Lecture 5 - Module 2

Today: * Superposition } Circuit jedi techniques

Note 15 * Equivalence

Goal: Want to design interesting systems
- Need the tools that provide insight

Reminder: Circuit analysis objective:

Find $\vec{x} = \begin{bmatrix} I_1 \\ \vdots \\ I_m \\ V_1 \\ \vdots \\ V_k \end{bmatrix}$ for some ckt matrix A and some stimulus vector

$\vec{b} = \begin{bmatrix} I_{s1} \\ \vdots \\ I_{sl} \\ V_{s1} \\ \vdots \\ V_{s\,m+k-l} \end{bmatrix}$ ← independent sources

$A\vec{x} = \vec{b} \Rightarrow \underbrace{\vec{x} = A^{-1} \cdot \vec{b}}_{\text{solution}}$

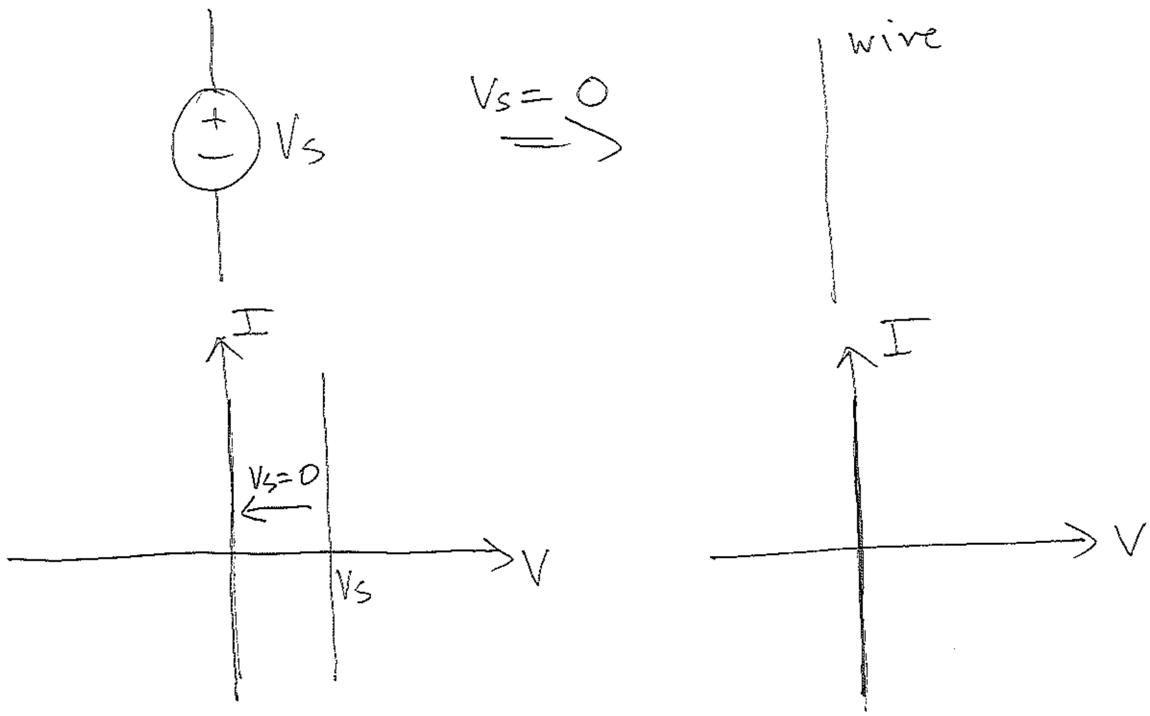
$\Rightarrow \begin{cases} I_i = \underline{\underline{\alpha_1}} I_{s1} + \dots + \underline{\underline{\alpha_l}} I_{sl} + \dots + \underline{\underline{\alpha_{m+k}}} V_{s\,m+k-l} \\ \text{and } V_j = \underbrace{\beta_1 I_{s1}}_{V_{j,1}} + \dots + \beta_{m+k} V_{s\,m+k-l} \end{cases}$

