EE16A

Nov 29, 2018.

Announcements:

- Review OH – Tue–Thu.
- Normal Discussion Monday.
- Review dis. Wednesday.
  - ↳ Topical.
- Extra OH.

Today

- OMP continued.
- m eqn's, n unknowns. $n > m$. but "sparse"

Setup:

2000 devices – home – toaster, fridge, pen...

Only a few devices are on at any given time.

1, 2, ...	2000.	device id.
$\vec{s_1}, \vec{s_2}$, ...	$\vec{s}_{2000}$	code.
α_1, α_2, ...	α_{2000}.	message.
τ_1, τ_2	$\tau, 2000$	shifts

$$\vec{y} = \alpha_1 \vec{s_1}^{(\tau_1)} + \alpha_2 \vec{s_2}^{(\tau_2)} + \dots + \alpha_{2000} \vec{s}_{2000}^{(\tau_{2000})}.$$

Most α_i's are zero

"Sparse signal"

$$N \left\{ \begin{bmatrix} | & & | \\ \vec{s_1}^{(\tau_1)} & \dots & \vec{s}_{2000}^{(\tau_{2000})} \\ | & & | \end{bmatrix} \right. \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{2000} \end{bmatrix} = \begin{bmatrix} | \\ \vec{y} \\ | \end{bmatrix}$$

"Sparse

Which α's are non-zero?

↳ What devices are "on"?

i_1, i_2, i_3, i_4

$\tau_{i_1}, \tau_{i_2}, \tau_{i_3}, \tau_{i_4}$.

$\alpha_{i_1}, \alpha_{i_2}, \alpha_{i_3}, \alpha_{i_4}$

# of equations = length of "Gold code"!

Alg 2

1. Find largest peak.

$$(i_1, \tau_{i_1}) = \underset{i,\tau}{\text{argmax}} \langle \vec{y}, \vec{s}_i^{(\tau)} \rangle.$$

2. Removing from $\vec{y}$ the "component" of $\vec{y}$ that is explained by $\vec{s}_{i_1}^{(\tau_{i_1})} = A_1 = \begin{bmatrix} | \\ \vec{s}_{i_1}^{(\tau_{i_1})} \\ | \end{bmatrix}$

$$\vec{y}_1 = A_1 (A_1^T A)^{-1} A_1^T \cdot \vec{y}$$

"Projection of $\vec{y}$ onto subspace spanned by A_1"

$$= \frac{\langle \vec{s}_{i_1}^{(\tau_{i_1})}, \vec{y} \rangle}{\| \vec{s}_{i_1}^{(\tau_{i_1})} \|^2} \cdot \vec{s}_{i_1}^{(\tau_{i_1})}$$

3. $$\vec{e}_1 = \vec{y} - \vec{y}_1$$

Find largest peak in $\vec{e}_1$.

$$(i_2, \tau_{i_2}) = \underset{i,\tau}{\text{argmax}} \langle \vec{e}_1, \vec{s}_i^{(\tau)} \rangle$$

4. Remove "component" explained by $A_2 = \begin{bmatrix} | & | \\ \vec{s}_{i_1}^{(\tau_{i_1})} & \vec{s}_{i_2}^{(\tau_{i_2})} \\ | & | \end{bmatrix}$

$$\vec{y}_2 = A_2 (A_2^T A_2)^{-1} \cdot A_2^T \cdot \vec{y}$$ "Projection onto subspace"

$$\vec{e}_2 = \vec{y} - \vec{y}_2$$

Scanned with CamScanner

Alg 1 .

① Found largest peak.

$$(i_1, \tau_{i_1}) = \underset{i,\tau}{\text{argmax}} \langle \vec{y}, \vec{s}_i^{(\tau)} \rangle$$

$$\rightarrow i_1, \tau_{i_1}$$

② Remove it from $\vec{y}$.

$$\vec{y}_1 = \underbrace{\langle \vec{y}, \vec{s}_{i_1}^{(\tau_{i_1})} \rangle}_{\text{height of peak}} \cdot \vec{s}_{i_1}^{(\tau_{i_1})}$$

$$\vec{e}_1 = \vec{y} - \vec{y}_1$$

③ Find the next peak.

