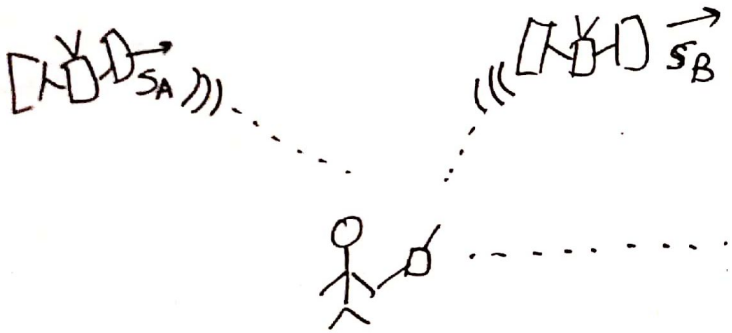
Module 3 Lecture 2

Nov 14, 2019

- Introduction to Research

- Which Satellite is transmitting?
  - Classification
  - Estimation of delay.
    - ↳ Inner product.
      - ↳ "Max" correlation



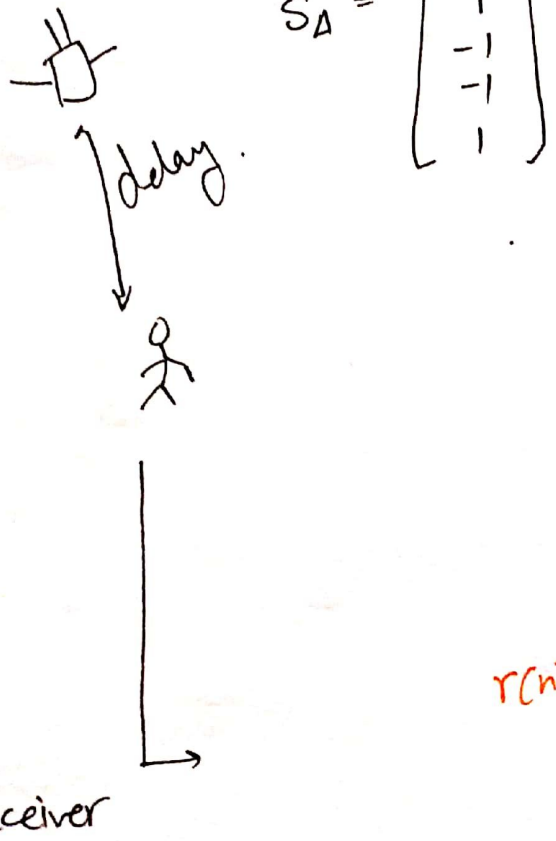
Superposition!

$\vec{r} = \vec{s}_A + \vec{s}_B + \vec{n}$ ← noise vector

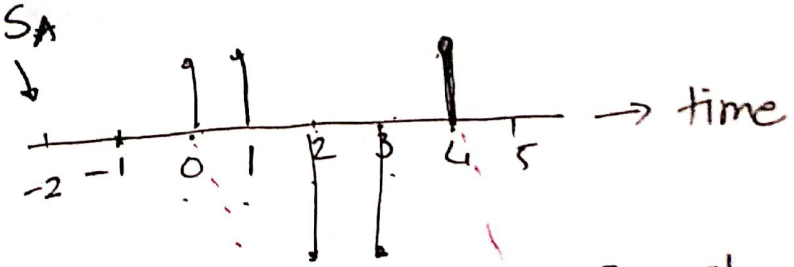
$$\langle \vec{r}, \vec{s}_A \rangle$$
$$= \langle \vec{s}_A + \vec{s}_B + \vec{n}, \vec{s}_A \rangle$$
$$= \vec{s}_A^T \vec{s}_A + \vec{s}_B^T \cdot \vec{s}_A + \vec{n}^T \cdot \vec{s}_A$$
$$= \langle \vec{s}_A, \vec{s}_A \rangle + \langle \vec{s}_B, \vec{s}_A \rangle + \langle \vec{n}, \vec{s}_A \rangle$$

LARGE SMALL small

How to Estimate Delay



$$\vec{S_A} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$



$S_A[-1] = 0$

$S_A[0] = 1$

$S_A[1] = 1$

$S_A[2] = -1$

$S_A[3] = -1$

$S_A[4] = 1$

$S_A[5] = 0$

$S_A[n]$ "signal"

delayed by 3

$r[n]$

$r[0] = 0$

$r[1] = 0$

$r[2] = 0$

$r[3] = 1 = S_A(0)$

$r[4] = 1 = S(1)$ $r[8] = 0$

$r[5] = -1 = S(2)$

$r[6] = -1 = S(3)$

$r[7] = 1 = S(4)$

$r[n] = s[n-3]$

- Correlation / Cross-correlation.