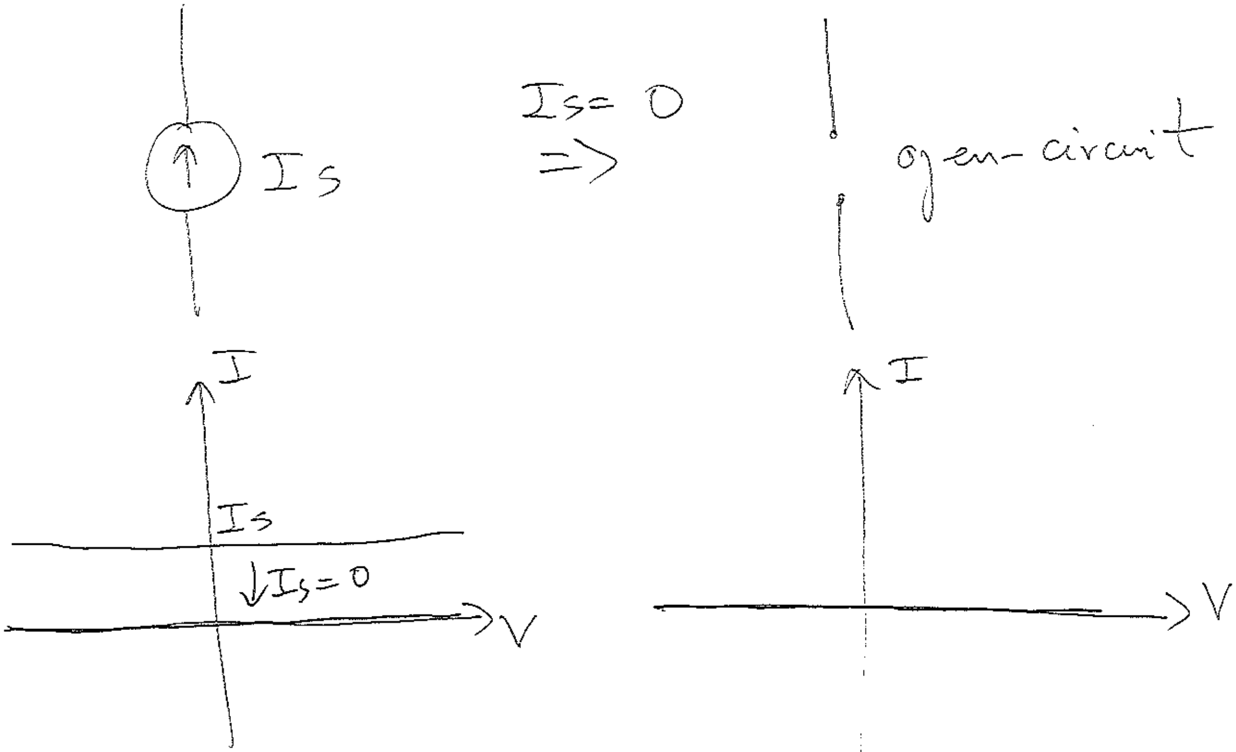
lin. combinations of sources

$I_i = \underbrace{I_{i,1}}_{\alpha_1 I_{s1}} + \dots + \underbrace{I_{i,l}}_{\alpha_l I_{sl}} + \dots + \underbrace{I_{i,m+k}}_{\alpha_{m+k} V_{s\,m+k-l}}$

$\Rightarrow$ can calculate I_i by calculating $I_{i,1} \dots I_{i,m+k}$ separately (i.e. by <u>nulling</u> the other sources).

(2) What does it really mean to "null" a source?



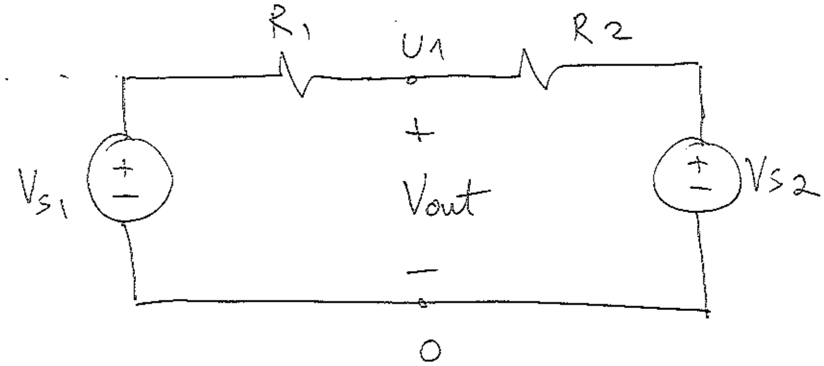


L3

Superposition: Find I's and U's by turning independent sources on one at a time and solving the ckt.

