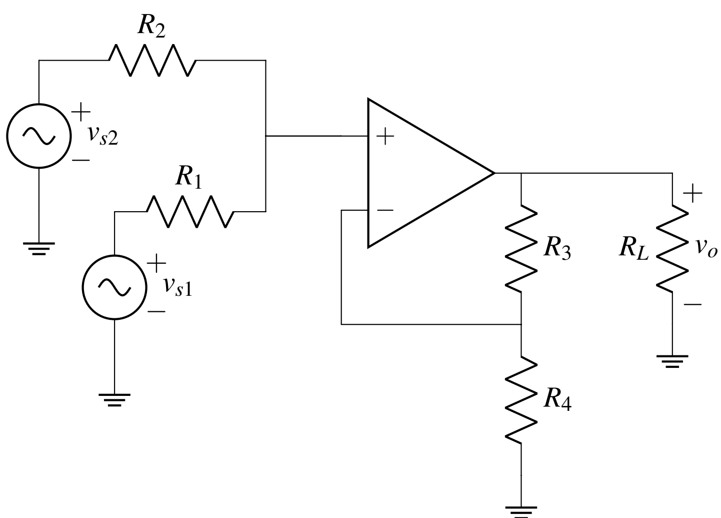
# EECS 16A Designing Information Devices and Systems I
# Fall 2021 Discussion 11A

**1. Multiple Inputs To One Op-Amp**

Estimated Time: 20 min

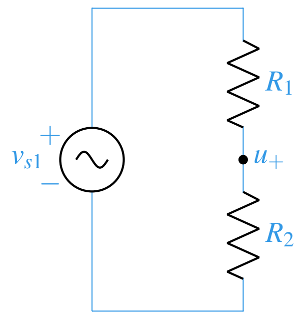
**Note to TAs:** Reminder that this is the composition of different blocks we have seen before: a summer, non-inverting amplifier, and load resistor. So you can simply apply the equations for each individual block, rather than doing analysis from scratch. Be sure to clarify why we can hook these blocks together and still get the expected output.

TAs should also mention the difference between voltage signal and voltage source. Voltage signals are usually mV level and don't dissipate much power. In contrast voltage sources are usually V level and are often used as power supplies (e.g. rails for op-amp).




(a) For the circuit above, find an expression for $v_o$. *(Hint: Use superposition.)*

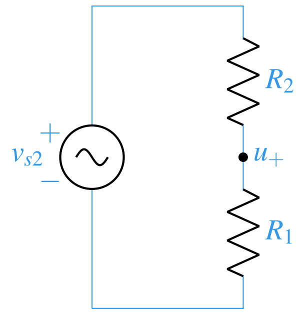
Let's call the potential at the positive input of the op-amp $u_+$. Using superposition, we first turn off $v_{s2}$ and find $u_+$. The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,vs1} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect $v_{s2}$ to have a similar circuit and expression. The circuit for $v_{s2}$ looks like:




The expression for $u_+$ with $v_{s2}$ is then:

$$u_{+,vs2} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}$$

With $u_+$ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.
From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express $u_-$ in terms of $v_o$:

$$u_- = \frac{R_4}{R_3 + R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set $u_+$ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right)$$

(b) How could you use this circuit to find the sum of different signals, i.e. $V_{s1} + V_{s2}$? What about taking the sum and adding multiplying by 2, i.e. $2(V_{s1} + V_{s2})$?

**Answer:**

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

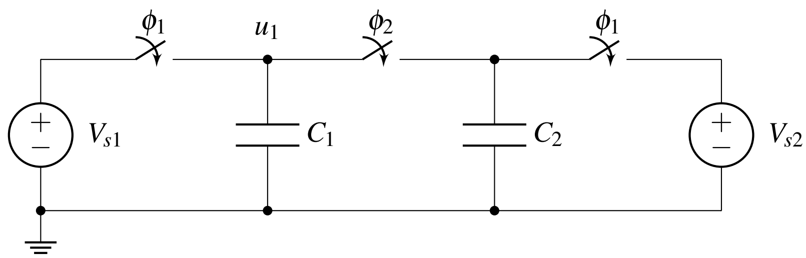
$$v_o = \left(1 + \frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right) = (1+1)\left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

Notice that the first half of this circuit ($R_1$ and $R_2$) form a voltage summer with coefficients less than one; the second half is just a non-inverting amplifier. Thus we can always use $R_1$ and $R_2$ to take an equally weighted sum of the inputs and then multiply greater than 1 using the non-inverting amplifier. If we set $R_1 = R_2$, we get $\left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right)$ into the op-amp. To get a total gain of 2, then the non-inverting op-amp needs a gain of 4, so we can pick $R_3 = 3R_4$.

**2. Capacitive Charge Sharing (from Spring 2020 Midterm 2)**

Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and three switches $\phi_1, \phi_2$. Suppose that initially the switches $\phi_1$ is closed and $\phi_2$ is open such that $C_1$ and $C_2$ are charged through the corresponding voltage sources $V_{s1} = 1\,\text{V}$ and $V_{s2} = 2\,\text{V}$.

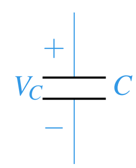



(a) How much charge is on $C_1$ and $C_2$? How much energy is stored in each of the capacitors? What is the total stored energy?

$$q_1 = C_1 V_1 = 1\,\mu\text{C}$$
$$q_2 = C_2 V_2 = 2\,\mu\text{C}$$

**Note to TAs:** Since capacitor energy is not covered in the letcture, TAs should derive it before diving into the solutions. One simple way can be:

Suppose we have an uncharged capacitor C.




Then the current on C becomes $I_C = \text{C}\frac{dV_C}{dt}$. Therefore, the power consumed by the capacitor is $P = V_C I_C = \text{C}\frac{V_C dV_C}{dt}$. The energy equals to the integral of the power over time, which is: $E = \int \text{C}\frac{V_C dV_C}{dt} dt = \int \text{C} V_C dV_C = \frac{1}{2} C V_C^2$.

Back to the problem. Energy:

$$E = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} Q V$$

Therefore, $E_1 = \frac{1}{2} C_1 V_1^2 = 0.5\,\mu\text{J}$, $E_2 = \frac{1}{2} C_2 V_2^2 = 2\,\mu\text{J}$, and the total energy is $2.5\,\mu\text{J}$.

(b) Now suppose that some time later, switch $\phi_1$ opens and switch $\phi_2$ closes. What is the value of voltage $u_1$ at steady state?

**Answer:** The total charge on capacitors $C_1$ and $C_2$ will be conserved after switch $S_1$ is opened. That charge is $Q_{tot} = q_1 + q_2$. Also note that during phase 2 the capacitors are connected in parallel so they will both have $V_{C1} = V_{C2} = u_1$.

$$Q_{tot} = C_1 u_1 + C_2 u_1$$
$$u_1 = \frac{q_1 + q_2}{C_1 + C_2} = 1.5\,\text{V}$$