EECS 16A Designing Information Devices and Systems I Fall 2021 Discussion 14B

1. Polynomial Fitting

Let's try an example. Say we know that the output, y, is a quartic polynomial in x. This means that we know that y and x are related as follows:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

We're also given the following observations:

X	У
0.0	24.0
0.5	6.61
1.0	0.0
1.5	-0.95
2.0	0.07
2.5	0.73
3.0	-0.12
3.5	-0.83
4.0	-0.04
4.5	6.42

- (a) What are the unknowns in this question?=
- (b) Can you write an equation corresponding to the first observation (x_0, y_0) , in terms of a_0 , a_1 , a_2 , a_3 , and a_4 ? What does this equation look like? Is it linear in the unknowns?
- (c) Now, write a system of equations in terms of a_0 , a_1 , a_2 , a_3 , and a_4 using all of the observations.
- (d) Finally, solve for a_0 , a_1 , a_2 , a_3 , and a_4 using IPython or any method you like. You have now found the quartic polynomial that best fits the data!

2. Orthogonal Subspaces

Two vectors are \vec{x} and \vec{y} are said to be orthogonal if their inner product is zero. That is $\langle \vec{x}, \vec{y} \rangle = 0$.

Two subspaces \mathbb{S}_1 and \mathbb{S}_2 of \mathbb{R}^N are said to be orthogonal if all vectors in \mathbb{S}_1 are orthogonal to all vectors in \mathbb{S}_2 . That is,

$$\langle \vec{v_1}, \vec{v_2} \rangle = 0 \ \forall \vec{v_1} \in \mathbb{S}_1, \vec{v_2} \in \mathbb{S}_2.$$

- (a) Recall that the *column space* of an $M \times N$ matrix **A** is the subspace spanned by the columns of **A** and that the *null space* of **A** is the subspace of all vectors \vec{v} such that $A\vec{v} = \vec{0}$.
 - Prove that for any matrix **A**, the column space of \mathbf{A}^T and null space of **A** are orthogonal subspaces. This can be denoted by $\operatorname{Col}(\mathbf{A}^T) \perp \operatorname{Null}(\mathbf{A}) \ \forall \mathbf{A} \in \mathbb{R}^{M \times N}$.
 - Hint: Use the row interpretation of matrix multiplication.
- (b) Now prove that for any matrix **A**, the column space and null space of \mathbf{A}^T are orthogonal subspaces. This can be denoted by $\operatorname{Col}(\mathbf{A}) \perp \operatorname{Null}(\mathbf{A}^T) \ \forall \mathbf{A} \in \mathbb{R}^{M \times N}$.