EECS 16A . Module 3 - Lecture 6 **Reminder:** HW14

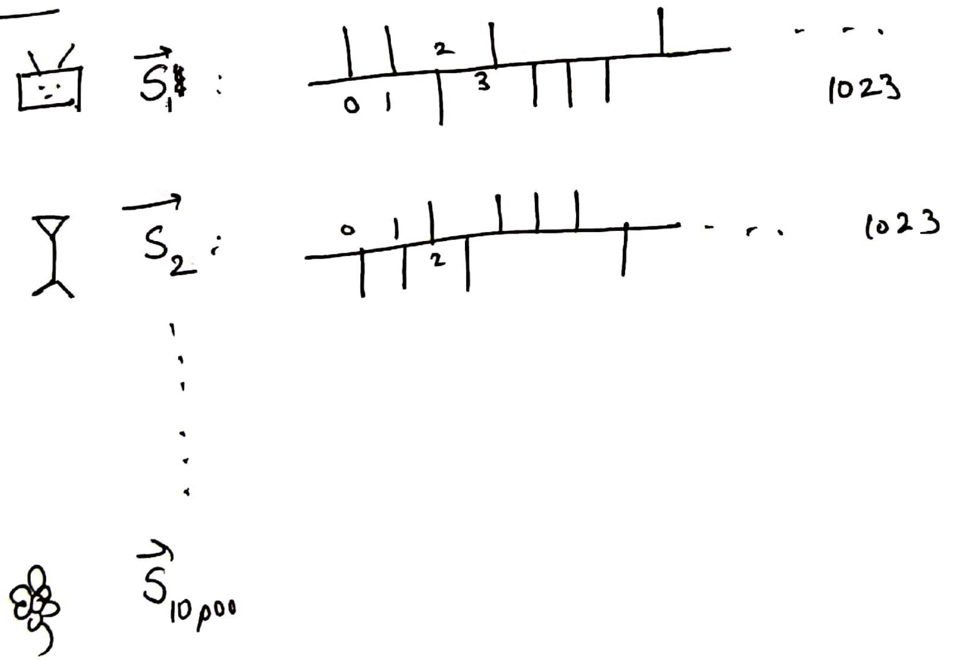
Dec 3, 2019

Today:

- Orthogonal Matching Pursuit (OMP)

Internet of Things

Gold Codes 1023 length.



$\langle \vec{S}_1, \vec{S}_1 \rangle$ = large

$\langle \vec{S}_1, \vec{S}_2 \rangle$ = small.

$\vec{r} = \alpha_1 \vec{S_1} + \alpha_2 \vec{S_2} + \cdots + \alpha_{10,000} \vec{S}_{10,000}$ 10,000 = N

$\boxed{\text{HOME BOX}}$

Unknowns: $\alpha_1, \alpha_2, \cdots, \alpha_{10,000}$

Use the structure of the signals to reduce number of measurements.

$$1023 \left\{ \underbrace{\begin{bmatrix} | & | & & | \\ \vec{S_1} & \vec{S_2} & \cdots & \vec{S_N} \\ | & | & & | \end{bmatrix}}_{10,000} \right. \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{10,000} \end{bmatrix} = \begin{bmatrix} \vec{r} \end{bmatrix} \Big\} 1023$$

Matrix A

$\uparrow$ 10,000

# of eqn: 1023.

# of unknowns: 10,000.

~~S^TS~~ → not be invertible.

A^TA

$S \vec{\alpha} = \vec{r}$

Cannot invert S. ??

$$\langle \vec{S_1}, \vec{r} \rangle \to \text{if } \alpha_1 = 0 \text{ is small } \langle \vec{S_1}, \vec{r} \rangle = \langle \vec{S_1}, \alpha_2 \vec{S_2} + \cdots \rangle$$

$$\to \text{if } \alpha_1 \neq 0 \quad \langle \vec{S_1}, \alpha_1 \vec{S_1} + \alpha_2 \vec{S_2} + \cdots \alpha_{10,000} \vec{S}_{10,000} \rangle$$

is large

$$= \underbrace{\alpha_1 \cdot \langle \vec{S_1}, \vec{S_1} \rangle}_{\text{large}} + \underbrace{\alpha_2 \langle \vec{S_1}, \vec{S_2} \rangle + \cdots}_{\text{small}}$$