Example:

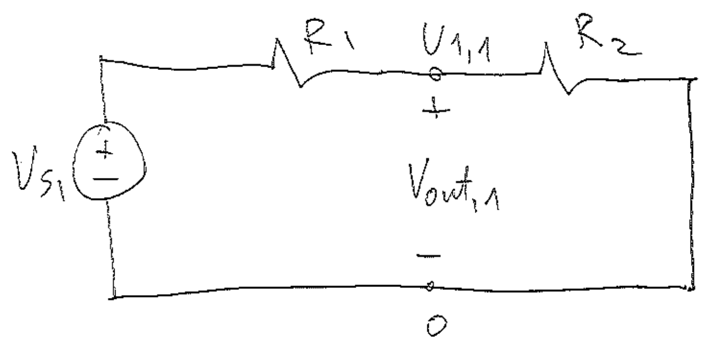
voltage summer



$$U_1 - 0 = V_{out}$$

$$U_1 = V_{out}$$

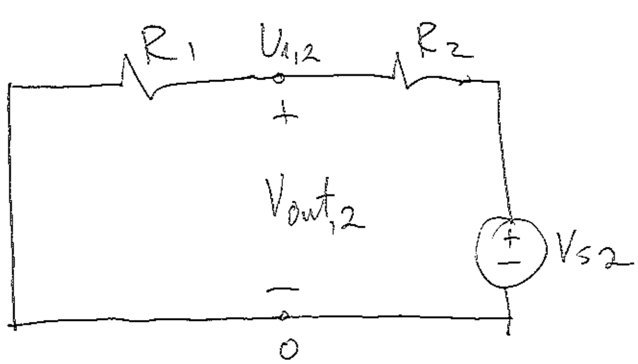
step 1: Compute a response to V_{s1} (set $\underline{V_{s2} = 0}$)



voltage divider ☺

$$V_{out,1} = \frac{R_2}{R_1 + R_2} \cdot V_{s1}$$

step 2: Compute a response to V_{s2} (set $V_{s1} = 0$)



voltage divider ☺

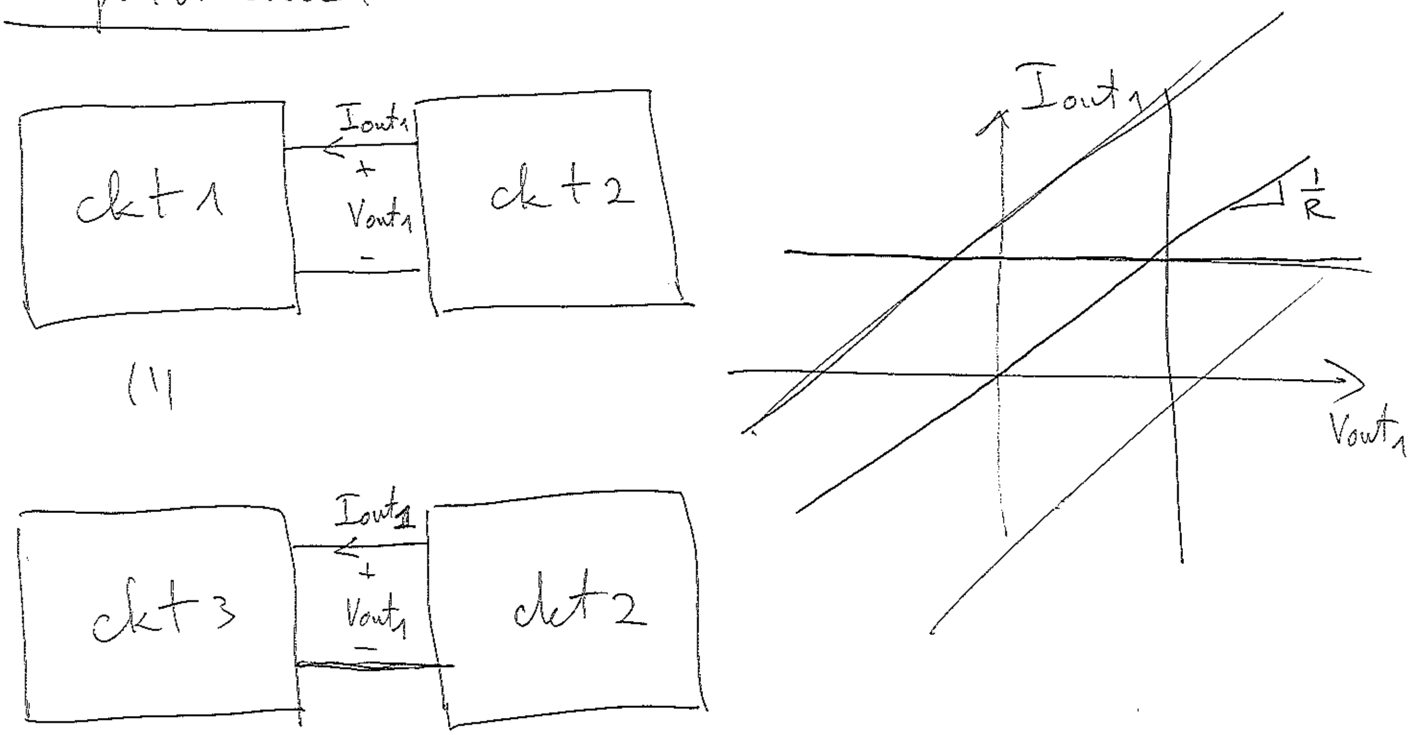
$$V_{out,2} = \frac{R_1}{R_1 + R_2} \cdot V_{s2}$$

