EECS 16A Dis 11A Notes

Tuesday, November 2, 2021 5:29 PM

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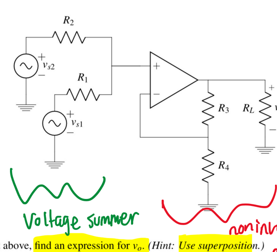
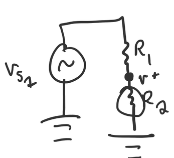
1. Multiple Inputs To One Op-Amp



Voltage summer | noninverting amplifier

Op-amp Golden Rules:
1. $I^+ = I^- = 0$
2. If neg. feedback, $v^+ = v^-$

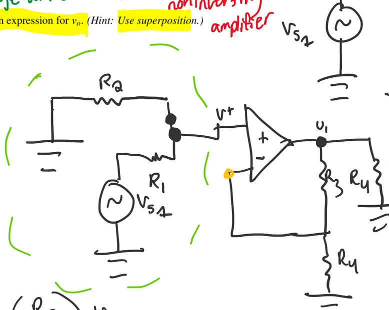
(a) For the circuit above, find an expression for v_o. (*Hint: Use superposition.*)



$v^+ = \frac{R_2}{R_1+R_2} \cdot V_{s1}$

Superposition:

V_{s2} off!



$V_o = u_1 - GND$

$V_o = u_1$

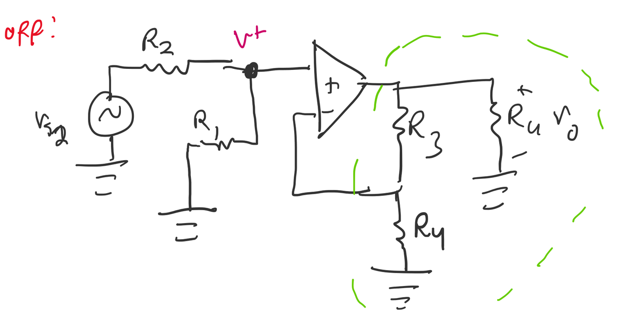
$v^+ = \left(\frac{R_2}{R_1+R_2}\right) V_{s1}$ $v^+ = v^-$

$v^- = \left(\frac{R_4}{R_3+R_4}\right) \cdot V_o$

$\left(\frac{R_2}{R_1+R_2}\right) V_{s1} = \left(\frac{R_4}{R_3+R_4}\right) V_o$

$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left(\frac{R_2}{R_1+R_2}\right) V_{s1}$$

V_{s1} off:



$v^+ = \left(\frac{R_1}{R_1+R_2}\right) \cdot V_{s2}$ $v^+ = v^-$

$v^- = \left(\frac{R_4}{R_3+R_4}\right) \cdot V_o$

$\left(\frac{R_1}{R_1+R_2}\right) \cdot V_{s2} = \left(\frac{R_4}{R_3+R_4}\right) \cdot V_o$

$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left(\frac{R_1}{R_1+R_2}\right) V_{s2}$$

+

$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left[\frac{R_2}{R_1+R_2} \cdot V_{s1} + \frac{R_1}{R_1+R_2} \cdot V_{s2}\right]$$

(b) How could you use this circuit to find the sum of different signals, i.e. $V_{s1}+V_{s2}$? What about taking the sum and adding multiplying by 2, i.e. $2(V_{s1}+V_{s2})$?

$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left[\frac{R_2}{R_1+R_2} \cdot V_{s1} + \frac{R_1}{R_1+R_2} \cdot V_{s2}\right]$$

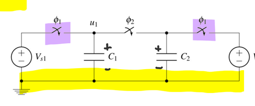
$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left(\frac{R_2}{R_1+R_2}\right) V_{s1} + \left(\frac{R_3+R_4}{R_4}\right)\left(\frac{R_1}{R_1+R_2} \cdot V_{s2}\right)$$

