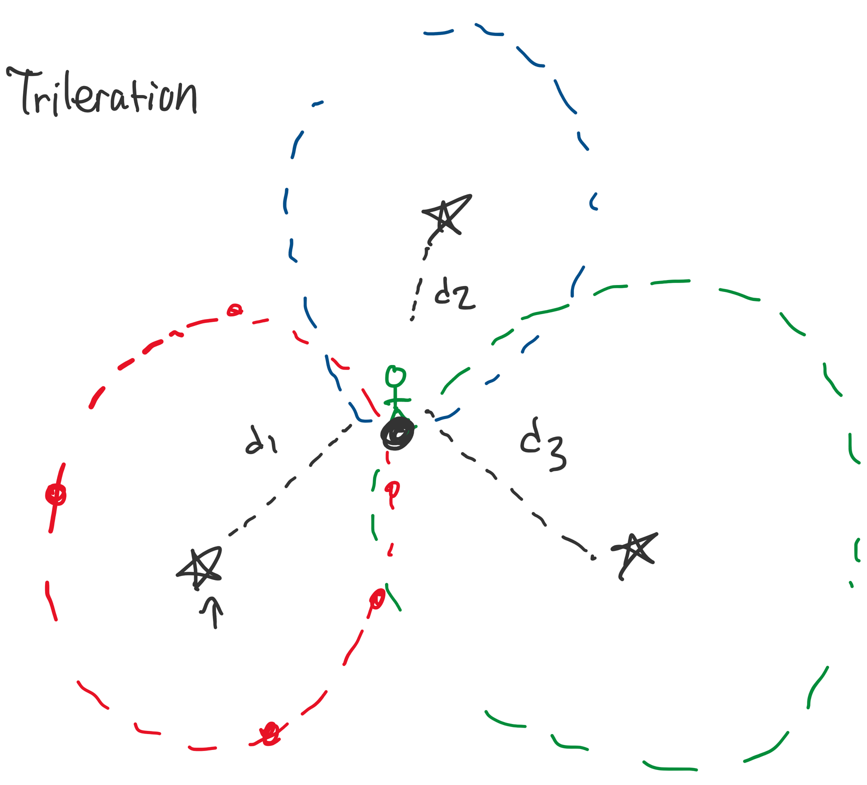
# EECS 16A Dis 13A Notes

Sunday, November 21, 2021 4:41 PM

Feedback form: tinyurl.com/anusha16afeedback

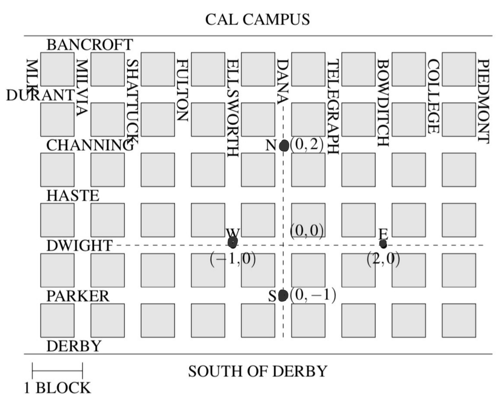
Trileration




**1. Search and Rescue Dogs**

Berkeley's Puppy Shelter needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the shelter have a collar that sends a Bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of five city blocks. Can you help the shelter locate their lost puppy?

**Note**: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map). Mr. Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Muffin?' it is sufficient to answer with his intersection or 'between these two intersections.'




