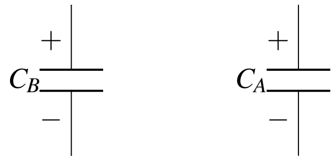
# EECS 16A Designing Information Devices and Systems I
# Fall 2021 Discussion 9A

## 1. Capacitors and Charge Sharing Revisited

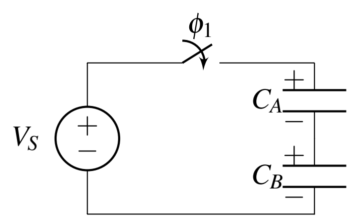
(a) Consider two capacitors, $C_A$ and $C_B$ that have been charged to $Q_{A_1}$ and $Q_{B_1}$ repectively. Compute the voltages across each one, $V_{A_1}$ and $V_{B_1}$.




$$V_{A_1} = \frac{Q_{A_1}}{C_A}$$

$$V_{B_1} = \frac{Q_{B_1}}{C_B}$$

(b) Consider the following circuit set up with an open switch. Given that these were the same charged capacitors as in the previous part, in phase 1, what is the charge and voltage across both capacitors, **before** the switch is closed?




They are the same as in part (a) since no current is allowed to flow. We have $V_{A_1} = \frac{Q_{A_1}}{C_A}$, and $V_{B_1} = \frac{Q_{B_1}}{C_B}$.

(c) Now the switch is closed, and the circuit is allowed to settle. Compute $V_{B_2}$, the final voltage drop across $C_B$, in terms of the given capacitances, the voltages computed in part (a), and $V_S$.

We can use charge conservation to solve this problem. Looking at the middle node between the two capacitors, we know that:

$$-Q_{A_1} + Q_{B_1} = -Q_{A_2} + Q_{B_2}$$

$$-C_A V_{A_1} + C_B V_{B_1} = -C_A V_{A_2} + C_B V_{B_2}$$

$$C_A(V_{A_2} - V_{A_1}) = C_B(V_{B_2} - V_{B_1})$$

From KVL, we get:

$$V_{A_2} + V_{B_2} = V_S \implies V_{A_2} = V_S - V_{B_2}$$

Plugging this into our prior equation, we get:

$$-C_A V_{A_1} + C_B V_{B_1} = -C_A V_S + (C_A + C_B) V_{B_2}$$

since we have $C_{eq} = \frac{C_A C_B}{C_A + C_B}$, we have:

$$V_{B_2} = \frac{C_A(V_S - V_{A_1}) + C_B V_{B_1}}{C_A + C_B} = \frac{C_{eq}}{C_B}(V_S - V_{A_1}) + \frac{C_{eq}}{C_A} V_{B_1}$$

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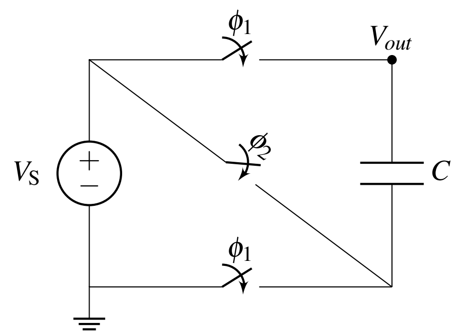
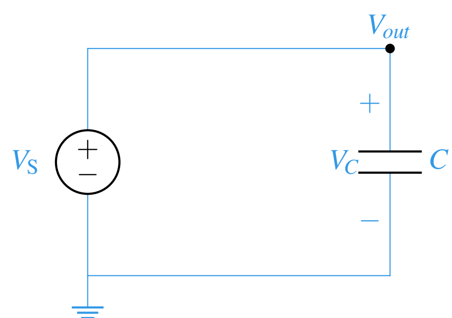
**2. Voltage Booster**

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_S = 5V$? We can do this with capacitors!




(a) In the circuit above switches $\phi_1$ are initially closed and switch $\phi_2$ is initially open. Calculate the value of the output voltage, $V_{out}$ with respect to ground, and the amount of charge stored on capacitor, $C$, at that state (phase 1).

In this setting we have the following equivalent circuit:

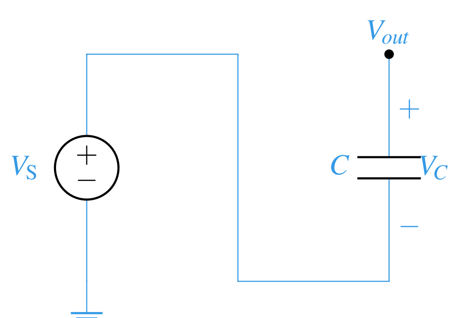



Hence,

$$V_{out} = V_S, \quad Q = CV_S.$$

(b) Now, after the capacitors are charged, switches $\phi_1$ are opened and switch $\phi_2$ is closed. Calculate the new voltage output voltage, $V_{out}$, at steady state.

Phase 2 equivalent ckt:



In phase 2 notice that the voltage source is connected to the *negative* plate of capacitor $C$, while the positive plate is left floating (since it is open). Hence, charge is going to be conserved on the top plate of $C$. However, in phase 2: $V_C^{\phi_2} = V_{out} - V_S$:

$$Q_C^{\phi_1} = Q_C^{\phi_2} \Rightarrow CV_S = C(V_{out} - V_S) \Rightarrow V_{out} = 2V_S = 10V!$$

We have created a voltage doubler!