$$V_{out} = V_{out,1} + V_{out,2} = \underbrace{\frac{R_2}{R_1 + R_2}}_{\alpha < 1} V_{s1} + \underbrace{\frac{R_1}{R_1 + R_2}}_{\beta < 1} V_{s2}$$

$$V_{out} = U_1, \quad V_{out,1} = V_{1,1}, \quad V_{out,2} = V_{1,2}$$

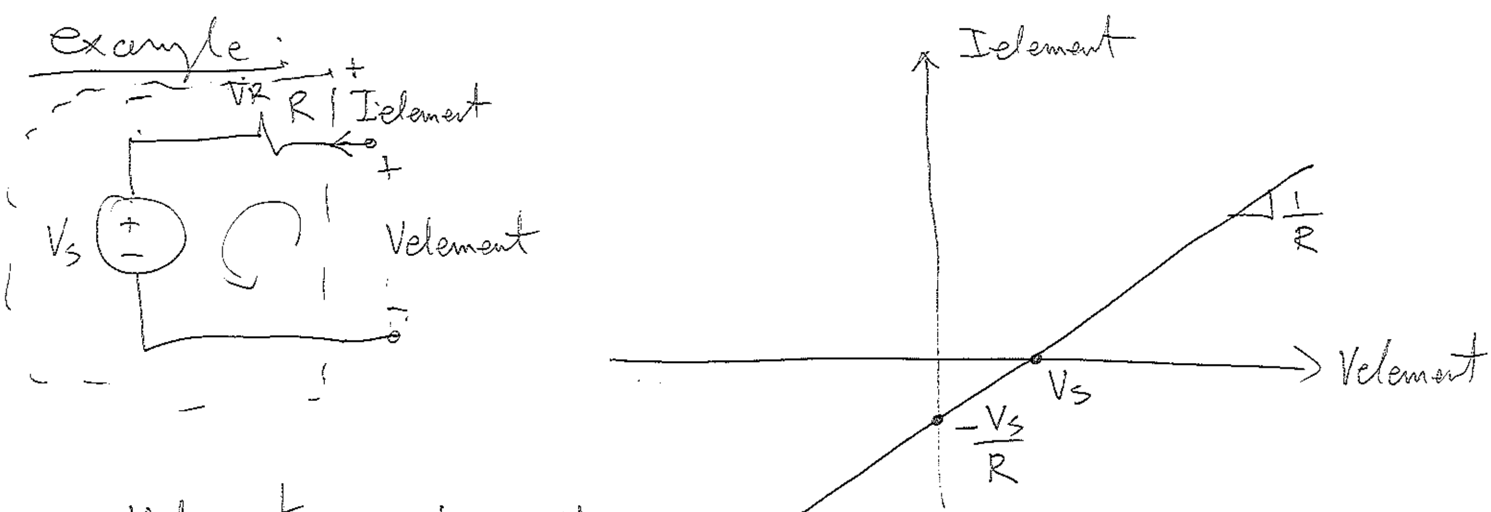
(4)

Equivalence:



In circuits, two elements are equivalent if they have the same I-V characteristics.