Define: $\text{corr}_r(S_A)[k] = \sum_{i=-\infty}^{\infty} r[i]\, S_A[i-k]$

$k=0$

$$\text{corr}_r(S_A)(0) = \sum_{i=-\infty}^{\infty} r[i]\, S_A[i-0] = \sum_{i=-\infty}^{\infty} r[i] \cdot S_A[i]$$

$$= r(3)\cdot s(3) + r(4)s(4) + r(5)s(5) + r(6)s(6) + r(7)s(7) + 0+0+0\cdots$$

$$= 1(-1) + 1\cdot 1 + (-1)\cdot 0 + (-1)(0) + (1)\cdot 0 + 0\cdots$$

$$= -1 + 1 = 0.$$

$$\text{corr}_r(S_A)[1] = \sum_{i=-\infty}^{\infty} r[i]\, S_A[i-1]$$

$$= r(3)\cdot S_A(2) + r(4)\cdot S_A(3) + r(5)\cdot S_A(4) + r(6)\cdot S_A(5) + r(7)\cdot S_A(6)$$

$$= 1(-1) + 1(-1) + (-1)(1) + (-1)(0) + 0$$

$$= -3$$

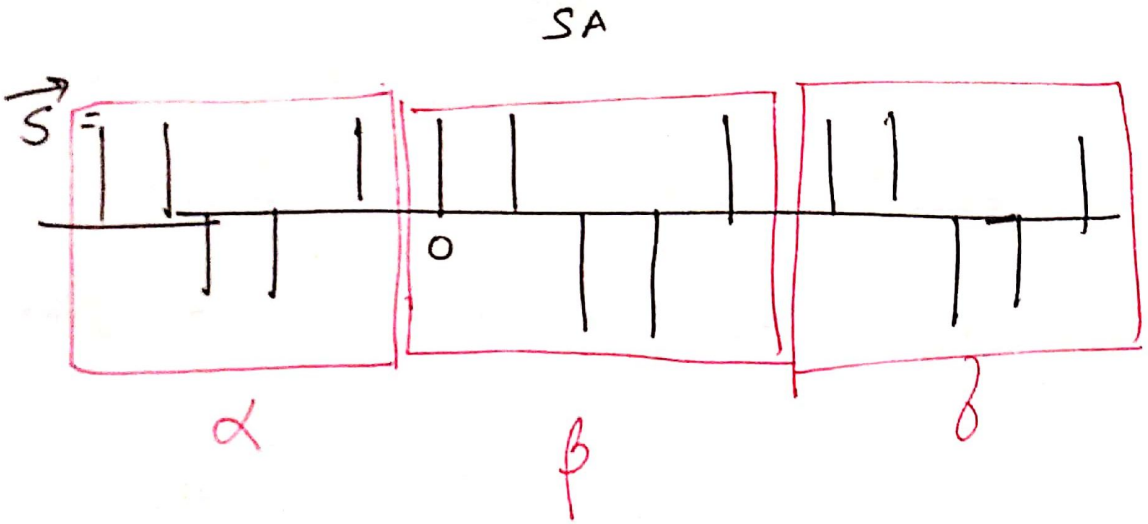
Cross correlation: $\text{Corr}_r(S_A)[k] = \sum_{i=-\infty}^{\infty} r(i)\, S_A(i-k)$

$$\text{Corr}_r(S_A)[3] = \sum_{i=-\infty}^{\infty} r[i]\, S_A[i-3].$$

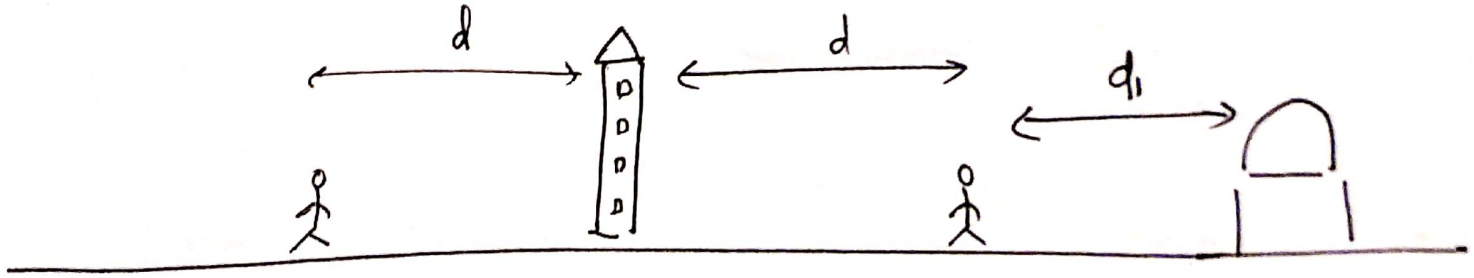
$$= r(3)s(0) + r(4)s(1) + r(5)s(2) + r(6)s(3) + r(7)s(4)$$

