# EECS 16A Designing Information Devices and Systems I
# Fall 2018 Midterm 1

Exam Location: Soda 373 (DSP)

PRINT your student ID: ________________________________

PRINT AND SIGN your name: ________________, ________________ ________________
(last name) (first name) (signature)

PRINT time of your Monday section and the GSI's name: ________________________________

PRINT time of your Wednesday section and the GSI's name: ________________________________

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Name and SID of the person to your right: ________________________________

Name and SID of the person in front of you: ________________________________

Name and SID of the person behind you: ________________________________

**1. What is one of your hobbies? (2 Points)**

**2. Tell us about something that makes you happy. (2 Points)**

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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Extra page for scratchwork.

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### 3. Pizza and Pirates! (18 points)

You are stuck on a deserted island and you need to find food everyday! From science class, you know that each day you need to eat:

Table 3.1: Daily doses

| Food [grams] | Daily dose |
|---|---|
| Fat | $4g$ |
| Carbs | $2g$ |
| Protein | $14g$ |
| Vitamins | $6g$ |

Thankfully, you find a pirate camp on the the island and they have 4 kinds of food; eggs, pineapple pizza, bananas, and carrots. Once again, you thank your science teacher, and remember that you know the composition of each of these foods:

Table 3.2: Food composition

| Food [grams] | 1 egg | 1 slice of pineapple pizza | 1 banana | 1 carrot |
|---|---|---|---|---|
| Fat | $1g$ | $2g$ | $0g$ | $0g$ |
| Carbs | $0g$ | $2g$ | $1g$ | $0g$ |
| Protein | $3g$ | $3g$ | $1g$ | $0g$ |
| Vitamins | $1g$ | $0g$ | $1g$ | $1g$ |

In order to get enough food, you decide to steal some from the pirates. But, since it is so dangerous to steal food, you want to take *exactly* what you need, no more no less. Each day, you must decide how how much food to steal; number of eggs, $x_e$, number of pineapple pizza slices, $x_p$, number of bananas, $x_b$, and number of carrots, $x_c$.

(a) (2 points) How many unknowns are there in this problem?

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(b) (6 points) Using Tables 3.1 and 3.2, write the equation for your daily dose of food groups in the form $\mathbf{A}\vec{x} = \vec{y}$ where $\vec{x} = [x_e, x_p, x_b, x_c]^T$. Clearly define $\mathbf{A}$ and $\vec{y}$ in your solution.

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(c) (10 points) Now let $\mathbf{A}$ and $\vec{y}$ be:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 4 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \text{ and } \vec{y} = \begin{bmatrix} 4 \\ 2 \\ 14 \\ 6 \end{bmatrix}, \tag{1}$$

where $\vec{y}$ is the daily dose of each food group needed.

Using the values from Equation (1), find the solution or the set of solutions for how much of each type of food you need to steal everyday, i.e. solve for $\vec{x}$ in $\mathbf{A}\vec{x} = \vec{y}$.

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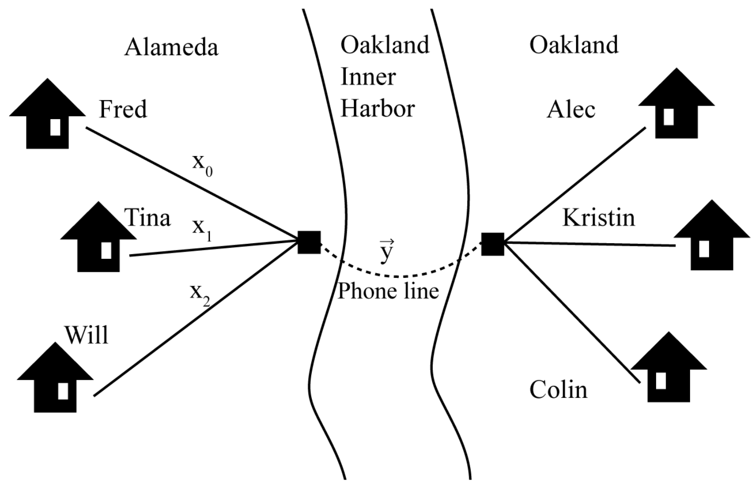
**4. Trouble in Telecomm (24 points)**

Fred ($x_0$), Tina ($x_1$), and Will ($x_2$) each are sending messages (where each message $x_0, x_1, x_2$ is a real number) at the same time to Alec, Kristin, and Colin respectively.