Mr. Muffin

[1]http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg

(a) You check the logs of the cell towers, and they have received the following messages:

| Sensor | Distance | |
|---|---|---|
| N | 5 | |
| W | $\sqrt{20}$ | $= 4.47$ |
| E | $\sqrt{5}$ | $= 2.36$ |
| S | $\sqrt{10}$ | $= 3.16$ |

**On the map provided above, identify where Mr. Muffin is!**

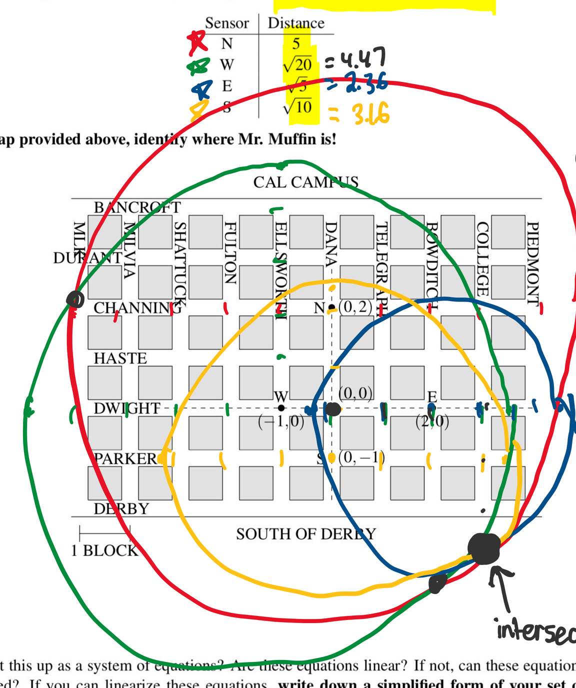
Circle eq. $(x-x_0)^2 + (y-y_0)^2 = r^2$

N: $(x-0)^2 + (y-2)^2 = 25$

W: $(x+1)^2 + (y-0)^2 = 20$

E: $(x-2)^2 + (y-0)^2 = 5$

S: $(x-0)^2 + (y+1)^2 = 10$




(b) Can you set this up as a system of equations? Are these equations linear? If not, can these equations be linearized? If you can linearize these equations, **write down a simplified form of your set of equations.**

*Hint:* Set (0,0) to be Dwight and Dana.

*Hint 2:* Distance $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

*Hint 3:* You don't need all 4 equations. You have two unknowns, $x$ and $y$. You know from lecture that you need at least three circles to uniquely find a point on a 2D plane. How can you use the third circle/equation to get two equations and two unknowns?

*Note:* Remember to check for consistency for all nonlinear equations after finding the coordinates.

$(x-0)^2 + (y-2)^2 = 25 = x^2 + y^2 - 4y + 4 = 25$ (1)

$(x+1)^2 + (y-0)^2 = 20 = x^2 + 2x + 1 + y^2 = 20$ (2)

$(x-2)^2 + (y-0)^2 = 5 = x^2 - 4x + 4 + y^2 = 5$ (3)

$(x-0)^2 + (y+1)^2 = 10 \quad x^2 + y^2 + 2y + 1 = 10$ (4)

2-1: $2x + 1 + 4y - 4 = -5 \quad 2x + 4y - 3 = -5 \quad 2x + 4y = -2 \quad x + 2y = -1$

3-1: $-4x + 4 + 4y - 4 = -20 \quad -4x + 4y = -20 \quad + \quad -x + y = -5$

$x + 2y = -1$

$-x + y = -5$

$3y = -6 \quad y = -2$

$-x + y = -5$

$-x = -3$

$x = 3$

$\boxed{\text{Mr. Muffin: } [3, -2]}$

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

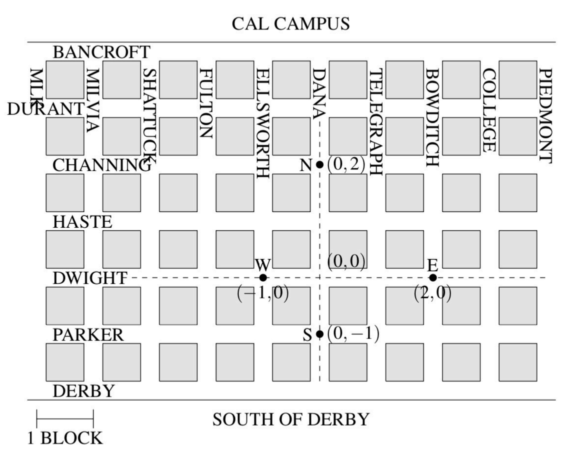
| Sensor | Distance |
|---|---|
| N | 5 |
| W | $\sqrt{20}$ |
| E | Out of Range |
| S | Out of Range |