$V_{s1}+V_{s2}$: $R_3+R_4 = R_1+R_2$; $R_4 = R_2 = R_1$

$2(V_{s1}+V_{s2})$: $R_3+R_4 = R_1+R_2$; $R_2 = 2R_4$; $R_1 = 2R_4$

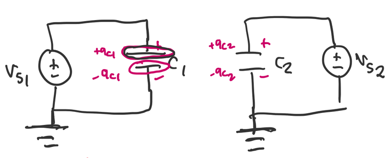
2. Capacitive Charge Sharing (from Spring 2020 Midterm 2)

Consider the circuit below with $C_1 = C_2 = 1\,\mu F$ and three switches ϕ_1, ϕ_2. Suppose that initially the switches ϕ_1 is closed and ϕ_2 is open such that C_1 and C_2 are charged through the corresponding voltage sources $V_{s1} = 1\,V$ and $V_{s2} = 2\,V$.



Charge sharing!
- Close switches for each phase
- Use floating nodes to solve for charge on caps

(a) How much charge is on C_1 and C_2? How much energy is stored in each of the capacitors? What is the total stored energy?



$Q = CV$ ★

$Q_{c1} = C_1 \cdot V_{s1}$ $Q_2 = C_2 \cdot V_{s2}$

$P = IV$ $I = \frac{dQ}{dt}$ $I = C \cdot \frac{dV_c}{dt}$

$P = \left(C \cdot \frac{dV_c}{dt}\right) \cdot V_c$ $\int P = \int \frac{dE}{dt}$ $\int P \cdot dt = \int dE$

$\int C \cdot V_c \cdot \frac{dV_c}{dt} \cdot dt = \int dE$

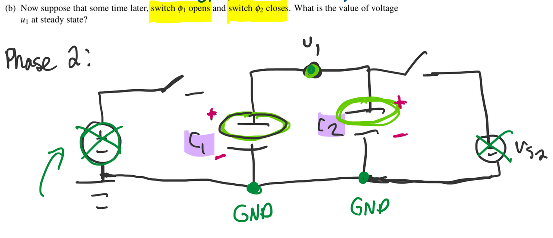
$\int C \cdot V_c \cdot dV_c = \int dE$

$\frac{1}{2} \cdot C \cdot V_c^2 = E$ ★

$E_{tot} = E_{C_1} + E_{C_2}$
$= 2.5\,\mu J$

(b) Now suppose that some time later, switch ϕ_1 opens and switch ϕ_2 closes. What is the value of voltage u_1 at steady state?

Phase 2:



$Q = CU$

$Q_{C_1}^{\phi_2} = C_1 u_1$ $Q_{C_2}^{\phi_2} = C_2 u_1$

$Q_{C_1}^{\phi_2} + Q_{C_2}^{\phi_2} = Q_{C_1}^{\phi_1} + Q_{C_2}^{\phi_1}$

$C_1 u_1 + C_2 u_1 = C_1 V_{s1} + C_2 V_{s2}$

$$u_1 = \frac{C_1 V_{s1} + C_2 V_{s2}}{C_1 + C_2}$$

$= 1.5\,V$

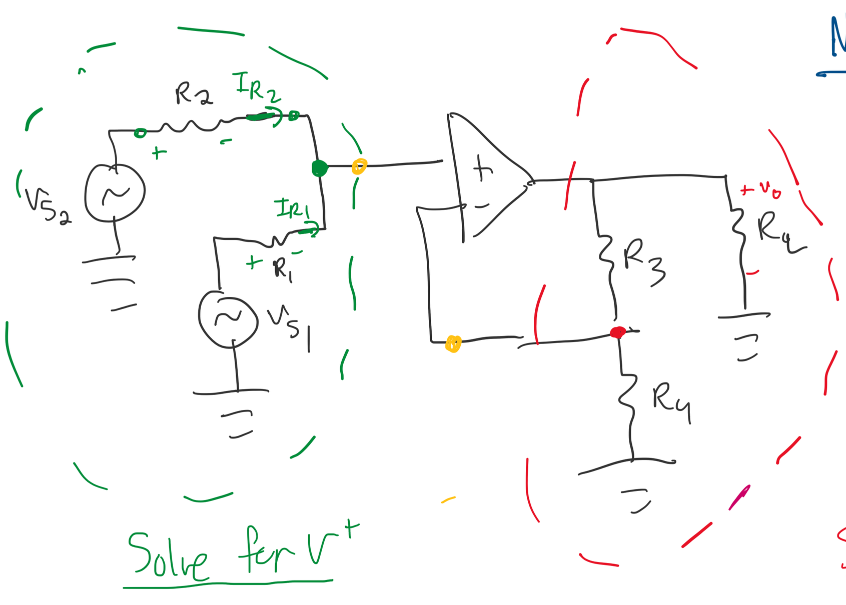
$E_{tot}^{\phi_2} \neq E_{tot}^{\phi_1}$