To achieve this, the phone company will transmit $\vec{y}$, which is a vector of linear combinations of $x_0$, $x_1$, $x_2$. Specifically,

$$\vec{y} = \mathbf{V}\vec{x} = \begin{bmatrix} | & | & | \\ \vec{c}_0 & \vec{c}_1 & \vec{c}_2 \\ | & | & | \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}. \tag{2}$$

$\mathbf{V}$ is the encoding matrix.

On the receiver side, Alec, Kristin and Colin need to recover $x_0, x_1, x_2$ respectively from $\vec{y}$. You are helping the phone company evaluate different choices for the columns $\vec{c}_0$, $\vec{c}_1$ and $\vec{c}_2$ of matrix $\mathbf{V}$:

$$\begin{aligned} \mathbf{V_0} &= \begin{bmatrix} | & | & | \\ \vec{c}_0 & \vec{c}_1 & \vec{c}_2 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 10 \\ 0 & 2 & 4 \end{bmatrix} \\ \mathbf{V_1} &= \begin{bmatrix} | & | & | \\ \vec{c}_0 & \vec{c}_1 & \vec{c}_2 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

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(a) (8 points) You decide to characterize $\mathbf{V_0}$ in terms of its null space. Find a basis for the nullspace of $\mathbf{V_0}$.

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(b) (8 points) If the matrix $\mathbf{V_0} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 10 \\ 0 & 2 & 4 \end{bmatrix}$ is invertible, find its inverse. If it is not invertible, why not? Given this, is $\mathbf{V_0}$ a good encoding matrix to use? Justify your answer.

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(c) (8 points) If the matrix $\mathbf{V_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is invertible, find its inverse. If it is not invertible, why not? Given this, is $\mathbf{V_1}$ a good encoding matrix to use? Justify your answer.

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**5. The Romulan Ruse (32 points)** While scanning parts of the galaxy for alien civilization, the starship USS Enterprise NC-1701D encounters a Romulan starship that is known for advanced cloaking devices.

(a) (6 points) The Romulan illusion technology causes a point $(x_0, y_0)$ to transform or *map* to $(u_0, v_0)$. Similarly, $(x_1, y_1)$ is mapped to $(u_1, v_1)$. Figure 5.1 and Table 5.1 show two points on a Romulan ship and the corresponding *mapped* points.

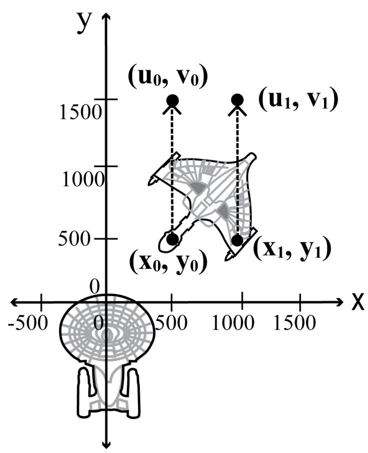


Figure 5.1: Figure for part (a)

| Original Point | Mapped Point |
|---|---|
| $(x_0, y_0) = (500, 500)$ | $(u_0, v_0) = (500, 1500)$ |

| Original Point | Mapped Point |
|---|---|
| $(x_1, y_1) = (1000, 500)$ | $(u_1, v_1) = (1000, 1500)$ |

Table 5.1: Original and Mapped Points

**Find a transformation matrix $\mathbf{A}_0$ such that**

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \text{ and } \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}.$$

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(b) (6 points) In this scenario, every point on the Romulan ship $(x_m, y_m)$ is mapped to $(u_m, v_m)$, such that vector $\begin{bmatrix} x_m \\ y_m \end{bmatrix}$ is rotated counterclockwise by 30° and then scaled by 2 in the x- and y-directions. This transformation is shown in Figure 5.2.

