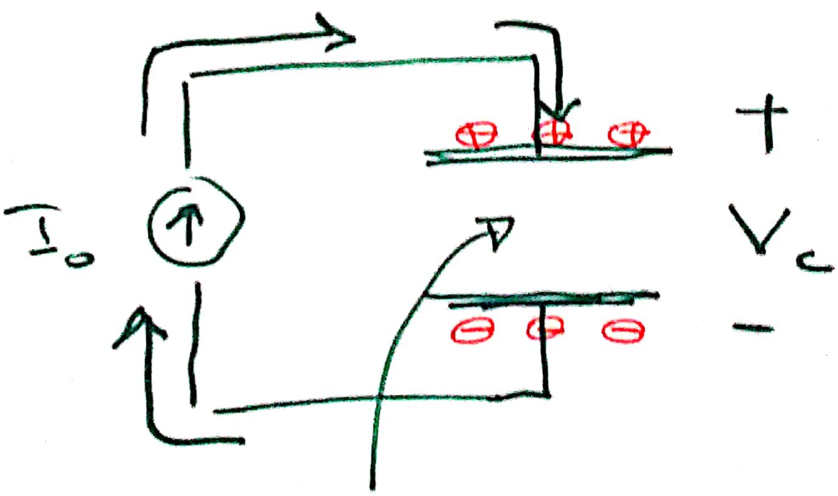
# EE 16 A, Lect 6, Mod 2

- Capacitance
  - Physics
  - Models (equations)
  - Circuits

## Virtual Experiment

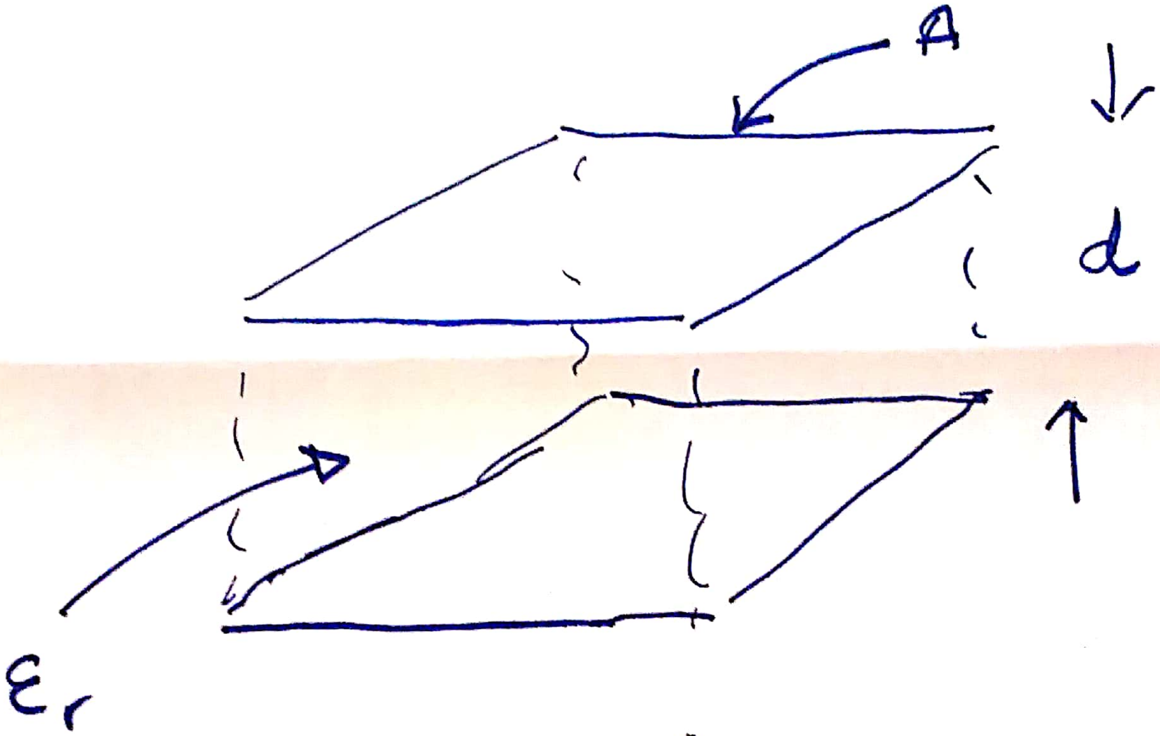



Voltage [V]

$$Q_c = C \cdot V_c$$

Capacitance
Farad [F]

