EECS 16A Designing Information Devices and Systems I
Fall 2021 Discussion 12B

1. Inner Product Properties

For this question we will verify our coordinate definition of the inner product

$$\langle \vec{x}, \vec{y} \rangle \;=\; x_1 y_1 + x_2 y_2 + \ldots + x_n y_n \;, \quad \text{for any } \vec{x}, \vec{y} \in \mathbb{R}^n$$

indeed satisfies the key properties required for all inner products, but presently for the 2-dimensional case. Suppose $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^2$ for the following parts:

(a) Show symmetry $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$:

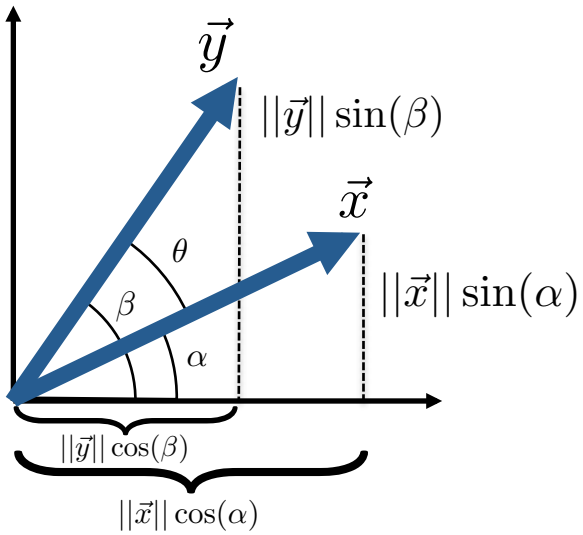
(b) Show linearity $\langle \vec{x}, c\vec{y} + d\vec{z} \rangle = c\langle \vec{x}, \vec{y} \rangle + d\langle \vec{x}, \vec{z} \rangle$, where $c \in \mathbb{R}$ is a real number.

(c) Show non-negativity $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$:

2. Geometric Interpretation of the Inner Product

In this problem we explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in $\mathbb{R}^2$.

(a) Derive a formula for the inner product of two vectors in terms of their magnitudes and the angle between them. The figure below may be helpful:



(b) For each sub-part, identify any two (nonzero) vectors $\vec{x}, \vec{y} \in \mathbb{R}^2$ that satisfy the stated condition and compute their inner product.

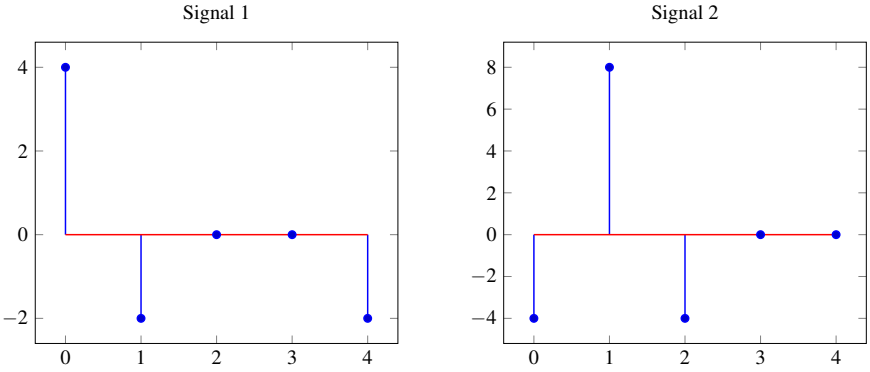
i. Identify a pair of parallel vectors:

ii. Identify a pair of anti-parallel vectors (vectors that point in opposite directions):

iii. Identify a pair of perpendicular vectors:

3. Correlation

You are given the following two signals:



(a) Sketch the linear cross-correlation of signal 1 with signal 2, that is find : $\text{corr}(\vec{s}_1, \vec{s}_2)$. Do not assume the signals are periodic.

(b) Assume signal $\vec{s}_2$ is periodic with period 5. Find the linear cross correlation $\text{corr}(\vec{s}_1, \vec{s}_2)$ of the two signals.