EECS 16A Designing Information Devices and Systems I Fall 2021 Discussion 1B

1. Gaussian Elimination

Use Gaussian elimination to solve the following systems. Does a solution exist? Is it unique?

(a)

| | $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$ |
|---|--|
| | $\begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 8 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$ |
| (c) | $\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$ |
| (d) True or False: A system of equation | ns with more equations that |

- (d) True or False: A system of equations with more equations than unknowns will always have either infinite solutions or no solutions.
- (e) (Practice)

| [3 | -1 | 2 | 1] |
|----|----|---|-----|
| 0 | 0 | 2 | 1 |

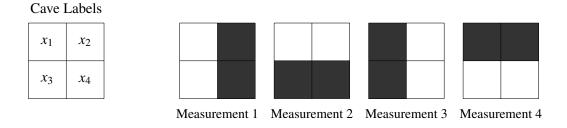
(f) (**Practice**)

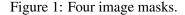
 $\begin{bmatrix} 2x + 4y + 2z = 8\\ x + y + z = 6\\ x - y - z = 4 \end{bmatrix}$

2. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoul fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):





- (a) Let \vec{x} be the four-element vector that represents the magnitude of light emanating from the four cave entrances. Write a matrix **K** that performs the masking process in Figure 1 on the vector \vec{x} , such that $\mathbf{K}\vec{x}$ is the result of the four measurements.
- (b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?
- (c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:



Does this additional measurement give them enough information to solve the problem? Why or why not?

2