Charge
Coulomb [C]




$$C = \varepsilon \cdot \frac{A}{d}$$

↑ permittivity

Material
(1) Vacuum, Air
$\varepsilon_r = 1$
(2) water
$\varepsilon_r \cong 80$

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$$

$\varepsilon_0$ = $8.85 \cdot 10^{-12}$ F/m

8.85 pF/m

$p = 10^{-12}$

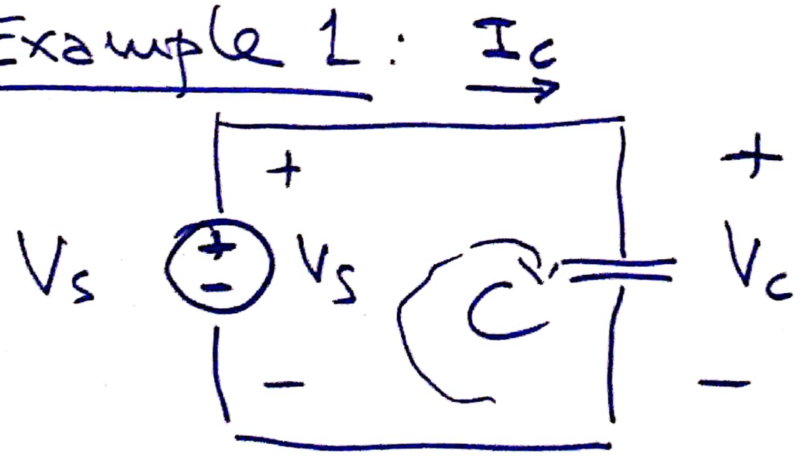
# I/V characteristics ?

$$Q = C \cdot V$$

$$I = \frac{dQ}{dt} = C \cdot \frac{dV}{dt}$$

$$I(t) = C \cdot \frac{dV(t)}{dt}$$

## Example 1:




steady state

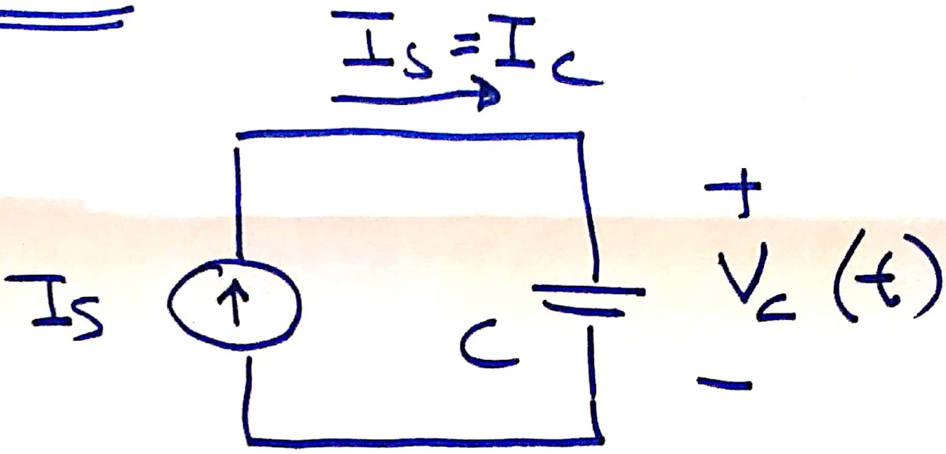
$$\frac{dV}{dt} = \frac{dI}{dt} = \emptyset$$