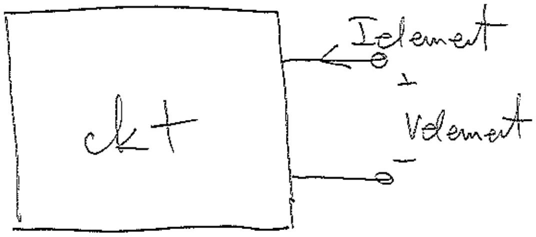
example:



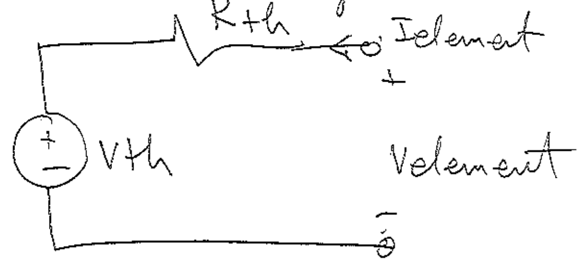
$$V_{element} = V_s + V_R$$

$$V_{element} = V_s + I_{element} \cdot R$$

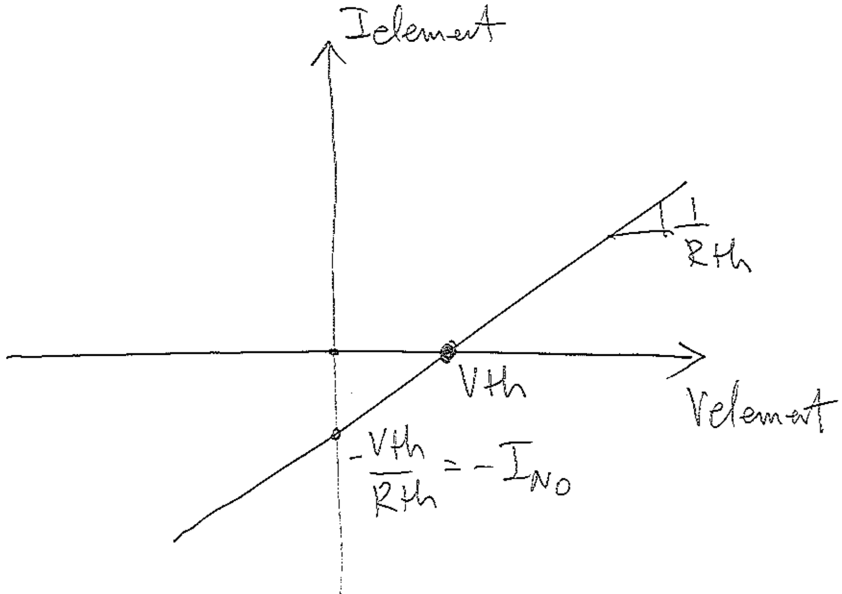
$$I_{element} = \frac{1}{R} \cdot V_{element} - \frac{V_s}{R}$$



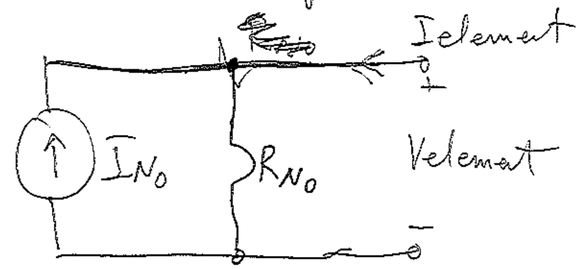
Thevenin equivalent:



Need a min. of two elements (a resistor and a source) to create any I−V <u>line</u>.



