EE16 A

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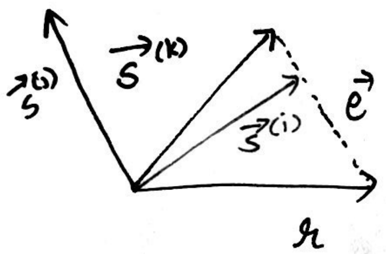
Today:

- Geometric interpretations of correlation and inner product.
- Projections.
- Multiple beacons.
- Trilateration.



$$\vec{s} = \begin{bmatrix} s[0] \\ \vdots \\ s[N] \end{bmatrix} \qquad \vec{s}^{(k)} = \begin{bmatrix} s[N-k] \\ \vdots \\ s[N-1] \\ s[0] \\ \vdots \\ s[N-k-1] \end{bmatrix} \leftarrow \text{kth position}$$

$$\vec{r} = \begin{bmatrix} r[0] \\ r[1] \\ \vdots \\ r[N-1] \end{bmatrix}$$

$$\vec{e} = \vec{r} - \vec{s}_k$$

Minimize length ~~len~~ $\|\vec{e}\|$

Find k

$$\boxed{\min_k \|\vec{e}\|^2} = \|\vec{r} - \vec{s}^{(k)}\|^2$$

$$= (\vec{r} - \vec{s}^{(k)})^T (\vec{r} - \vec{s}^{(k)})$$

$$= \vec{r}^T\vec{r} + \vec{s}^{(k)T}\vec{s}^{(k)} - \vec{r}^T\vec{s}^{(k)} - \vec{s}^{(k)T}\vec{r}$$

$$= \|\vec{r}\|^2 + \|\vec{s}\|^2 - 2\langle \vec{r}, \vec{s}^{(k)} \rangle$$

–ve sign.

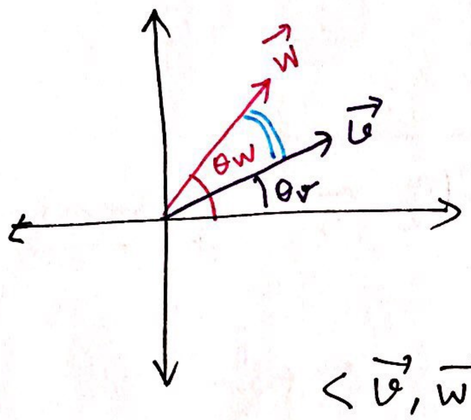
$$\langle \vec{r}, \vec{s}^{(k)} \rangle = \langle \vec{s}^{(k)}, \vec{r} \rangle$$

$$\max_k \langle \vec{r}, \vec{s}^{(k)} \rangle$$

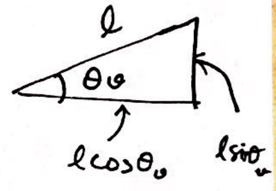
Geometric interpretation.

Intuitively $\langle \vec{s}^{(k)}, \vec{r} \rangle$ should be larger if $\vec{s}^{(k)}_{\theta}$, $\vec{r}$ are close.

2D Geometric.



$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \|\vec{v}\| \cdot \cos\theta_v \\ \|\vec{v}\| \sin\theta_v \end{bmatrix}$$



$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \|\vec{w}\| \cos\theta_w \\ \|\vec{w}\| \sin\theta_w \end{bmatrix}$$

$$\begin{aligned}
\langle \vec{v}, \vec{w} \rangle &= \langle \vec{w}, \vec{v} \rangle \\
&= v_1 w_1 + v_2 w_2 \\
&= \|\vec{v}\| \cos\theta_v \cdot \|\vec{w}\| \cos\theta_w + \|\vec{v}\| \sin\theta_v \cdot \|\vec{w}\| \sin\theta_w \\
&= \|\vec{v}\| \|\vec{w}\| \left(\cos\theta_v \cos\theta_w + \sin\theta_v \sin\theta_w \right) \\
&= \|\vec{v}\| \|\vec{w}\| \left(\cos(\theta_v - \theta_w) \right) \\
&= \|\vec{v}\| \|\vec{w}\| \left(\cos(\underline{\theta_w - \theta_v}) \right).
\end{aligned}$$

$-1 \leq \cos\theta \leq 1$ $\quad$ $\cos 0 = 1.$ $\quad$ $\cos \frac{\pi}{2} = 0.$

Properties.

1. $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle.$
2. $\langle \vec{x} + \vec{y}, \vec{z} \rangle = \langle \vec{x}, \vec{z} \rangle + \langle \vec{y}, \vec{z} \rangle$
3. $\langle \alpha\vec{x}, \vec{y} \rangle = \alpha \langle \vec{x}, \vec{y} \rangle.$
4. $\langle \vec{x}, \vec{x} \rangle \geq 0$
5. $\langle \vec{x}, \vec{x} \rangle = 0$ if and only if $\vec{x} = \vec{0}$.