KVL : $-V_s + V_c = \emptyset$

$$V_c = V_s = \text{const}$$

$$I_c = C \frac{dV_c}{dt} \to \emptyset = \emptyset$$

## Ex 2:



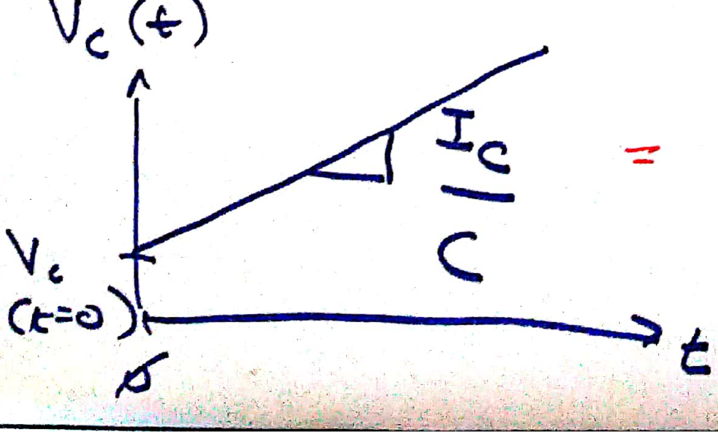

$$I_c = I_s = C \cdot \frac{dV_c(t)}{dt}$$

$$\therefore \frac{dV_c(t)}{dt} = \frac{I_c}{C}$$

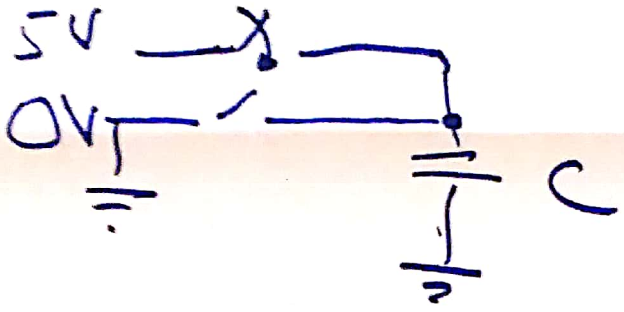
$$\int_0^T dV_c(t) = \int_0^T \frac{I_c}{C} \cdot dt$$

$$V_c(t) = \frac{I_c}{C} \int_0^T dt$$

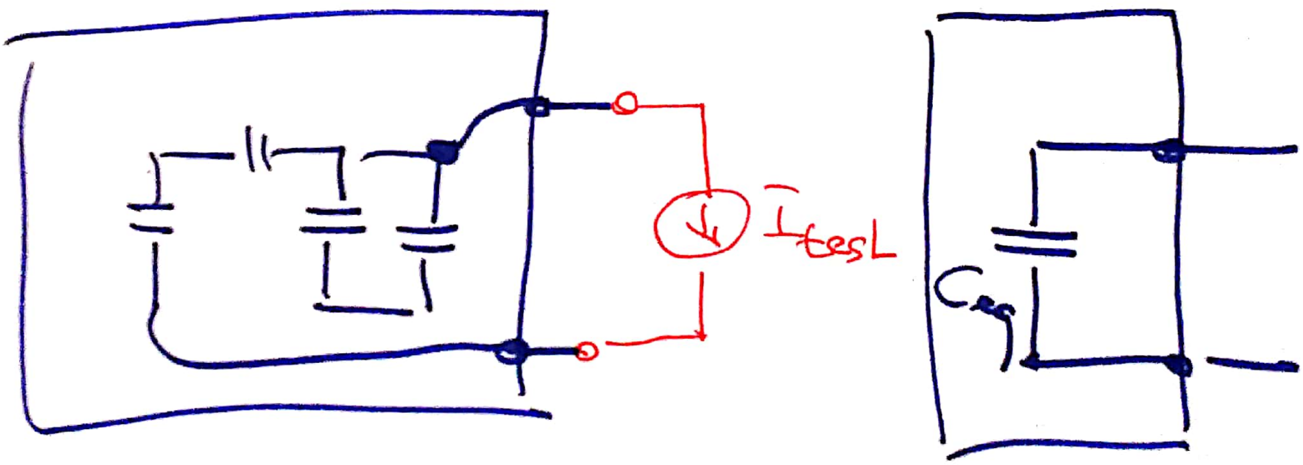
$$= \frac{I_c}{C} \cdot T + V_c(t=\varnothing)$$



