Module 3 - Lecture 7.

Dec 6, 2019

. Today:

- OMP for Imaging.
- Validation.

Say you have image : 100 x 100 pixels. $\rightarrow$ Module 1: 10^4 measurements.

Can we do with less?

What if: we knew our image is SPARSE.

Most pixels are black (ie. 0).

Represent image $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{10,000} \end{bmatrix}$

$$\begin{array}{c} 4 \end{array} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\ \begin{array}{cccccccccc} \vec{c_1} & \vec{c_2} & \vec{c_3} & \vec{c_4} & \vec{c_5} & \vec{c_6} & \vec{c_7} & \vec{c_8} & \vec{c_9} & \vec{c_{10}} \end{array} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}}_{\vec{r}}$$

Only 2 of the 10 pixels are non-zero.

$$\vec{c_1} x_1 + \vec{c_2} x_2 + \cdots \vec{c_3} x_3 + \cdots + \vec{c_{10}} x_{10} = \vec{r}$$

The OMP algorithm

$\vec{r}$: received signal $\vec{r} = \alpha_1 \vec{s}_1 + \alpha_2 \vec{s}_2 + \ldots + \alpha_N \vec{s}_N$

$\vec{s}_i$: Candidate signatures of transmitting devices

Millions of these.

$$1023 \; \underbrace{\left[\begin{array}{cccc} | & | & & | \\ \vec{s}_1 & \vec{s}_2 & \cdots & \vec{s}_N \\ | & | & & | \end{array}\right]}_{1M} \left[\; \vec{\alpha} \;\right] = \left[\; \vec{r} \;\right]$$

1M unknowns.

<u>Initialize:</u> $S = [\;]$ empty matrix, we don't know any devices yet

$\vec{e} = \vec{r}$ The entire signal is unexplained.

① Find signature with max correlation.

$$\vec{s}_{max} = \underset{\vec{s}_i}{\operatorname{argmax}} \langle \vec{e}, \vec{s}_i \rangle$$ In the first round $\vec{e} = \vec{r}$

② Add $\vec{s}_{max}$ to the set of signatures present. ie. add it as a column in S.

$$S = S_{update} = \left[\begin{array}{c:c} S_{old} & \vec{s}_{max} \end{array}\right]$$ add new column.

Project $\vec{r}$ onto S $\quad \hat{\vec{\alpha}} = (S^T S)^{-1} S^T \vec{r}$

Update $\vec{e} = \vec{r} - S\hat{\vec{\alpha}}$

Go back to ①.

① Take inner products.

$\langle \vec{r}, \vec{c_5} \rangle = 3$, $\qquad \langle \vec{r}, \vec{c_7} \rangle = 3$.

$\vec{c_5}$ is present $\qquad \vec{c_5} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad S = \begin{bmatrix} \vec{c_5} \\ 1 \end{bmatrix}$.

Now! Project $\vec{r}$ onto $\vec{c_5}$

$$\hat{x}_5 = (\vec{c_5}^T \vec{c_5})^{-1} \vec{c_5}^T \cdot \vec{r}$$

$$= \left(\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \left(\frac{1}{2}\right) \cdot 3 = 1.5$$

Remove $1.5 \cdot \vec{c_5}$ from $\vec{r}$ $\qquad \vec{e} = \vec{r_1} = \vec{r} - 1.5 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1.5 \\ 1.5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -.5 \\ .5 \\ 0 \\ 1 \end{bmatrix}$

② Consider: $\langle \vec{r_1}, \vec{c_1} \rangle, \langle \vec{r_1}, \vec{c_2} \rangle \ldots \langle \vec{r_1}, \vec{c_{10}} \rangle$

$\vec{c_7}$ has maximum inner product.

$$S = \begin{bmatrix} \vec{c_5} & \vec{c_7} \end{bmatrix}$$

Explain $\vec{r}$ using both $\vec{c_5}$ and $\vec{c_7}$

$$S = \begin{bmatrix} \vec{c_5} & \vec{c_7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Project $\vec{r}$ onto S

$$\begin{bmatrix} \hat{x}_5 \\ \hat{x}_7 \end{bmatrix} = (S^T S)^{-1} S^T \cdot \vec{r}$$

$$= \left[\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

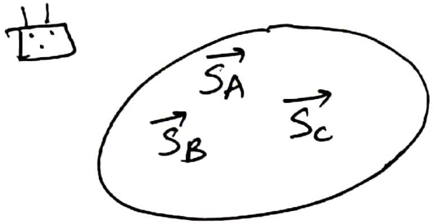
$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

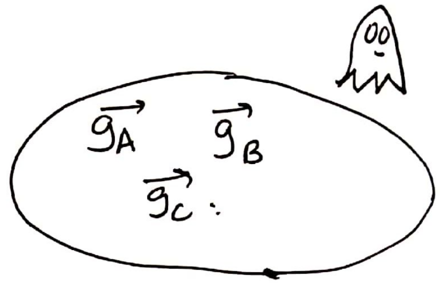
$$\vec{r_1} = \hat{x}_5 \cdot \vec{c_5} + \hat{x}_7 \vec{c_7} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \vec{r}$$

$$1 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 1 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \vec{r}$$

Unexplaind $\vec{r} - \vec{r_1} = 0$.

- **Validation:** Back to devices transmitting $\vec{r}$: Microphone 1.





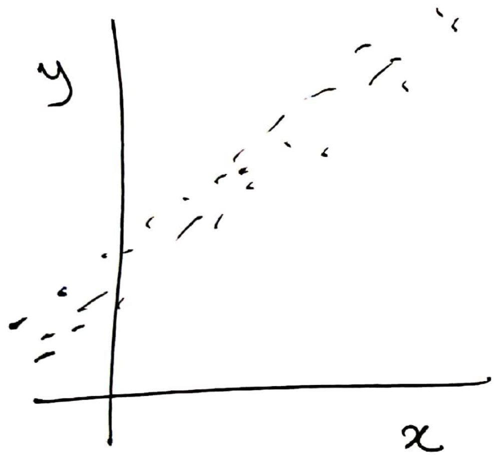
How do I know when to stop?

As soon as I pick out a device from my fake "ghost" set.

Validation set: $\vec{r}_{Primary}$ $\vec{r}_{validation}$.

Strongest signals + devices will be present in both.

- **ML:** How do I know if my model is any good?



Model 1: $y = \alpha_1 x + \alpha_0$

Model 2: $y = \alpha_2 x^2 + \alpha_1 x + \alpha_0$

Model 3: $y = \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0$