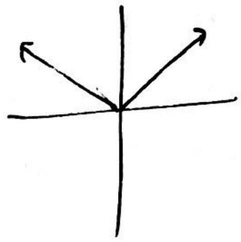
Cauchy-Schwarz inequality. : CS70.

$$|\langle \vec{v}, \vec{w} \rangle| = \left| \|\vec{v}\| \|\vec{w}\| \cos(\theta_v - \theta_w) \right| \leq \|\vec{v}\| \cdot \|\vec{w}\|.$$

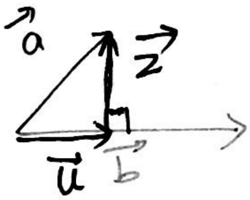
Minimum Value

Orthogonality $\vec{v}, \vec{w}$ are orthogonal if $\langle \vec{v}, \vec{w} \rangle = 0$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



Projection $\vec{a}, \vec{b}$. "Project" $\vec{a}$ onto the subspace spanned by $\vec{b}$



Find: $\vec{u}, \vec{z}$ such that.

1. $\vec{u} + \vec{z} = \vec{a}$.
2. $\vec{u}$ is in the same direction as $\vec{b}$
   $\vec{u} = \alpha \cdot \vec{b}$
3. $\vec{z}$ to be orthogonal to $\vec{b}$: $\langle \vec{z}, \vec{b} \rangle = 0$

Known: $\vec{a}, \vec{b}$

Unknown: $\vec{u}, \vec{z}$.

$\downarrow$

$\vec{u}$

$\boxed{\text{Find: unknown: } \alpha}$

$$\vec{z} = \vec{a} - \vec{u}$$

$$\langle \vec{z}, \vec{b} \rangle = 0$$

$$\Rightarrow \langle \vec{a} - \vec{u}, \vec{b} \rangle = 0$$

$$\Rightarrow \langle \vec{a}, \vec{b} \rangle - \langle \vec{u}, \vec{b} \rangle = 0.$$

$$\Rightarrow \langle \vec{a}, \vec{b} \rangle - \langle \alpha \vec{b}, \vec{b} \rangle = 0.$$

$$\Rightarrow \langle \vec{a}, \vec{b} \rangle - \alpha \langle \vec{b}, \vec{b} \rangle = 0.$$

$$\Rightarrow \langle \vec{a}, \vec{b} \rangle$$

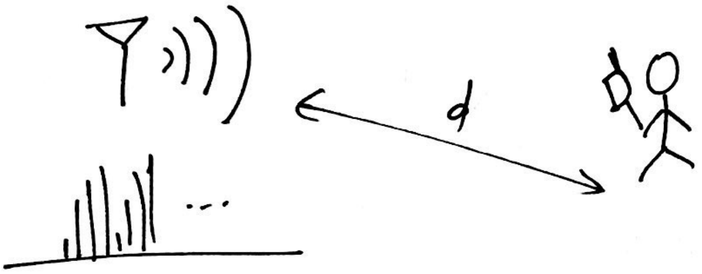
$$\Rightarrow \qquad \alpha = \frac{\langle \vec{a}, \vec{b} \rangle}{\langle \vec{b}, \vec{b} \rangle} = \frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{b}\|^2}$$

$$\vec{u} = \frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{b}\|^2} \cdot \vec{b}$$

Find co-ordinates of $\vec{a}$ along basis given by $\vec{b}, \vec{z}$

Co-ord: $\left(\frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{b}\|^2}, 1 \right)$

Connecting to locationing



Summary.

- Becon sends periodic signal.
- Use correlation to find the best aligned "shift"
- Transmission delay.
- Compute · distance.

Periodic signals.

Periodic corr.

$$\text{corr}_N(\vec{r},\vec{s})[k] = \sum_{i=0}^{N-1} r[i]\cdot s[i-k] = \langle \vec{r}, \vec{s}^{(k)} \rangle$$

For period 4.

$y(-4) = y[0] = y[4] \ldots$

$y[-3] = y(1) = y[5] \ldots$

$y[-2] = y[2] = y(2)$

$y[-1] = y[3] = y[7] \ldots$

Circular corr

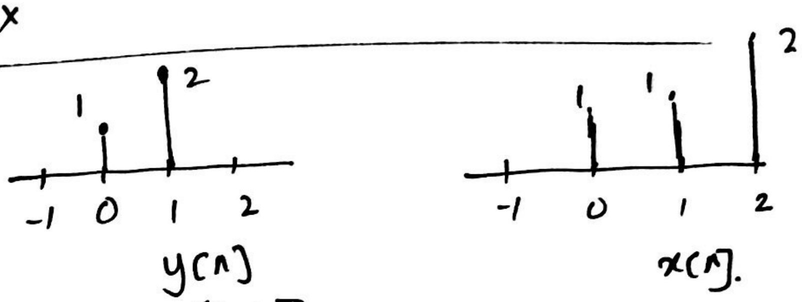
$$\text{circcorr}(\vec{r},\vec{s}) = \sum_{i=0}^{N-1} r[i]\, s[(i-k)_N]$$