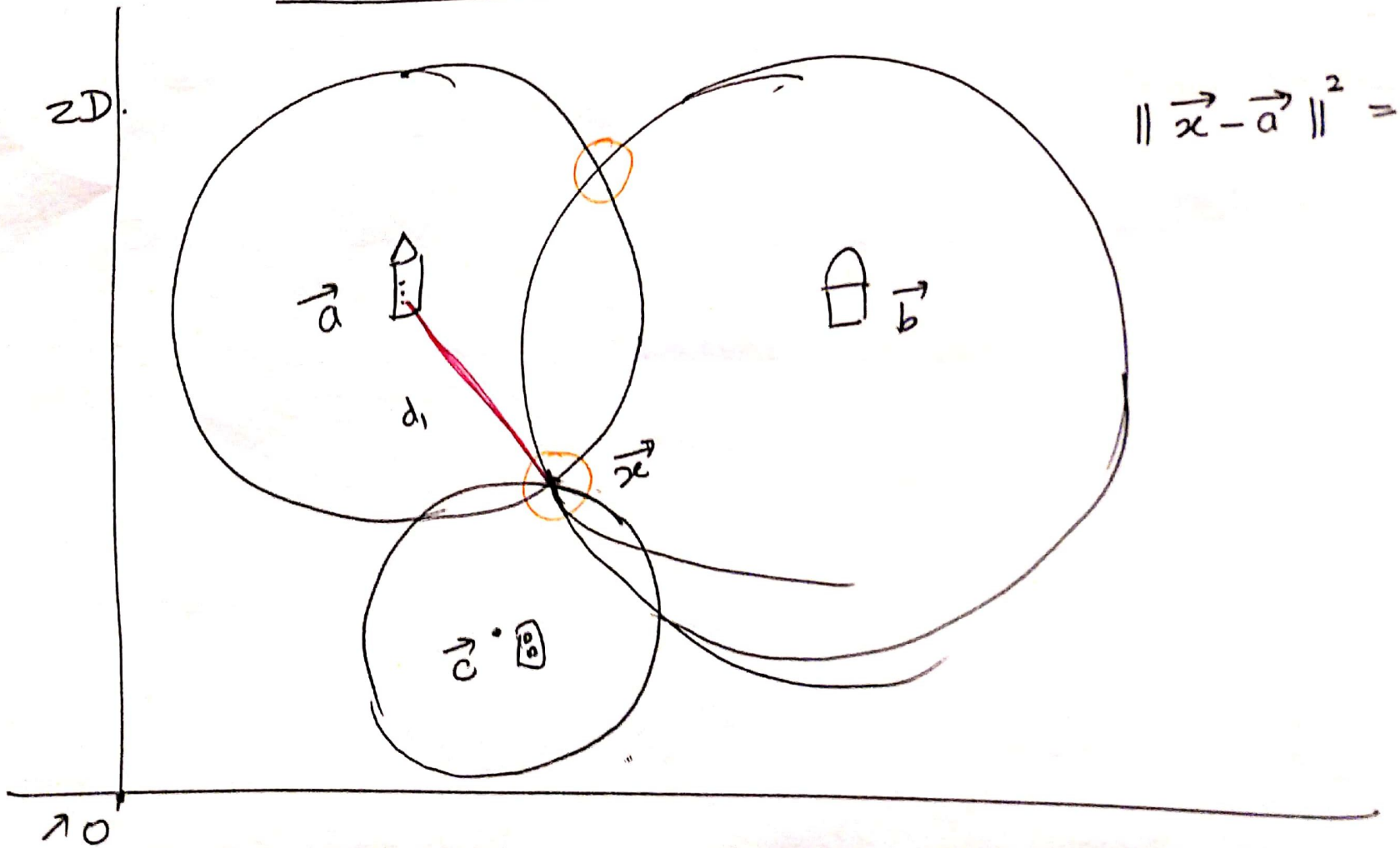
$$= 5$$

$\Rightarrow$.



How do we go from distances to position?





$$\| \vec{x} - \vec{a} \|^2 = d_1^2$$