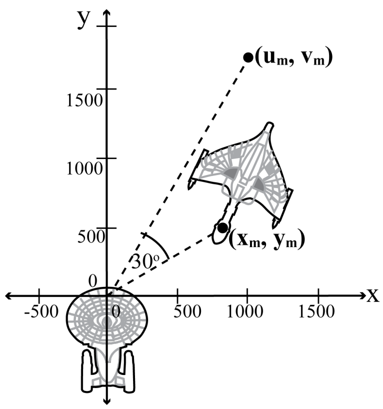


Figure 5.2: Figure for part (b)

| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|
| 0° | 0 | 1 | 0 |
| 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ |
| 45° | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 |
| 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| 90° | 1 | 0 | $\infty$ |

Table 5.2: Trigononometric Table

**Find a transformation matrix R such that** $\begin{bmatrix} u_m \\ v_m \end{bmatrix} = \mathbf{R} \begin{bmatrix} x_m \\ y_m \end{bmatrix}$**.**

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The Romulan ship has launched a probe into space and the Enterprise is trying to destroy the probe by firing a photon torpedo along a straight line from point $(0,0)$ towards the probe.

(c) (10 points) The Romulan generals found a clever way to hide the probe by transforming (mapping) its position with a *cloaking* (transformation) matrix $\mathbf{A}_p$:

$$\mathbf{A}_p = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}.$$

They positioned the probe at $(x_p, y_p)$ so that it maps to $(u_p, v_p) = (0,0)$, where $\begin{bmatrix} u_p \\ v_p \end{bmatrix} = \mathbf{A}_p \begin{bmatrix} x_p \\ y_p \end{bmatrix}$. This scenario is shown in Figure 5.3. The initial position of the torpedo is $(0,0)$ and the torpedo cannot be fired on its initial position! Impressive trick indeed!

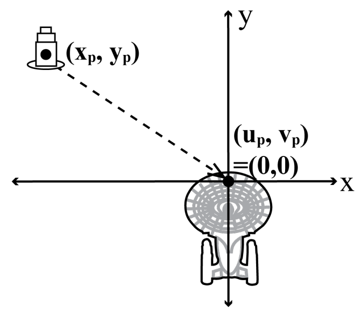


Figure 5.3: Figure for part (c)

**Find the possible positions of the probe $(x_p, y_p)$ so that $(u_p, v_p) = (0,0)$.**

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(d) (10 points) It turns out the Romulan engineers were not as smart the Enterprise engineers. Their calculations did not work out and they positioned the probe at $(x_q, y_q)$ such that the *cloaking* (transformation) matrix, $\mathbf{A}_p$, mapped it to $(u_q, v_q)$, where

$$\begin{bmatrix} u_q \\ v_q \end{bmatrix} = \mathbf{A}_p \begin{bmatrix} x_q \\ y_q \end{bmatrix}, \text{ and } \mathbf{A}_p = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}.$$

As a result, the torpedo while traveling along a straight line from $(0,0)$ to $(u_q, v_q)$, hit the probe at $(x_q, y_q)$ on the way!

The scenario is shown in Figure 5.4. For the torpedo to hit the probe, we must have $\begin{bmatrix} u_q \\ v_q \end{bmatrix} = \lambda \begin{bmatrix} x_q \\ y_q \end{bmatrix}$, where $\lambda$ is a real number.