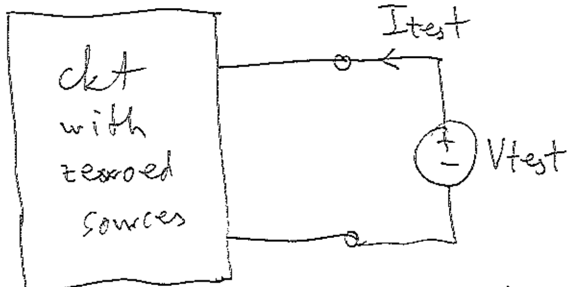
Norton equivalent:



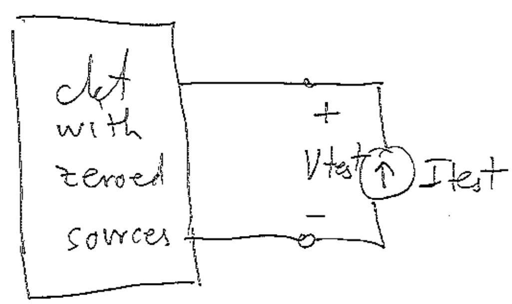
$R_{th} = R_{No}$

To find Vth: "Connect" an "open-circuit", across two terminals and measure $V_{open\text{-}circuit} = V_{th}$

To find Rth: Zero-out <u>independent</u> sources (to find a slope)

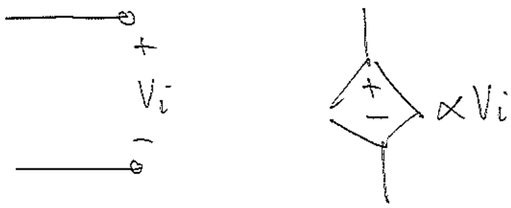


or

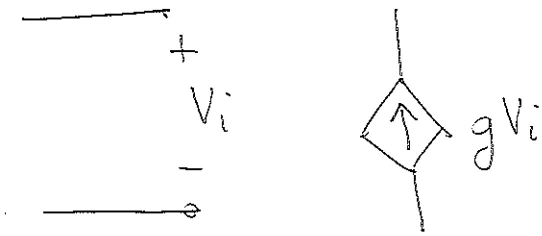


$$R_{th} = \frac{V_{test}}{I_{test}}$$

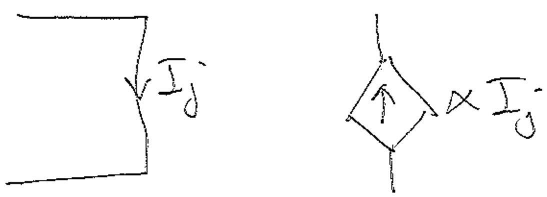
Dependent sources:



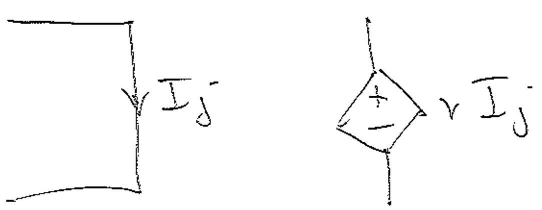
Voltage-controlled voltage source
(VCVS)



Voltage-controlled current-source
(VCCS)

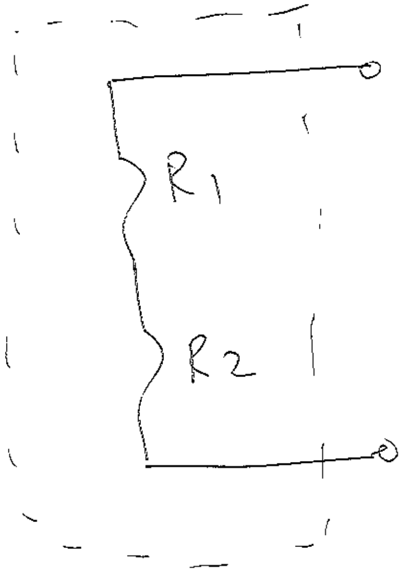


Current-controlled current source
(CCCS)



Current-controlled voltage source
(CCVS)

Example 1:



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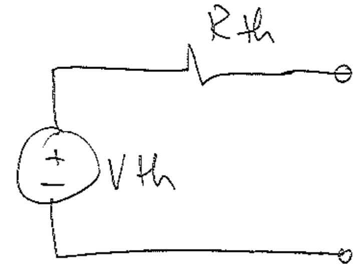