App: Memory




## Equivalent Capacitance

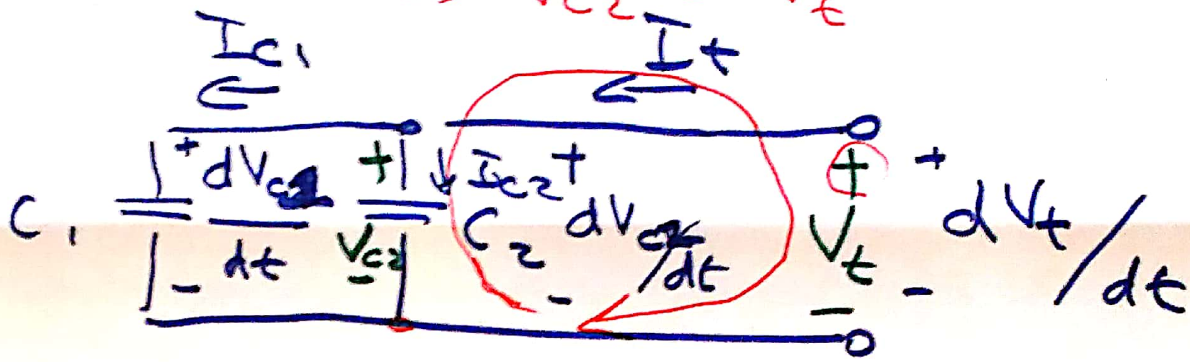



$$\frac{dV_{test}}{dt} = 0 \Rightarrow I = \emptyset$$

**Ex:**

KVL $-V_{c2} + V_t = \phi$

$\Rightarrow V_{c2} = V_t$



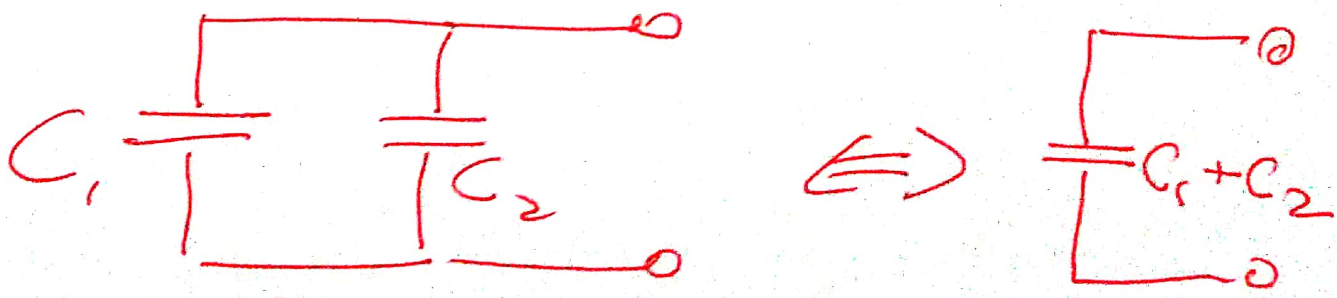

$$I_{c1} = C_1 \cdot \frac{dV_{c1}}{dt} = C_1 \cdot \frac{dV_t}{dt}$$

$$I_{c2} = C_2 \cdot \frac{dV_t}{dt}$$

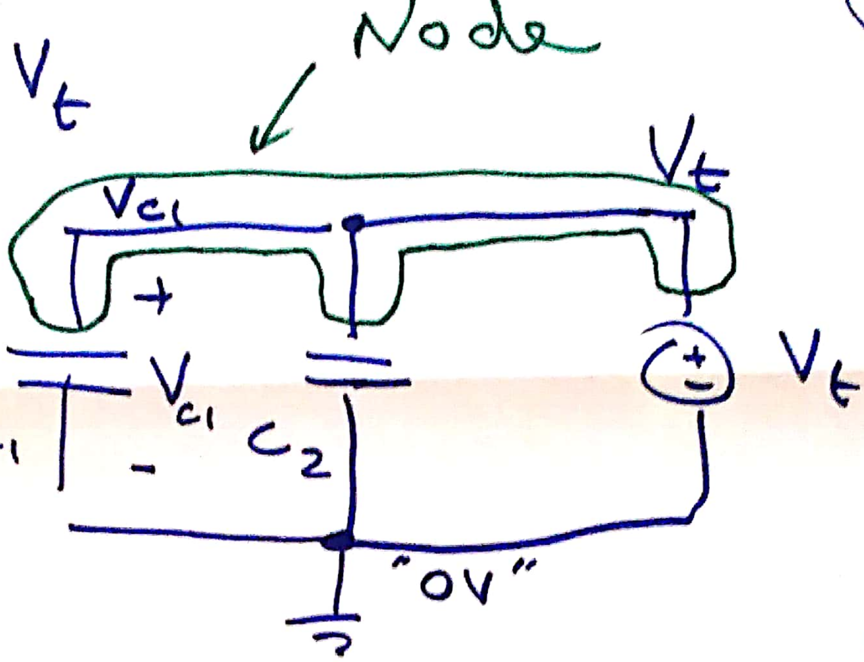
$$I_{c1} + I_{c2} = (C_1 + C_2) \cdot \frac{dV_t}{dt}$$