- **Consider:**

$$\left.\begin{array}{l}\langle \vec{r}, \vec{s_1} \rangle \\ \langle \vec{r}, \vec{s_2} \rangle \cdots \\ \vdots \\ \langle \vec{r}, \vec{s}_{10,000} \rangle \end{array}\right\}$$

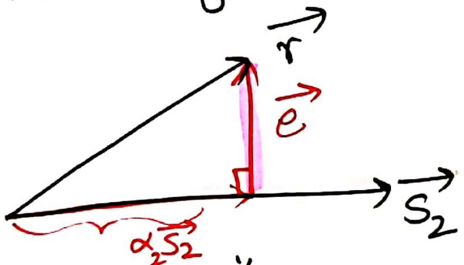
Find that signature with max inner product.
Declare the device is present.

Pretend $\langle \vec{r}, \vec{s_2} \rangle$ was the largest.

Declare device 2 is present. $\vec{s_2} \cdot \alpha_2 \approx \vec{r}$

Question: How much of $\vec{r}$ is explained by $\vec{s_2}$?

$\vec{r} \approx \alpha_2 \vec{s_2} + \cdots$



"Projections"

Project $\vec{r}$ onto $\vec{s_2}$.

$$\hat{\alpha}_2 = \frac{\langle \vec{r}, \vec{s_2} \rangle}{\| \vec{s_2} \|^2}$$

What's left: $\vec{e} = \vec{r} - \hat{\alpha}_2 \vec{s_2}$

"error/ residual"

Now:
Consider inner prods. $\langle \vec{e}, \vec{s_1} \rangle, \ldots \langle \vec{e}, \vec{s}_{10,000} \rangle$

- $\vec{e} = \vec{r} - \hat{\alpha}_2 \cdot \vec{s_2}$

$$\langle \vec{e}, \vec{s_2} \rangle = \langle \vec{r} - \hat{\alpha}_2 \vec{s_2}, \vec{s_2} \rangle$$

$$= \langle \vec{r}, \vec{s_2} \rangle - \hat{\alpha}_2 \cdot \langle \vec{s_2}, \vec{s_2} \rangle$$

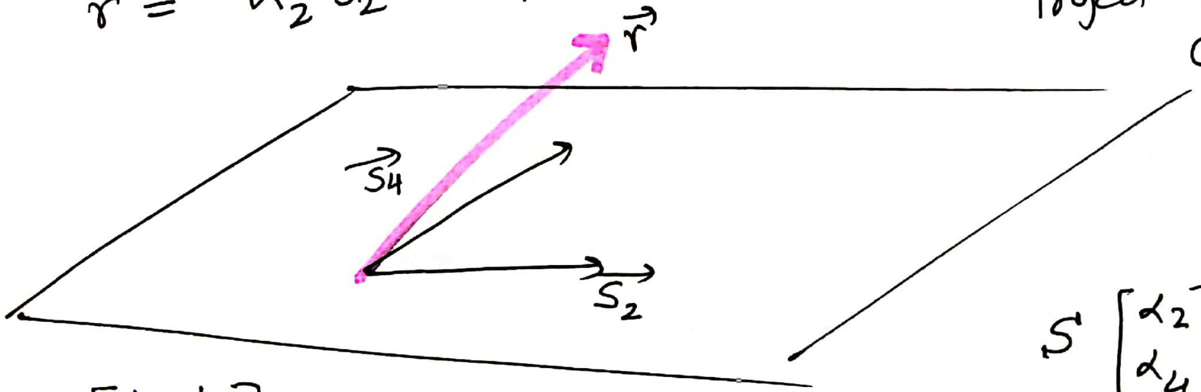
$$= \langle \vec{r}, \vec{s_2} \rangle - \frac{\langle \vec{r}, \vec{s_2} \rangle}{\| \vec{s_2} \|^2} \cdot \| \vec{s_2} \|^2$$

$$= 0$$

Say: Next max was $\vec{s_4}$ — Device 4 is present.