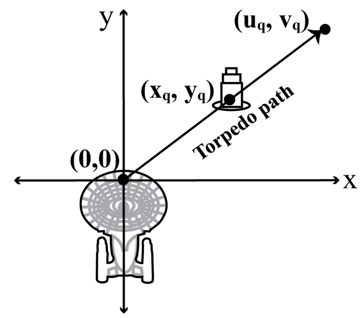


Figure 5.4: Figure for part (d)

**Find the possible positions of the probe $(x_q, y_q)$ so that $(u_q, v_q) = (\lambda x_q, \lambda y_q)$. Remember that the torpedo cannot be fired on its initial position $(0,0)$.**

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**6. A Tropical Tale of Triumph: Does Pineapple Come Out on Top? (52 points)**

(Based on a true story) During a discussion section, one of your TAs, Nick, makes the claim that pineapple belongs on pizza. Another TA, Elena, strongly disagrees. Naturally, a war starts and students begin to flock to the TA they agree with, switching discussion sections every week. Some students don't have an opinion and go to Lydia's section since she is neutral in the matter. As a 16A student, you want to analyze this war to see how it will play out.

(a) (6 points) You manage to capture the behavior of the students as a transition matrix, but want to visualize it. You've written out the transition matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.25 & 0.5 & 1 \\ 0.25 & 0.5 & 0 \end{bmatrix}$$

such that

$$\begin{bmatrix} x_{\text{Elena}}[n+1] \\ x_{\text{Nick}}[n+1] \\ x_{\text{Lydia}}[n+1] \end{bmatrix} = \mathbf{M} \begin{bmatrix} x_{\text{Elena}}[n] \\ x_{\text{Nick}}[n] \\ x_{\text{Lydia}}[n] \end{bmatrix}.$$

Each element of the state vector $\vec{x}[n] = \begin{bmatrix} x_{\text{Elena}}[n] & x_{\text{Nick}}[n] & x_{\text{Lydia}}[n] \end{bmatrix}^T$ represents the number of students attending that section at timestep $n$. **Fill in values in the boxes in Figure 6.1 below** such that the diagram represents the transition matrix $\mathbf{M}$.

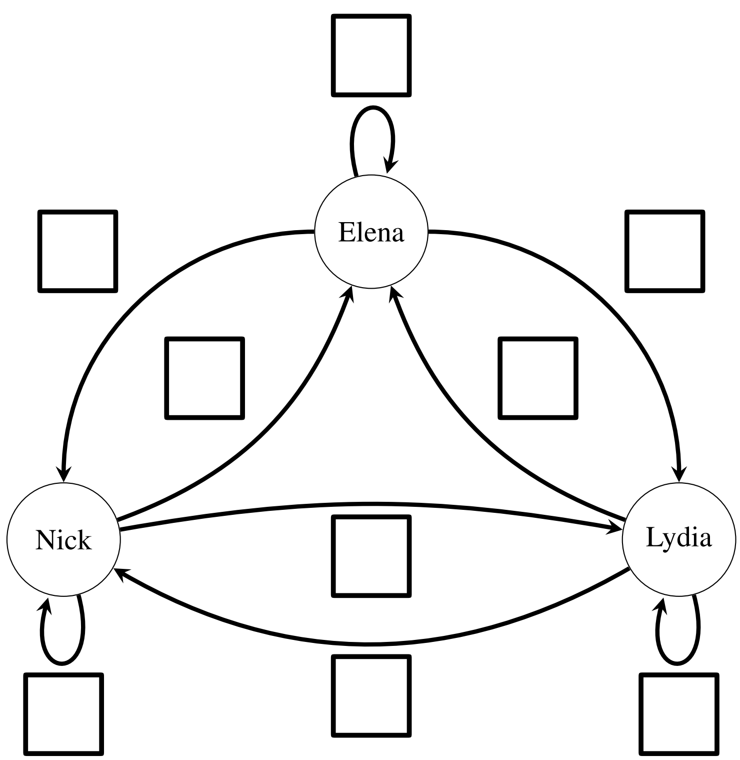


Figure 6.1: Flow diagram for discussion sections

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(b) (10 points) Your friend Vlad tells you that your transition matrix **M** was wrong, and gives you a new transition matrix **S**, **which has a steady state**. In order to find who wins the war, you need to find how many students end up in each section after everything has settled. **Find a vector $\vec{x}$ that represents a steady state of S**.