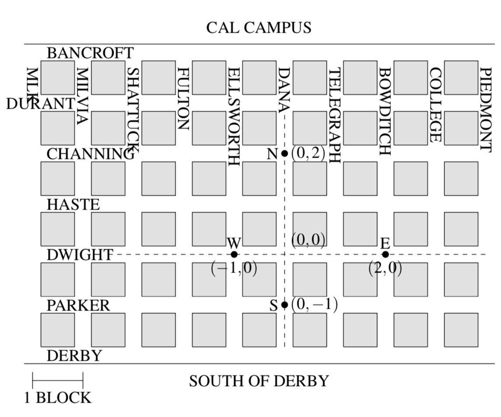
**Can you find Mr. Muffin? With a system of linear equations? Other methods? If so, on the map provided above, identify where Mr. Muffin is!**

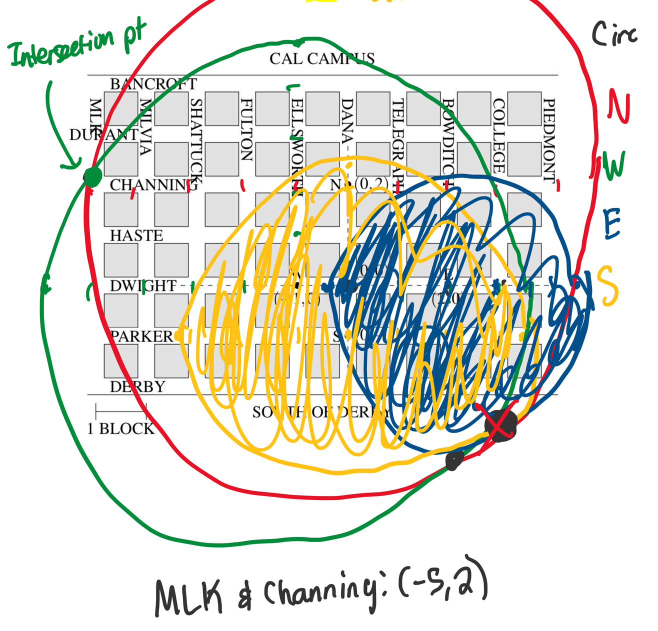
N: 5

W: $\sqrt{20}$

E, S: Out of range



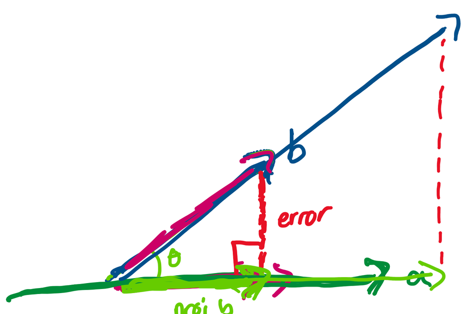

MLK & Channing: $(-5, 2)$

**2. Mechanical Projection**

In $\mathbb{R}^n$, the vector valued projection of vector $\vec{b}$ onto vector $\vec{a}$ is defined as:

$$\text{proj}_{\vec{a}}\left(\vec{b}\right) = \frac{\langle \vec{a}, \vec{b} \rangle}{\|\vec{a}\|^2}\vec{a}.$$

Recall $\|\vec{a}\|^2 = \langle \vec{a}, \vec{a} \rangle$.




proj$_a$b: projection of b onto a

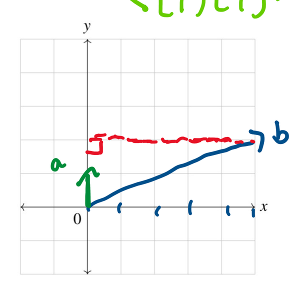
closest vector to b in span(a)

min error: $\|b - \text{proj}_a b\|$

$$\boxed{\text{proj}_a b = \frac{\langle a, b \rangle}{\|a\|^2}\vec{a}}$$

(a) Project $\underset{b}{\begin{bmatrix} 5 \\ 2 \end{bmatrix}}$ onto $\underset{a}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$ – that is, onto the $y$-axis. Graph these two vectors and the projection.