$$\vec{r} = \alpha_2 \vec{s_2} + \alpha_4 \vec{s_4} + \cdots$$

Project $\vec{r}$ onto Col $\{\vec{s_2}, \vec{s_4}\}$.



$$S \begin{bmatrix} \alpha_2 \\ \alpha_4 \end{bmatrix} \approx \vec{r}$$

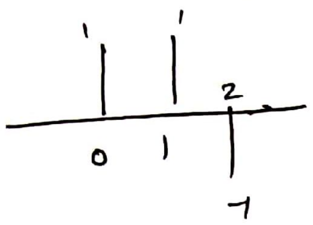
$$S \vec{\alpha} = \vec{r}$$

$$S = \begin{bmatrix} | & | \\ \vec{s_2} & \vec{s_4} \\ | & | \end{bmatrix}$$

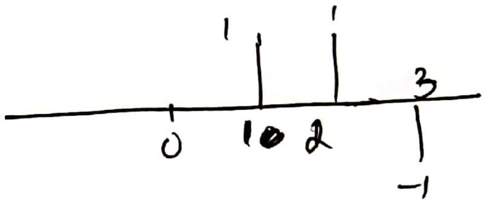
Least-squares / Projection: $\hat{\vec{\alpha}} = (S^T S)^{-1} S^T \cdot \vec{r}$

- Delays in transmission.

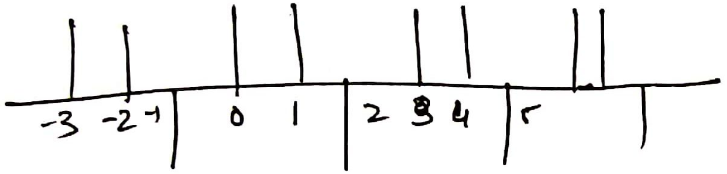
$\vec{s}_1[n]$

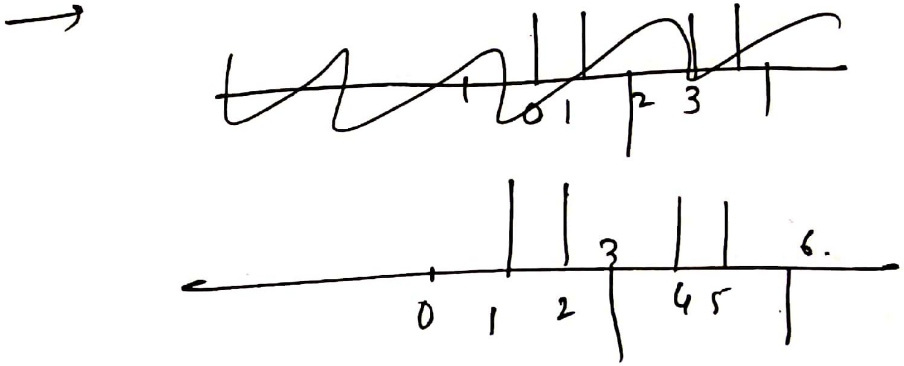


$\vec{s}_1[n]$ delayed by 1.



$\vec{s}_1[n]$





$\langle \vec{s}_1, \vec{s}_1 \rangle$ = large

$\langle \vec{s}_1, \vec{s}_2 \rangle$ = small

$\langle \vec{s}_1$, delayed $\vec{s}_1 \rangle$ = small

$\vec{s}_1^{\,(1)} = \vec{s}_1$ delayed by 1.

$\langle \vec{r}, \vec{s}_1 \rangle$ $\langle \vec{r}, \vec{s}_1^{\,(1)} \rangle$ $\langle \vec{r}, \vec{s}_1^{\,(2)} \rangle$, $\langle \vec{r}, \vec{s}_1^{\,(3)} \rangle$ . . . . . .

$\langle \vec{r}, \vec{s}_2 \rangle$ $\langle \vec{r}, \vec{s}_2^{\,(1)} \rangle$ $\langle \vec{r}, \vec{s}_2^{\,(2)} \rangle$, ...