p0[2]  p0[1]  p0[0]
+                      ~p1[3]  p1[2]  p1[1]  p1[0]    0
+               ~p2[3]  p2[2]  p2[1]  p2[0]    0      0
+   1   ~p3[3]  ~p3[2] ~p3[1] ~p3[0]    0      0      0
------------------------------------------------------------
P[7]    P[6]    P[5]    P[4]   P[3]   P[2]   P[1]   P[0]
```

Latch

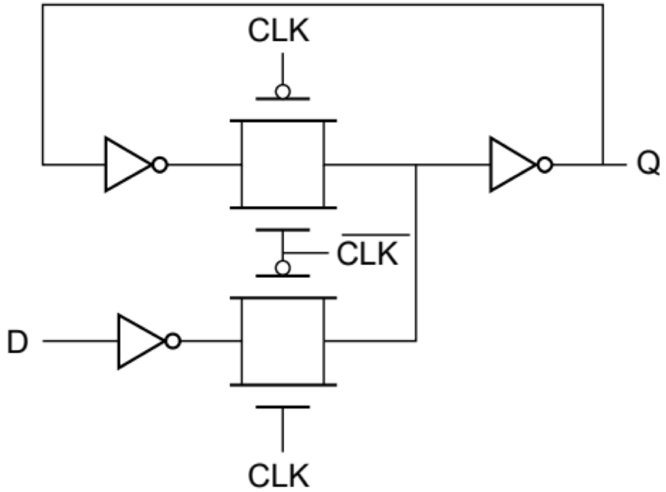
Latch Timing

- A positive latch is **transparent** (q = d) when the **clock is high** and **opaque** (q = d, sampled at negedge clock) when the **clock is low**
- $t_{d\text{->}q}$: delay from d to q when the latch is transparent
- $t_{clk\text{->}q}$: delay from the rising clock edge to d propagating to q

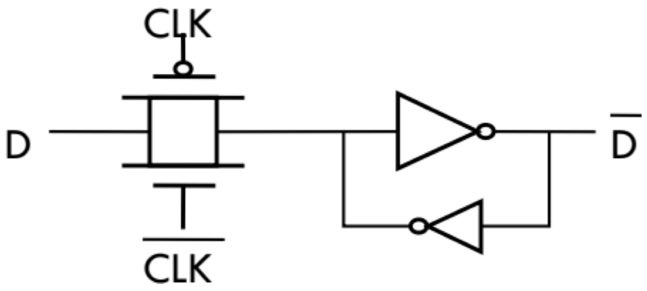
```
endmodule
```



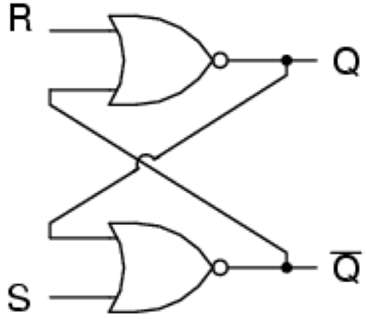
Latch Circuits

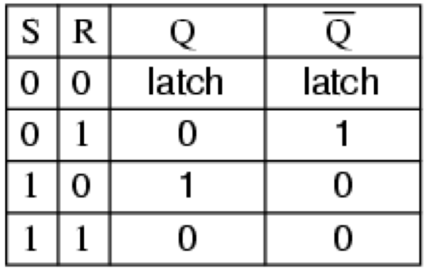


‘Feedback-breaking’ latch
Transparent high



‘State-forcing’ latch
Transparent low



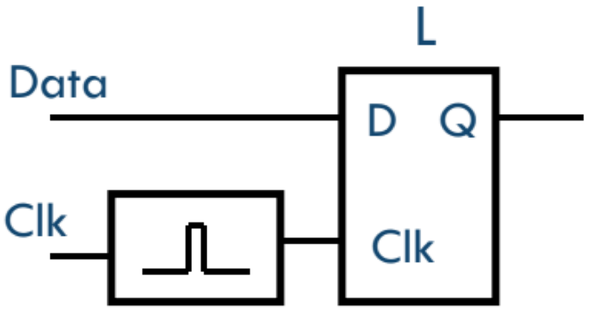


S	R	Q	$\overline{Q}$
0	0	latch	latch
0	1	0	1
1	0	1	0
1	1	0	0

SR latch
Common interview question

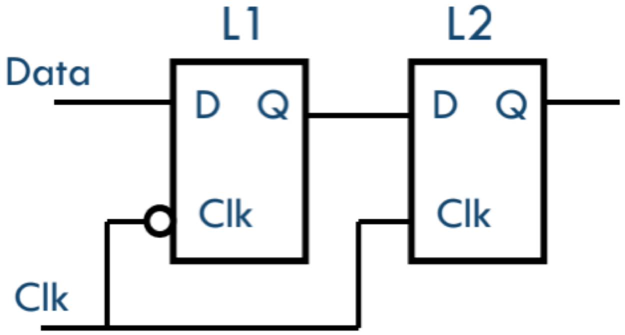
Building a Flip-Flop from Latches

- Clock pulsed latch
  - Latch becomes transparent for the pulse duration only, then holds data
  - Not common anymore, sometimes used in high performance circuits
  - Positive hold time



Pair of latches – edge triggered (posedge clk!!!)

- Commonly used technique
- L2 holds output data stable when clock is high.
- Negative hold time



Questions?