matrix - vector multiplication

$$\begin{bmatrix} s[0] & s[1] & \cdots & s[N-1] \\ s[N-1] & s[0] & \cdots & s[N-2] \\ & \ddots & & \vdots \\ s[1] & \cdots & s[N-1] & s[0] \end{bmatrix} \begin{bmatrix} r[0] \\ r[1] \\ \vdots \\ r[N-1] \end{bmatrix}$$

circulant matrix

Linear correlation

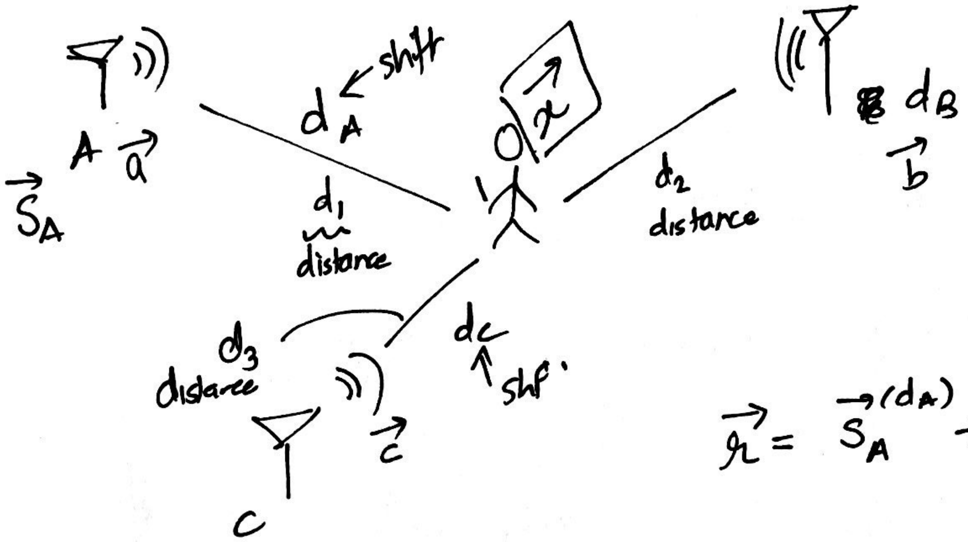


$$\text{corr}(\vec{x},\vec{y})[k] = \sum_{i=-\infty}^{\infty} x[i]\cdot y[i-k]$$

$$\text{corr}(\vec{x},\vec{y})[0] = 1 + 2 + 0 = 3.$$

$$\text{corr}(x,\vec{y})[1] = 0 + 1 + 4 = 5$$

Multiple beacons



$$\vec{r} = \vec{S}_A^{(d_A)} + \vec{S}_B^{(d_B)} + \vec{S}_C^{(d_C)}$$

$$\langle \vec{r}, \vec{S}_A^{(k)} \rangle = \langle \vec{S}_A^{(d_A)} + \vec{S}_B^{(d_B)} + \vec{S}_C^{(d_C)}, \vec{S}_A^{(k)} \rangle$$

$$= \underbrace{\langle \vec{S}_A^{(d_A)}, \vec{S}_A^{(k)} \rangle}_{\text{large}} + \langle \vec{S}_B^{(d_B)}, \vec{S}_A^{(k)} \rangle + \langle \vec{S}_C^{(d_C)}, \vec{S}_A^{(k)} \rangle$$

small

$$\|\vec{x}-\vec{a}\|^2 = d_1^2 \rightarrow \|\vec{x}\|^2 - 2\langle\vec{a},\vec{x}\rangle + \|\vec{a}\|^2 = d_1^2 \quad ①$$

$$\|\vec{x}-\vec{b}\|^2 = d_2^2 \qquad \|\vec{x}\|^2 - 2\langle\vec{b},\vec{x}\rangle + \|\vec{b}\|^2 = d_2^2 \quad ②$$

$$\|\vec{x}-\vec{c}\|^2 = d_3^2 \qquad \|\vec{x}\|^2 - \cdots\cdots = d_3^2 \quad ③.$$

$$② - ① : 2\langle\vec{x},\vec{b}\rangle - 2\langle\vec{x},\vec{a}\rangle = (d_1^2 - d_2^2) - (\|\vec{a}\|^2 - \|\vec{b}\|^2)$$

$$③ - ① : 2\langle\vec{x},\vec{c}\rangle - \text{ — '' — } = (d_1^2 - d_2^2) - (\|\vec{a}\|^2 - \|\vec{c}\|^2)$$

$$2\begin{bmatrix}(\vec{b}-\vec{a})^T \\ (\vec{c}-\vec{a})^T\end{bmatrix}\vec{x} = \begin{bmatrix} \\ \end{bmatrix}$$

$\boxed{\text{Trilateration}}$