$$\mathbf{S} = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.3 & 0 & 1 \end{bmatrix}$$

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(c) (6 points) Your other friend Gireeja points out that the arguments are causing new people to join the sections and others to leave entirely. In other words, the system is not conservative! The new system can be modeled with a state transition matrix $\mathbf{A}$ that has the following eigenvalue/eigenvector pairings:

$$\lambda_1 = 1 : \vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_2 = \frac{1}{2} : \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_3 = 2 : \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

You want the number of students in sections to stabilize. Which of the vectors below represent **steady states** of the system, i.e. $\vec{x}$ such that $\mathbf{A}\vec{x} = \vec{x}$? **Fill in the circle(s) to the left of these vector(s).**

○ $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$ ○ $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ○ $\begin{bmatrix} 513 \\ 513 \\ 0 \end{bmatrix}$ ○ $\begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$

○ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ○ $\begin{bmatrix} 1026 \\ 0 \\ 0 \end{bmatrix}$ ○ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ○ $\begin{bmatrix} 0 \\ 1026 \\ 0 \end{bmatrix}$

Space for scratchwork, **will NOT be graded**

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(d) (6 Points) Assume we are still working with the same state transition matrix **A** as in part (c). Which of the vectors below represent **initial states** such that the number of students in the sections keeps growing? **Fill in the circle(s) to the left of these vector(s).**

○ $\begin{bmatrix}5\\0\\0\end{bmatrix}$ ○ $\begin{bmatrix}0\\0\\1\end{bmatrix}$ ○ $\begin{bmatrix}513\\513\\0\end{bmatrix}$ ○ $\begin{bmatrix}0\\12\\0\end{bmatrix}$

○ $\begin{bmatrix}1\\1\\0\end{bmatrix}$ ○ $\begin{bmatrix}1026\\0\\0\end{bmatrix}$ ○ $\begin{bmatrix}1\\2\\3\end{bmatrix}$ ○ $\begin{bmatrix}0\\1026\\0\end{bmatrix}$

Space for scratchwork, **will NOT be graded**

(e) (6 points) Again assume we are still working with the same state transition matrix **A** as in part (c). Which of the vectors below represent **initial states** such that everyone leaves the system, i.e. $\lim_{n\to\infty} \mathbf{A}^n\vec{x} = \vec{0}$? **Fill in the circle(s) to the left of these vector(s).**

○ $\begin{bmatrix}5\\0\\0\end{bmatrix}$ ○ $\begin{bmatrix}0\\0\\1\end{bmatrix}$ ○ $\begin{bmatrix}513\\513\\0\end{bmatrix}$ ○ $\begin{bmatrix}0\\12\\0\end{bmatrix}$

○ $\begin{bmatrix}1\\1\\0\end{bmatrix}$ ○ $\begin{bmatrix}1026\\0\\0\end{bmatrix}$ ○ $\begin{bmatrix}1\\2\\3\end{bmatrix}$ ○ $\begin{bmatrix}0\\1026\\0\end{bmatrix}$

Space for scratchwork, **will NOT be graded**

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(f) (16 Points) Let us generalize the idea of convergence. Consider the following system:

$$\vec{x}[n+1] = \mathbf{T}\vec{x}[n]$$

where $\vec{x}$ is a vector with $N$ elements and $\mathbf{T}$ is any $N \times N$ matrix unrelated to the previous parts. $\mathbf{T}$ has $N$ distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_N$, and $N$ associated eigenvectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_N$ such that $\mathbf{T}\vec{v}_i = \lambda_i \vec{v}_i$ for $1 \leq i \leq N$. Let $|\lambda_i| > 1$. Prove that there exists at least one initial state $\vec{x}[0]$ for this system such that it does not converge to a steady state.