$$(i_2, \tau_{i_2}) = \underset{i,\tau}{\text{argmax}} \langle \vec{e}_1, \vec{s}_i^{(\tau)} \rangle$$

$$\vec{y}_2 = \ldots$$

Remove $\vec{y}_2$

$$\vec{e}_2 = \vec{y} - \vec{y}_1 - \vec{y}_2$$

$\vdots$

Iteration	Signal Found	A matrix	Explained	Un-explained	Measurement
			$\vec{0}$	$\vec{y}$	$\vec{y}$
0	[ ]	[ ]			
1	$\vec{s}_1^{(\tau_1)}$	$A_1 = \begin{bmatrix} \mid \\ \vec{s}_1^{(\tau_1)} \\ \mid \end{bmatrix}$	$\vec{y}_1 = A_1 (A_1^T A_1)^{-1} A_1^T \cdot \vec{y}$	$\vec{y} - \vec{y}_1$	$\vec{y}$
2	$\vec{s}_1^{(\tau_1)}, \vec{s}_2^{(\tau_2)}$	$A_2 = \begin{bmatrix} \mid & \mid \\ \vec{s}_1^{(\tau_1)} & \vec{s}_2^{(\tau_2)} \\ \mid & \mid \end{bmatrix}$	$\vec{y}_2 = A_2 (A_2^T A_2)^{-1} A_2^T \cdot \vec{y}$	$\vec{y} - \vec{y}_2$	$\vec{y}$

Connect full circle. Back to module 1

Imaging ~~Photo~~ ~~Sparse~~ OMP only works for large vectors.

"Sparse image" few non-zero pixels.

e.g.:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\vec{c}_1\ \vec{c}_2\ \vec{c}_3\ \vec{c}_4\ \vec{c}_5\ \vec{c}_6\ \vec{c}_7\ \vec{c}_8\ \vec{c}_9} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = \overset{\alpha}{\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}}.$$

A pixels $\vec{x}$ $\vec{y}$.

Only 2 non-zero's in $\vec{x}$.

Think of column-view of matrix multiplication.

Step (1):

Inner products:

$\langle \vec{y}, \vec{c}_1 \rangle = 3$ $\langle \vec{y}, \vec{c}_5 \rangle = 2$ $\langle \vec{y}, \vec{c}_9 \rangle = 1$

$\langle \vec{y}, \vec{c}_2 \rangle = 2$ $\langle \vec{y}, \vec{c}_6 \rangle = 3$

$\langle \vec{y}, \vec{c}_3 \rangle = 5$ $\langle \vec{y}, \vec{c}_7 \rangle = 1$

$\langle \vec{y}, \vec{c}_4 \rangle = 2$ $\langle \vec{y}, \vec{c}_8 \rangle = 1$

$\vec{c}_3$ has "most in common" "Greedy" strategy. $A = \vec{c}_3$

What part is explained by $\vec{c}_3$: $\vec{y}_1 = \vec{c}_3 (\vec{c}_3^T \vec{c}_3)^{-1} \cdot \vec{c}_3^T \cdot \vec{y} = \frac{5}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Residue / error: $\vec{e}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \frac{5}{4} = \begin{bmatrix} 3/4 \\ -1/4 \\ -1/4 \\ -1/4 \end{bmatrix}$

$\vec{y}$ compared to $\vec{c_1} \dots \vec{c_9}$

component in direction $\vec{c_3}$

$$\vec{y}_1 = \vec{c_3}\,(\vec{c_3}^{\,T}\vec{c_3})^{-1}\,\vec{c_3}^{\,T}\cdot\vec{y}$$

$$= \begin{bmatrix}1\\1\\1\\1\end{bmatrix}\left(\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\right)^{-1}\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}2\\1\\1\\1\end{bmatrix}$$

$$= \begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\frac{5}{4}$$

$$\vec{e} = \vec{y} - \vec{y}_1 = \begin{bmatrix}2\\1\\1\\1\end{bmatrix} - \frac{5}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \underbrace{\begin{bmatrix}3/4\\-1/4\\-1/4\\-1/4\end{bmatrix}}\ \vec{e}$$

$\langle \vec{e}, \vec{c_1} \rangle \dots$

$\frac{1}{2},\ -\frac{1}{2},\ 0,\ \frac{1}{2},\ -\frac{1}{2},\ \frac{3}{4},\ -\frac{1}{4},\ -\frac{1}{4},\ -\frac{1}{4}$

$\uparrow \vec{c_6}$

$$A_2 = \begin{bmatrix}| & | \\ \vec{c_3} & \vec{c_6} \\ | & |\end{bmatrix} \dots$$

$$\vec{y}_2 = A_2(A_2^T A_2)^{-1} A_2 \cdot \vec{y}$$