EECS 16A: Module 3: Machine Learning.

Nov 12, 2019.

Module 1: Systems + modeling.
- Segway.
- PageRank
- Quadcopter

Module 2: Analysis + Design.
- Touchscreen
- LIDAR.

Machine Learning

Problems
- Classification.
- Estimation
- Prediction.
- Clustering

Techniques to solve:
- Model.
- Optimization.
  - ↳ Optimize an error metric.

Today:
- Classification.
- Intro GPS problem
- Inner Product
- Cauchy Schwartz inequality.
- Satellite classification.

- Design problem: GPS system
  - 24 satellites.
  -

Things to figure out

- Distance between satellites.
- — " ——— you + satellite.
- How many satellites are enough?
- Location of the Satellites.
- Information processing to compute distance
- Access data sent from satellite.
- Synchronize time across satellite + phone.
- Distances ⟷ Position
- Signalling to satellites
- How accurate is the data
  - ↳ How to deal with noise

- Which satellite am I talking to?

Inner Product:

$\vec{v}, \vec{w}$ be two vectors. $\in \mathbb{R}^n$ $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$

Def: Inner prod. $\langle \vec{v}, \vec{w} \rangle = \vec{v}^T \cdot \vec{w}$

$$= [v_1 \; v_2 \cdots v_n] \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

Dot product

Correlation

$$= v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$

$$= \sum_{i=1}^{n} v_i w_i$$

$$\langle \vec{w}, \vec{v} \rangle = \langle \vec{v}, \vec{w} \rangle.$$

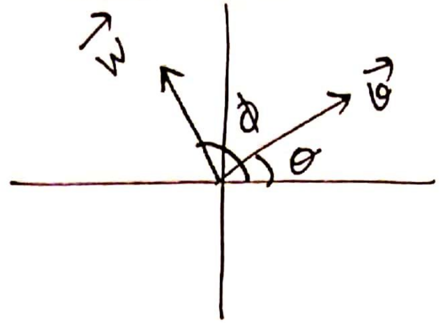
$$\langle \vec{v}, \vec{v} \rangle = \sum_{i=1}^{n} v_i^2 = \|\vec{v}\|^2$$ Magnitude / norm.

$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\langle \vec{v}, \vec{w} \rangle = 2$

$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\langle \vec{v}, \vec{w} \rangle = 0$ "Orthogonal"

Consider general 2D vectors.

$$\vec{v} = \alpha \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad \vec{w} = \beta \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$



$$\langle \vec{v}, \vec{w} \rangle = (\alpha\cos\theta)(\beta\cos\phi) + (\alpha\sin\theta)(\beta\sin\phi)$$

$$= \alpha\cdot\beta\cdot(\cos\theta\cos\phi + \sin\theta\cdot\sin\phi)$$

$$= \alpha\beta\cdot\cos(\theta-\phi). \quad \text{Trig identity.}$$

$$= \|\vec{v}\|\cdot\|\vec{w}\|\cdot\cos(\theta-\phi) \le \|\vec{v}\|\cdot\|\vec{w}\|\cdot 1$$

$\|\vec{v}\| = \alpha$

$\|\vec{w}\| = \beta.$

$\cos(\theta-\phi) \le 1$

Inner product between $\langle \vec{v}, \vec{w} \rangle$ is maximized when $\theta = \phi$.

If $\theta - \phi = 90^\circ$, $\cos(\theta-\phi) = 0$, then $\langle \vec{v}, \vec{w} \rangle = 0$ "Orthogonal"

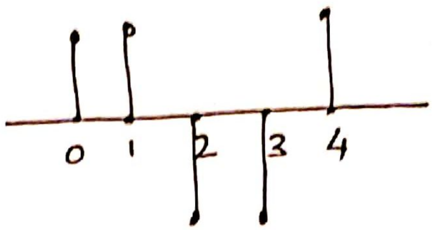
~~Sat~~ State : Cauchy Schwartz inequality.

$$\langle \vec{v}, \vec{w} \rangle \le \|\vec{v}\|\cdot\|\vec{w}\|$$

• Satellite Classification

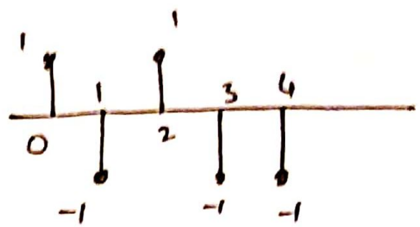
A 

$$\vec{s}_A = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$



(B)

$$\vec{s}_B = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$



$$\underbrace{\|\vec{s}_A\|^2 = \|\vec{s}_B\|^2}_{\text{fixed}}$$

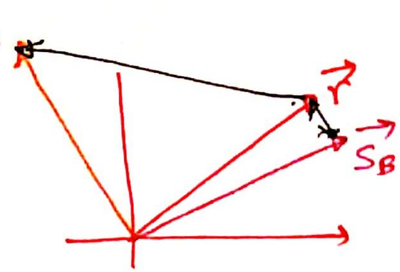
Signatures "Gold codes"

$$\underset{\text{sent}}{\vec{s}} + \underset{\text{noise}}{\vec{n}} = \underset{\text{received.}}{\vec{r}}$$

$$\vec{r} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad \vec{r} = \begin{bmatrix} 0.9 \\ 1.1 \\ -1.2 \\ 0 \\ 1 \end{bmatrix}$$

Classify in the presence of noise.

• $\vec{r}$ Find which signature, $\vec{r}$ is closest to?



$\boxed{\text{Choose error metric}}$

$\vec{e_A} = \vec{r} - \vec{s_A}$, $\vec{e_B} = \vec{r} - \vec{s_B}$.

Want! Find that satellite such that ~~the~~ the error is minimized.

$\|\vec{e}\|^2$

$\boxed{\text{minimize } \|\vec{e}\|^2 \text{ over all satellites}}$

Optimization problem.

$$\|\vec{e}\|^2 = \langle \vec{e}, \vec{e} \rangle = \vec{e}^T \cdot \vec{e}$$
$$= (\vec{r} - \vec{s})^T (\vec{r} - \vec{s})$$
$$= (\vec{r}^T - \vec{s}^T)(\vec{r} - \vec{s})$$
$$= \vec{r}^T \cdot \vec{r} + \vec{s}^T \vec{s} - \vec{s}^T \cdot \vec{r} - \vec{r}^T \cdot \vec{s}$$
$$= \underbrace{\|\vec{r}\|^2}_{\text{fixed.}} + \underbrace{\|\vec{s}\|^2}_{\text{fixed}} - 2\langle \vec{r}, \vec{s} \rangle$$

minimize $\|\vec{e}\|^2$

$\Leftrightarrow$ minimize $-2\langle \vec{r}, \vec{s} \rangle$

$\Leftrightarrow$ maximize $\langle \vec{r}, \vec{s} \rangle$

- <u>Design</u> <u>algorithm:</u>

For all satellites $\vec{s_i}$

Compute $\langle \vec{r}, \vec{s_i} \rangle$.

Return index i of satellite that maximizes $\langle \vec{r}, \vec{s_i} \rangle$.