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(g) (2 points) Does pineapple belong on pizza? (*Hint: Full points for honesty!*)

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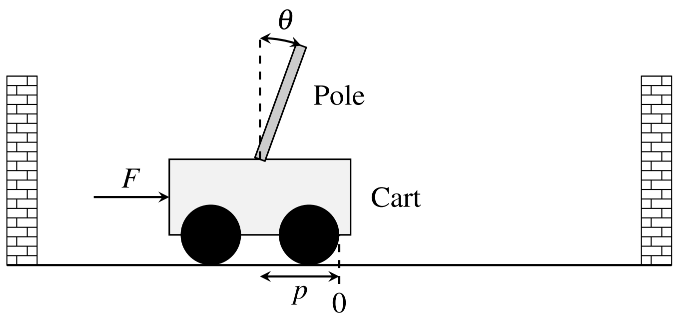
## 7. A Balancing Act! (46 points)

Your friend has started a San Francisco tour business using segways and she needs your help to find which sensor to use for her segways.

We model the segway as a cart-pole system:




A cart-pole system can be fully described at any time step $n$ by its position $p[n]$, velocity $\dot{p}[n]$, angle $\theta[n]$, and angular velocity $\dot{\theta}[n]$. We write this as a "state vector":

$$\vec{x}[n] = \begin{bmatrix} p[n] \\ \dot{p}[n] \\ \theta[n] \\ \dot{\theta}[n] \end{bmatrix}$$

In this case, the cart-pole system can be represented by the following linear model:

$$\vec{x}[n+1] = \mathbf{A}\vec{x}[n], \tag{4}$$

where $\mathbf{A} \in \mathbb{R}^{4\times 4}$.

Since no sensor can measure all four elements of the state vector, the following model can be used to represent the measurements made by a certain sensor:

$$\vec{y}[n] = \mathbf{C}\vec{x}[n], \tag{5}$$

where $\mathbf{C} \in \mathbb{R}^{2\times 4}$ expresses the measurement matrix of the sensor. Each sensor has an output: $\vec{y}[n] \in \mathbb{R}^2$.

We have a few different sensors and we need to determine if the initial state $\vec{x}[0]$ can be uniquely identified using each sensor.

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(a) (4 points) Express $\vec{y}[0]$ in terms of $\vec{x}[0]$.

(b) (6 points) Express $\vec{x}[1]$ and $\vec{y}[1]$ in terms of $\vec{x}[0]$.

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(c) (10 points) The first sensor measures $p[n]$ and $\theta[n]$, and has the following measurement matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

For this part, use the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now:

- For any $\vec{x}[0]$ we have $\vec{y}[0] \in \mathbb{R}^2$ and $\vec{y}[1] \in \mathbb{R}^2$.
- Stacking $\vec{y}[0]$ and $\vec{y}[1]$ on top of each other gives a $4 \times 1$ column vector: $\vec{z} = \begin{bmatrix} \vec{y}[0] \\ \vec{y}[1] \end{bmatrix} \in \mathbb{R}^4$.
- Expressing $\vec{y}[0]$ and $\vec{y}[1]$ in terms of $\vec{x}[0]$, we can write a system of equations:

$$\mathbf{Q}\vec{x}[0] = \begin{bmatrix} \vec{y}[0] \\ \vec{y}[1] \end{bmatrix} = \vec{z}, \tag{6}$$

  where $\mathbf{Q} \in \mathbb{R}^{4\times4}$, $\vec{z} \in \mathbb{R}^4$ and $\vec{x}[0] \in \mathbb{R}^4$.

If your friend can recover $\vec{x}[0]$ from any given $\vec{z}$, calculate $\vec{x}[0]$ for $\vec{y}[0] = \begin{bmatrix} 5.0 \\ 0.1 \end{bmatrix}$, and $\vec{y}[1] = \begin{bmatrix} 6.0 \\ 0.2 \end{bmatrix}$. If $\vec{x}[0]$ cannot be recovered, explain why.