$I_t$

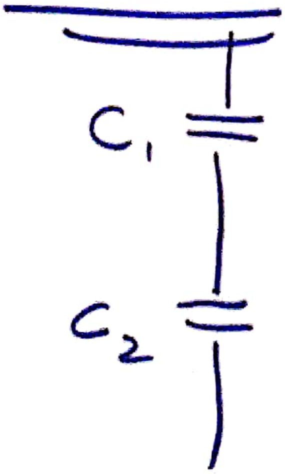
$$C_{eq} = \frac{I_{c1} + I_{c2}}{\frac{dV_t}{dt}} = C_1 + C_2$$



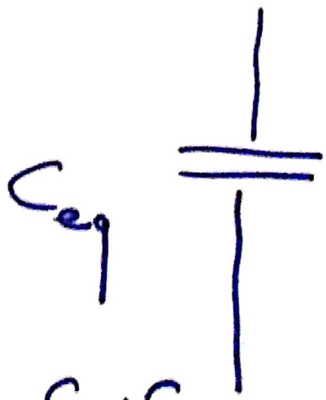
$V_{c1} = V_t$




Series:



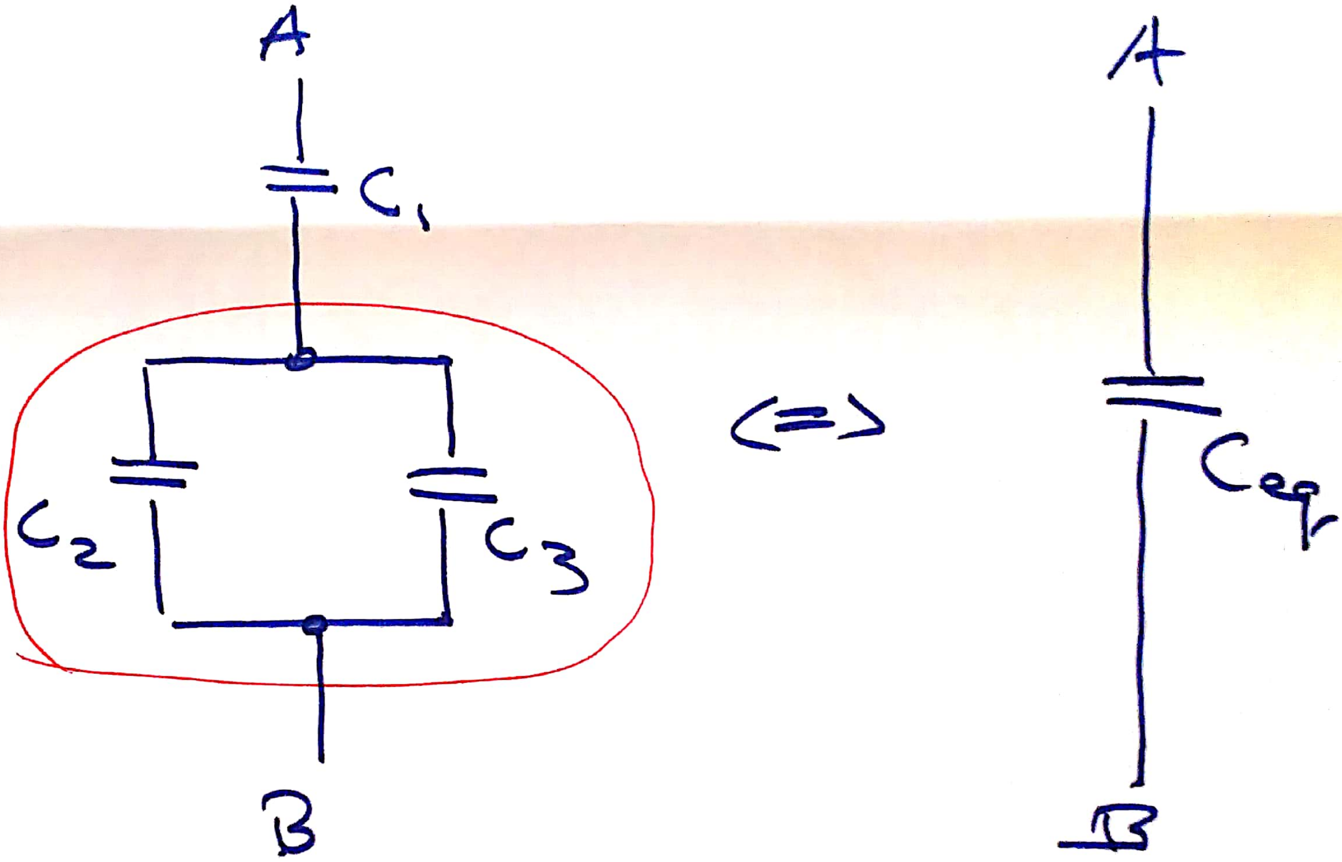

$\Longleftrightarrow$



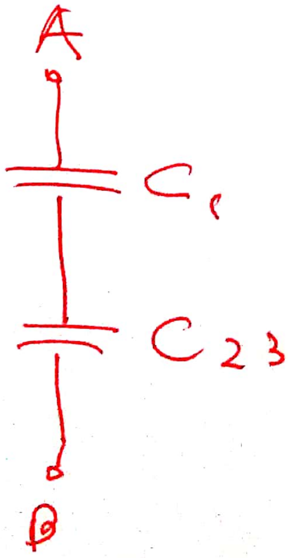

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$= C_1 // C_2$$

