EECS 16A Dis 6B Notes

Wednesday, September 22, 2021 5:47 PM

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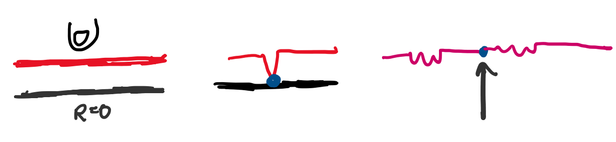
1. Material Resistivity

(a) Recall the 1D resistive touch screen model introduced in class. In this model, the top layer can be thought of as a resistor, while the bottom layer can be thought of as a wire. When the top layer is touched, it flexes at the touch point and makes contact with the bottom layer. This results in a voltage divider.



Material	Resistivity ρ (Ω cm)	Conductivity 1/ρ (Ω^{-1} cm^{-1})
Silver	1.6 x 10^{-5}	6.3 x 10^{4}
Aluminum	2.7 x 10^{-5}	3.7 x 10^{4}
Carbon (graphite)	10 x 10^{-2}	10
Rubber	100 x 10^{12}	1 x 10^{-14}

Given the following list of materials and their resistivity/conductivity, which materials would be good to use as a top layer, and which would be good to use as a bottom layer? Why?

Top layer: Carbon, rubber
Bottom layer: Silver, aluminum

(b) Let's say you want to make your own **10 cm** long resistor out of **graphite**. You need the resistance to be **1** Ω. Recall the equation for resistance: $R = \rho\frac{L}{A} = \rho\frac{L}{W*H}$

i. What are some possible widths and heights of your resistor?

ii. Can you think of advantages of having a wide and thin resistor? How about advantages of a narrow and thick resistor?

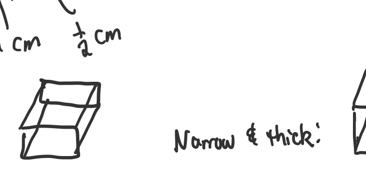
$R = \rho \cdot \frac{L}{A}$ ← length; ρ: Material; A: W×H

i. $R = \rho\frac{L}{A} = \frac{10\times10^{-2} \times 10\text{ cm}}{W\times H} = \frac{10\times10^{-2} \times 10\times10^{-2}}{W\times H} = 1\,\Omega$

$W\times H = 10\times10^{-2} \times 10\text{ cm}$ $\qquad$ 10 cm = 10×10^{-2}

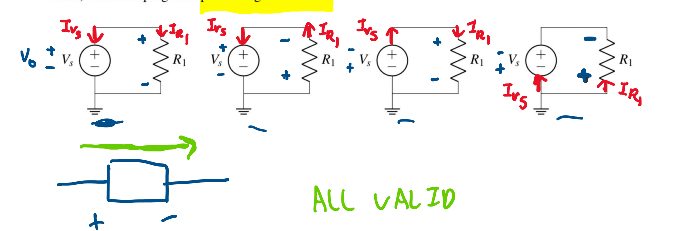
W × H: 2 cm, 1 cm, 1 cm, ½ cm

ii. Wide & thin: $\qquad$ Narrow & thick:



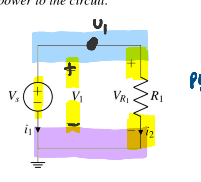
2. Passive Sign Convention and Power

(a) Below are four copies of a the same single-resistor circuit. On each copy, provide a distinct choice of labels for each circuit's voltage polarities and current directions (there should be 4 possible choices in total!) while keeping with passive sign convention.



ALL VALID

(b) Suppose we consider one of the possible labelings you have found above. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5$ V and let $R_1 = 5\,\Omega$. *Recall that the power dissipated is the rate of electric energy converted into other forms and is given by the equation $P = IV$. When the power dissipated by an element is a negative value, it signifies that element is actually supplying electrical power to the circuit.*



$P = IV$ ← power dissipated by an element

$V = IR$

$P = \frac{V^2}{R} = I^2R$

Power: Rate of change of energy

$V_1 = u_1 - 0$

$V_1 = u_1$

$V_{R_1} = u_1 - 0 = u_1 = V_S$ $\qquad$ $V_{R_1} = I_2 \cdot R_1$, $I_2 \cdot R_1 = u_1$