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(d) (10 points) The second sensor measures $\dot{p}[n]$ and $\dot{\theta}[n]$, and has the following measurement matrix:

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

For this part, use the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Again:

- For any $\vec{x}[0]$ we have $\vec{y}[0] \in \mathbb{R}^2$ and $\vec{y}[1] \in \mathbb{R}^2$.
- Stacking $\vec{y}[0]$ and $\vec{y}[1]$ on top of each other gives a $4 \times 1$ column vector: $\vec{z} = \begin{bmatrix} \vec{y}[0] \\ \vec{y}[1] \end{bmatrix} \in \mathbb{R}^4$.
- Expressing $\vec{y}[0]$ and $\vec{y}[1]$ in terms of $\vec{x}[0]$, we can write a system of equations:

$$\mathbf{Q}\vec{x}[0] = \begin{bmatrix} \vec{y}[0] \\ \vec{y}[1] \end{bmatrix} = \vec{z}, \tag{7}$$

  where $\mathbf{Q} \in \mathbb{R}^{4\times 4}$, $\vec{z} \in \mathbb{R}^4$ and $\vec{x}[0] \in \mathbb{R}^4$.

If your friend can recover $\vec{x}[0]$ from any given $\vec{z}$, calculate $\vec{x}[0]$ for $\vec{y}[0] = \begin{bmatrix} 2.0 \\ 0.2 \end{bmatrix}$, and $\vec{y}[1] = \begin{bmatrix} 2.0 \\ 0.2 \end{bmatrix}$. If $\vec{x}[0]$ cannot be recovered, explain why.

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(e) (16 points) The third sensor measures $p[n]$ and $\dot{\theta}[n]$, and has the following measurement matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

For this part, use the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now:

- For $n$ measurements, we have $\vec{y}[0] \in \mathbb{R}^2$, $\vec{y}[1] \in \mathbb{R}^2$, ... , $\vec{y}[n-1] \in \mathbb{R}^2$.
- Stacking the measurements $\vec{y}[0]$, $\vec{y}[1]$,... , $\vec{y}[n-1]$ on top of each other gives a $2n \times 1$ column vector:

$$\vec{z} = \begin{bmatrix} \vec{y}[0] \\ \vec{y}[1] \\ \dots \\ \vec{y}[n-1] \end{bmatrix} \in \mathbb{R}^{2n}. \tag{8}$$

- Expressing $\vec{y}[0]$, $\vec{y}[1]$, ... , $\vec{y}[n-1]$ in terms of $\vec{x}[0]$, we can write the system of equations as:

$$\mathbf{Q}\vec{x}[0] = \vec{z}, \tag{9}$$

  where $\mathbf{Q} \in \mathbb{R}^{2n \times 4}$, $\vec{z} \in \mathbb{R}^{2n}$ and $\vec{x}[0] \in \mathbb{R}^4$.

What is the minimum number of measurements needed to recover $\vec{x}[0]$ from $\vec{z}$? Show the work to justify your answer.

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Doodle page!
Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

EECS 16A Designing Information Devices and Systems I
Fall 2018 Midterm 1 Instructions

Read the following instructions before the exam.

**There are 7 problems of varying numbers of points.** You have 120 minutes for the exam. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

**There are 34 pages on the exam, so there should be 17 sheets of paper in the exam.** The exam is printed double-sided. Do not forget the problems on the back sides of the pages! Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

**No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.**

**Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.**

You may consult ONE handwritten 8.5” $\times$ 11” note sheet (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed.

**Please write your answers legibly in the boxed spaces provided on the exam.** The space provided should be adequate. **If you still run out of space, please use a boxed space for another part of the same problem and clearly tell us in the original problem space where to look.**

In general, show all of your work in order to receive full credit.

Partial credit will be given for substantial progress on each problem.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

**Our advice to you:** if you can’t solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

**Good luck!**

Do not turn this page until the proctor tells you to do so.