**Ex :**

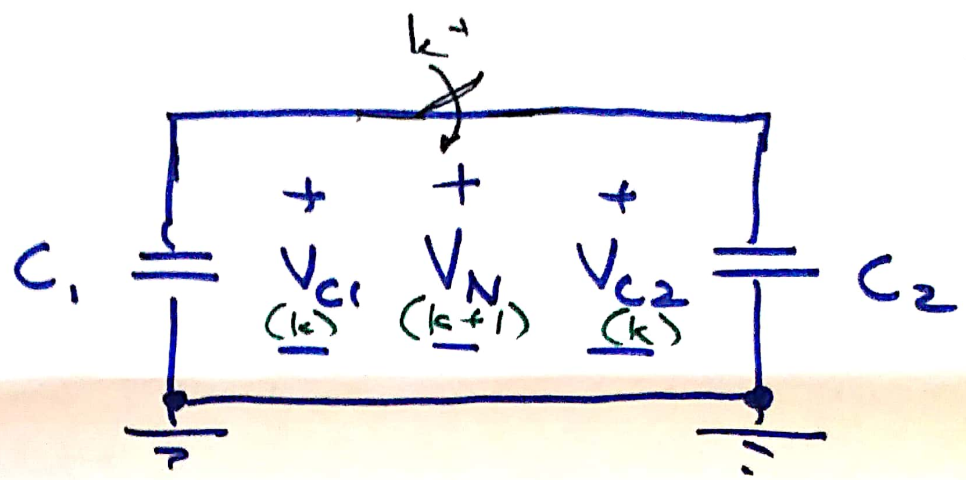



$$C_2 + C_3 = C_{23}$$




$$C_{eq} = C_1 \,//\, (C_2 + C_3)$$

$$= \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3}$$




$$Q_1(k) = C_1 \cdot V_{c1}(k)$$

$$Q_2(k) = C_2 \cdot V_{c2}(k)$$

$$Q_T(k+1) = Q_1(k) + Q_2(k)$$

$$= C_1 \cdot V_{c1}(k) + C_2 \cdot V_{c2}(k)$$