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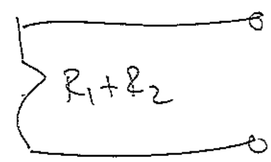
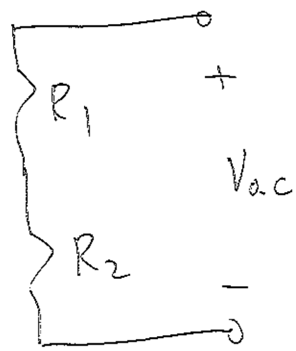
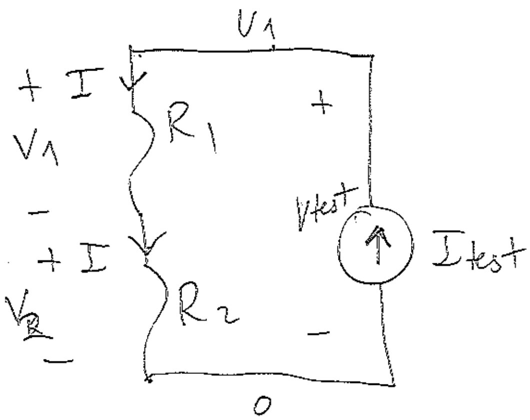


step 1:



$V_{a.c} = 0 \Rightarrow V_{th} = 0$

step 2: No sources & apply test current



$$V_{test} = (R_1 + R_2) \cdot I_{test}$$

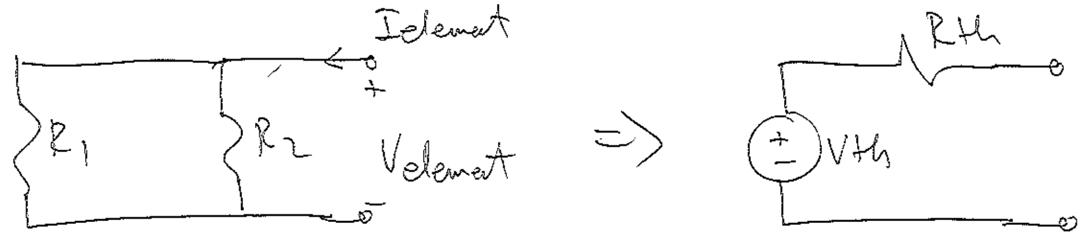
$$= I \cdot R_1 + I \cdot R_2$$

$$= I_{test} R_1 + I_{test} R_2$$

$I = I_{test}$

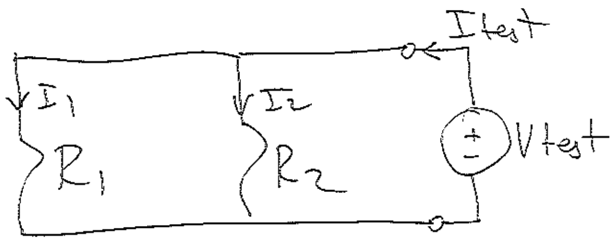
$$R_{th} = \frac{V_{test}}{I_{test}} = \boxed{R_1 + R_2}$$

"In series" means that the same current flows through the elements.



step 1: $V_{th} = 0$

step 2:



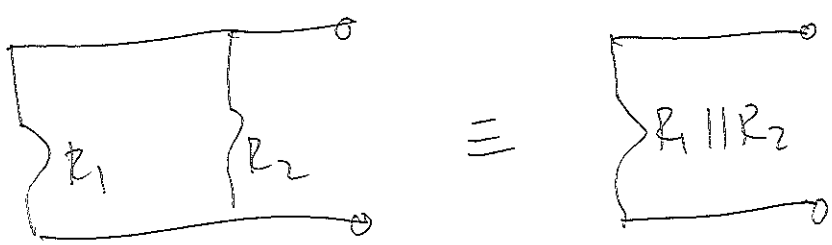
$$I_1 = \frac{V_{test}}{R_1}$$

$$I_2 = \frac{V_{test}}{R_2}$$

$$I_{test} = I_1 + I_2 = V_{test}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_{th} = \frac{V_{test}}{I_{test}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2$$

parallel operator



"in parallel" means voltage across them is the same.