$E_{tot}^{\phi_1} < 2.5\,\mu J$

Questions:

1. How to solve for V_{out} in problem 1 without using superposition?

NVA+KCL



Solve for v^+

$I_{R2} + I_{R1} = I^+ = 0$

$\frac{V_{s2} - v^+}{R_2} = I_{R2}$ $\frac{V_{s1} - v^+}{R_1} = I_{R1}$

$\frac{V_{s2} - v^+}{R_2} + \frac{V_{s1} - v^+}{R_1} = 0$

$\frac{V_{s2}}{R_2} - \frac{v^+}{R_2} + \frac{V_{s1}}{R_1} - \frac{v^+}{R_1} = 0$

$\frac{V_{s2}}{R_2} + \frac{V_{s1}}{R_1} = v^+\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$

$= v^+\left(\frac{R_1+R_2}{R_1 R_2}\right)$

$v^+ = \left(\frac{R_1 R_2}{R_1+R_2}\right)\left(\frac{R_1 V_{s2} + R_2 V_{s1}}{R_1 R_2}\right)$

$v^+ = \left(\frac{R_1 V_{s2} + R_2 V_{s1}}{R_1+R_2}\right)$

Solve for v^-

$v^- = \frac{R_4}{R_3+R_4} \cdot V_o$

Equate v^+ and v^-

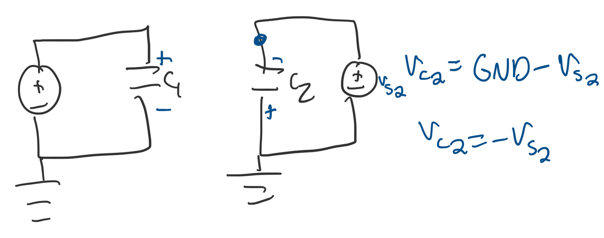
$v^+ = v^-$

$\frac{R_1 V_{s2} + R_2 V_{s1}}{R_1+R_2} = \frac{R_4}{R_3+R_4} \cdot V_o$

$$V_o = \left(\frac{R_3+R_4}{R_4}\right)\left[\frac{R_1 V_{s2}}{R_1+R_2} + \frac{R_2 \cdot V_{s1}}{R_1+R_2}\right]$$

2. How to solve for u_1 in problem 2 w/ different polarities?

Phase 1:



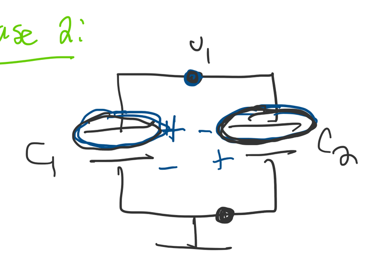
$V_{c2} = GND - V_{s2}$

$V_{c2} = -V_{s2}$

$Q_{c1} = C_1 \cdot V_{s1}$ $Q_{c2} = C_2 \cdot V_{c2}$

$Q_{c2} = -C_2 V_{s2}$

Phase 2:



u_1: $Q_{c_1}^{\phi_2} - Q_{c_2}^{\phi_2} = Q_{c_1}^{\phi_1} - Q_{c_2}^{\phi_1}$

$C_1 \cdot u_1 - C_2 \cdot (0 - u_1)$

$C_1 u_1 + C_2 u_1 = C_1 V_{s1} - (-C_2 V_{s2})$

$C_1 u_1 + C_2 u_1 = C_1 V_{s1} + C_2 V_{s2}$

$$u_1 = \frac{C_1 V_{s1} + C_2 V_{s2}}{C_1 + C_2}$$