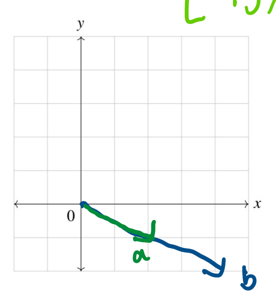
proj$_a$b: $\dfrac{\left\langle \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\rangle} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \dfrac{2}{1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}$




(b) Project $\underset{b}{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}$ onto $\underset{a}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}}$. Graph these two vectors and the projection.

proj$_a$b: $\dfrac{\left\langle \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\rangle} \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \dfrac{10}{4+1}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}$

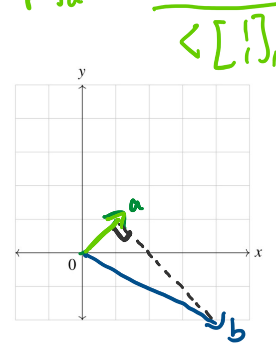



proj$_a$b = b

Since $\vec{b} \in$ span($\vec{a}$), proj$_{\vec{a}}\vec{b} = \vec{b}$

(c) Project $\underset{b}{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}$ onto $\underset{a}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$. Graph these two vectors and the projection.

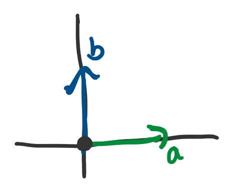
proj$_a$b: $\dfrac{\left\langle \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\rangle} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \dfrac{2}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$




proj$_a$b $= \vec{a}$

What is the projection of two orthogonal vectors?

let $\vec{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$




$\left\langle \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\rangle = 0$

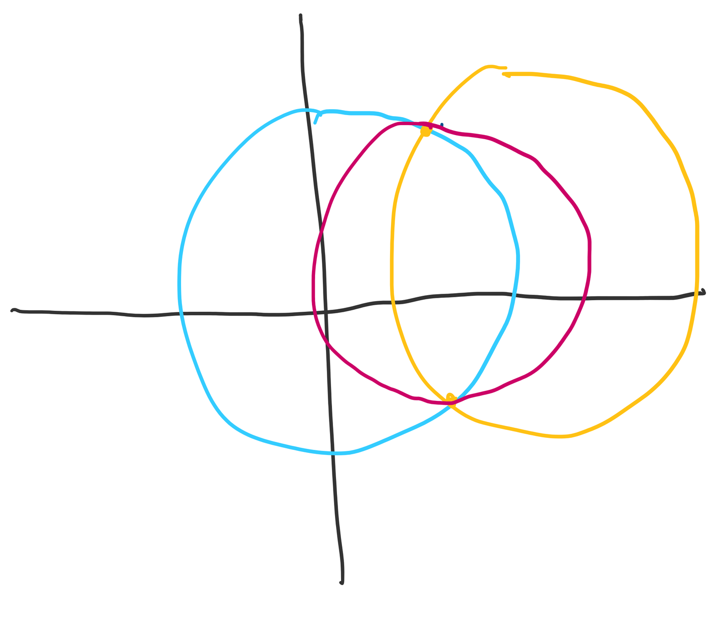
$= \vec{0}$

$\left\langle \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle = 0$

$= \vec{0}$

# Questions:

1. Can three circles have two intersection points?



Yes, it is possible as seen by the graph