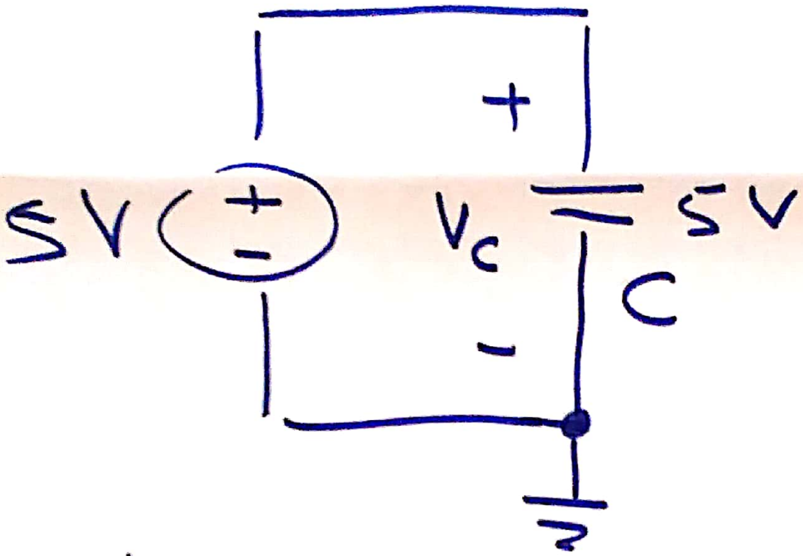
$$= V_N(k+1) \cdot (C_1 + C_2)$$

$$\parallel$$

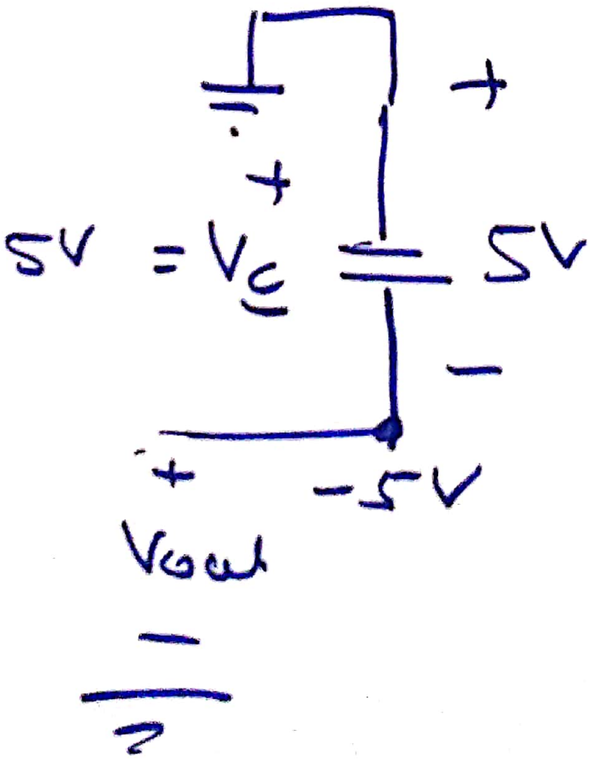
$$V_{c1}(k+1) = V_{c2}(k+1)$$

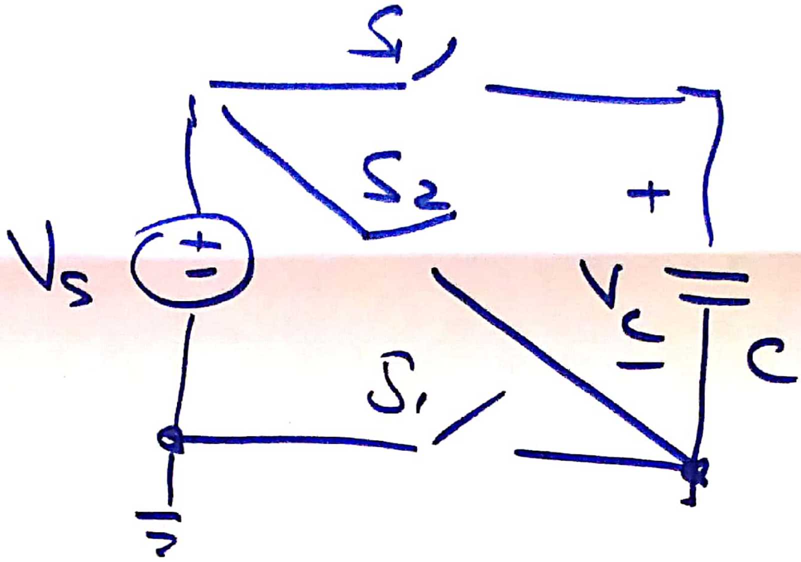
$$\therefore \quad V_N(k+1) = \frac{C_1 \cdot V_{c1}(k) + C_2 \cdot V_{c2}(k)}{C_1 + C_2}$$

## phase 1




## phase 2






+

$$V_{out}(k+1) = V_S + V_C = 2 \cdot V_S.$$