EE16 A

Prof. Gireeja Ranade

OH. Tu-Th 11-12 (212 Cory).

Reminders

- No Tech
- Read Notes
- Attend one discussion Mon + one Wed.
- HW 1 due Friday midnight
- H.W2 Released tomorrow.

Today:
- Matrix - vector multiplication.
- "Shorthand" for sys. of linear equations.
- Interpretations
- Linear independence + dependence.

Gaussian Elimination.

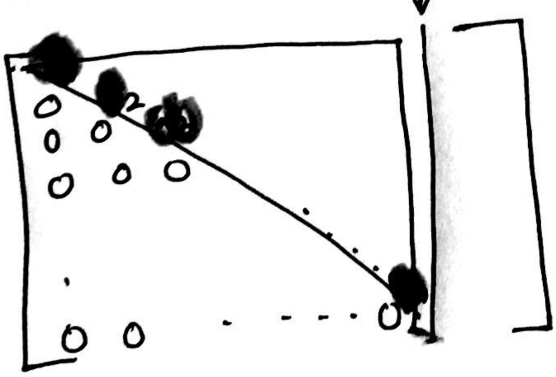
$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

$\longrightarrow$ Augmented Matrix.

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ & \ddots & & & b_2 \\ & & \ddots & & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} & b_m \end{array}\right]$$

m equations, n variables.

Linear combination: x, y $\quad 2x+3y$



α_0 : elements on the diagonal in upper triangular form.

Pivot

$\boxed{m=n}$ # of eqns = # of unknowns.

Stopping condition:

① Get a row $[0\ 0\ 0 \cdots 0 \mid b \neq 0]$

No solutions!

② Get a row $[0 \cdots\cdots 0 \mid 0]$

Infinite solutions.

③ Get rows ~~[0 0 0 0 ⋯ $a_{ii}=1$ stuff ⋯]~~

$[0\ 0\ 0\ 0\ \ 1\ c\ d \cdot e \mid b]$

$\uparrow$
(i,i)

Last row

$[0\ 0\ 0 \ \cdots\ 0\ \ 1 \mid b]$

$\Rightarrow$ Unique solution.

$y=1$
$x=1$

$m>n$.

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 2 & 5 \end{array}\right] \xrightarrow{R2-2(R1)} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 3 & 2 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 2 & 5 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 5 \\ 3 & 2 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 3 & 2 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 2 & 5 \\ 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{array}\right]$$

$n \times n$ case

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & \ddots & & \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix}$$

← Experimental design.

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \longrightarrow \text{measurement}$$

Matrix-vector multiplication.

$$\begin{bmatrix} [redacted] & a_{12} & \cdots & a_{1n} \\ [redacted] & a_{22} & & [redacted] \\ [redacted] & \cdots & & a_{nn} \\ & & & [redacted] \end{bmatrix} \begin{bmatrix} [redacted] \\ x_2 \\ \vdots \\ x_n \\ [redacted] \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$A \vec{x} = \vec{b}$$

arrow.

capital

Column : $\vec{a}_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{bmatrix}$ $\quad \vec{a}_2 = \begin{bmatrix} \\ \\ \end{bmatrix}$

$$A = \begin{bmatrix} 100 & 1 & 100 \\ 1 & 100 & 1 \\ 1 & 1 & 100 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$1 \cdot x_1 + 100 \cdot x_2 + 1 \cdot x_3 = 2$$

$\cancel{\text{if } x_2 = 0}$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad 1x_1 + 101 x_2 + 1 \cdot x_3 = 102$$

Scalar - vector multiplication.

$$c \cdot \vec{x} \overset{\text{def}}{=} c \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix}$$

Vector - vector addition.

$$\vec{a} + \vec{b} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \overset{\text{def}}{=} \begin{bmatrix} a_1+b_1 \\ a_2+b_2 \\ \vdots \\ a_n+b_n \end{bmatrix}$$

Commutative

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Associativity

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

Extra Credit

What happens if # of eqn's is 1 $m = 1$.

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_{1n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \qquad b_1 = a_{11} x_1 + a_{12} x_2 + \cdots a_{1n} x_n.$$

row vector

column vector.

Column-view of matrix-vector mult.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Claim: $A\vec{x} = \vec{a}_1 \cdot x_1 + \vec{a}_2 \cdot x_2 + \vec{a}_3 \cdot x_3$

Lin. comb of $\vec{a}_1, \vec{a}_2, \vec{a}_3$

Proof:

$$\vec{a}_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

$$x_1 \vec{a}_1 = \begin{bmatrix} x_1 a_{11} \\ x_1 a_{21} \\ x_1 a_{31} \end{bmatrix} \qquad x_2 \cdot \vec{a}_2 = \begin{bmatrix} x_2 \cdot a_{12} \\ x_2 \cdot a_{22} \\ x_2 \cdot a_{32} \end{bmatrix} \qquad x_3 \cdot \vec{a}_3 = \begin{bmatrix} x_3 \cdot a_{13} \\ x_3 \cdot a_{23} \\ x_3 \cdot a_{33} \end{bmatrix}$$

$$x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + x_3 a_{13} \\ x_1 a_{21} + x_2 a_{22} + x_3 a_{23} \\ x_1 a_{31} + x_2 a_{32} + x_3 a_{33} \end{bmatrix}$$

$$A \cdot \vec{x} =$$

Linear combination:

$$A\vec{x} = \vec{b}$$

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