EECS 16A

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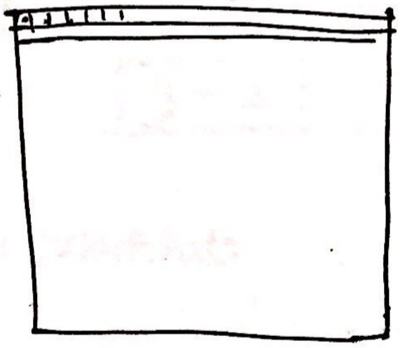
$$\boxed{\begin{array}{l} M1 \\ M2 \end{array}} \begin{array}{l} = (\alpha 1)\cdot(\text{Image}1) + (\alpha 2)(\text{Image}2) \\ = (\beta 1)(\text{Image}1) + (\beta 2)(\text{Image}2) \end{array}$$

Unknown

Measurements/
Data
Known

Known.



512×512

Vector.
Ordered list of elements.

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$n = 512 \times 512$

How do we solve?

16A Strategy: SIMPLIFY

$$2x + 3y = 8 \quad (E1)$$
$$3x - y = 1 \quad (E2)$$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 3 & -1 & 1 \end{array}\right]$$

"Augmented Matrix form"

Scanned with CamScanner

Solve systematically.

① Set the coefficient for x to be 1 "Normalize" (eq. 1.)

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 3 & -1 & 1 \end{array}\right]$$

$(E1)/2 \Rightarrow \quad x + \frac{3}{2}y = 4 \quad (E1^*)$

② Use $(E1^*)$ to eliminate x from $(E2)$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & -\frac{11}{2} & -11 \end{array}\right]$$

$(E2) - 3(E1^*)$

$\cancel{3x} - y - \cancel{3(x} + \frac{3}{2}y) = 1 - 3 \cdot 4 = -11$

$0x + -\frac{11y}{2} = -11$

③ Solve for y

$\boxed{y = 2}$

Upper triangular matrix.

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 1 & 2 \end{array}\right]$$

④ Back substitut

Gaussian Elimination!

Another representation:

Matrix-vector form.

$$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

Matrix Vector Vector.

Example 2: $\quad 2x+3y = 8$

$\qquad\qquad\quad 2x+3y = 6$

$$\left[\begin{array}{cc|c} 2 & 3 & 8 \\ 2 & 3 & 6 \end{array}\right]$$

$\rightarrow$ Normalize

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 2 & 3 & 6 \end{array}\right]$$ Divide R1 by 2

$\rightarrow$ Eliminate x from E2.

$R2 - 2(R1)$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 4 \\ 0 & 0 & \boxed{-2} \end{array}\right] \rightarrow 0x+0y=-2$$

non zero

Row of zeros on the left, and

non-zero on the right

$\Rightarrow$ NO SOLUTION.

Example 3: $\quad x+4y = 6$

$\qquad\qquad\quad 2x+8y = 12$

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 2 & 8 & 12 \end{array}\right]$$

(1) Normalize. ✓

(2) Eliminate x from E2.

$(R2) - 2(R1)$

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 0 & 0 & 0 \end{array}\right]$$

$\rightarrow$ Infinitely many solutions

- Gaussian Elimination

n variables ..
m equations.

- $$\left[\begin{array}{cccc|c} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} & \beta_1 \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} & \beta_2 \\ & & \vdots & & \vdots \\ \alpha_{m} & \alpha_{m2} & & \alpha_{mn} & \beta_m \end{array}\right]$$

$$\alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n = \beta_1$$
$$\alpha_{21}x_1 + \alpha_{22}x_2 + \quad + \alpha_{2n}x_n = \beta_2$$
$$\vdots$$
$$\alpha_{m1}x_1 + \alpha_{m2}x_2 \quad + \alpha_{mn}x_n = \beta_m$$

Coefficients "Known"

variables.

~~measur~~ measurement

Start with row $i = 1$

- Swap rows so you have equation with x_i in ith row.
- Coefficient of x_i in row i should be 1.
  → Multiply divide as necessary.
- For all rows $j = i+1$ to m
  Use the ith row to cancel x_i from row j
- Get upper triangular matrix
- Backsubstitute

Stopping conditions

① $[0 \ 0 \cdots 0 | 0]$
Infinite solutions

② $[0 \cdots\cdots 0 | b]$ $b \neq 0$
No solution.

③ $[0 \cdots\cdots 0 \ 1 | b]$
→ Unique solution.

Geometric perspective.

$2x + 3y = 8$ (E1) $\longrightarrow$ $x = 0 \Rightarrow y = 8/3 = 2.66$

$y = 0 \Rightarrow x = 4.$

$3x - y = 1.$ (E2)

$x = 0, \; y = -1$ $(0, -1)$

$y = 0, \; x = 1/3$

$(1/3, 0).$