$u_1 = I_2 \cdot R_1$

$V_S = I_2 R_1$

$I_2 = \frac{V_S}{R_1}$

KCL @ u1: $I_1 + I_2 = 0$ $\quad I_1 = -I_2$

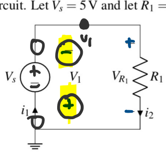
$I_1 = \frac{-V_S}{R_1}$

Power:

$P_{R_1} = I_2 \cdot V_{R_1} = \left(\frac{V_S}{R_1}\right) \cdot V_S = \frac{V_S^2}{R_1}$ dissipated

$P_{V_S} = I_1 \cdot V_1 = \frac{-V_S}{R_1} \cdot V_S = \frac{-V_S^2}{R_1}$ supplied

(c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5$ V and let $R_1 = 5\,\Omega$.



$V_S = u_1 - 0$

$V_1 = 0 - u_1$ $\qquad$ $V_1 = -V_S$

$V_1 = 0 - u_1$

$V_1 = -u_1$

$V_1 = -V_S$

$V_S = u_1$

$V_{R_1} = u_1 - 0$

$V_{R_1} = u_1$

$I_2 \cdot R_1 = V_{R_1}$

$I_2 \cdot R_1 = u_1 = V_S$

$I_2 = \frac{V_S}{R_1}$

KCL @ u_1: $i_1 = i_2$

$I_1 = \frac{V_S}{R_1} = I_2$

Power

$P_{R_1} = I_2 \cdot V_{R_1} = \frac{V_S}{R_1} \cdot V_S = \frac{V_S^2}{R_1}$

$P_{V_S} = I_1 \cdot V_1 = \frac{V_S}{R_1} \cdot -V_S = -\frac{V_S^2}{R_1}$

(d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

Element voltages/currents ✓

Power ✗ — Circuit changes or component values change

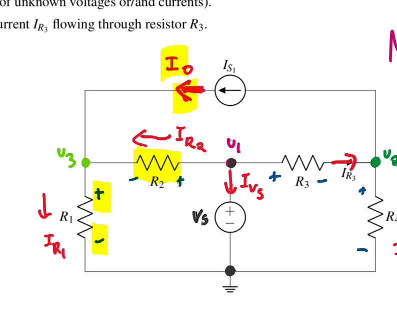
Node voltages/potentials ✗ — Circuit changes or component values change or GND changes

3. Circuit Analysis

(a) Use nodal analysis to solve for all node voltages.

(b) Practice writing out your expressions in matrix vector form. (Recall the form $\mathbf{A}\vec{x} = \vec{b}$, where $\vec{x}$ is your vector of unknown voltages or/and currents).

(c) Find current I_{R_3} flowing through resistor R_3.



NVA: Solve for node potentials

1. Select ground node
2-3. Label all nodes
4. Label element voltages & currents
5. Write out $A\vec{x} = \vec{b}$
6. Use KCL to fill in $A\vec{x} = \vec{b}$
7. Use Ohm's Law to fill in $A\vec{x} = \vec{b}$
8. Solve $A\vec{x} = \vec{b}$ using Gaussian elimination (or substitution)

(a) KCL @ u_3: $I_{R_2} + I_0 = I_{R_1}$ (1)

KCL @ u_1: $I_{R_2} + I_{V_S} + I_{R_3} = 0$ (2)

KCL @ u_2: $I_{R_3} - I_0 - I_{R_4} = 0$ (3)

$V_{R_1} = u_3 - 0$, $V_{R_1} = I_{R_1} \cdot R_1$

$V_{R_3} = u_1 - u_2$, $V_{R_3} = R_3 \cdot I_{R_3}$

$V_{R_4} = u_2 - 0$, $V_{R_4} = I_{R_4} \cdot R_4$

$V_{R_2} = u_1 - u_3$, $I_{R_2} \cdot R_2 = u_1 - u_3$

$I_{R_1} \cdot R_1 = u_3$ (4) $\quad u_1 - u_2 = R_3 \cdot I_{R_3}$ (5) $\quad u_2 = I_{R_4} \cdot R_4$ (6)

$I_0 = I_{S_1}$ (7)

Use KCL eqs to solve for node potentials

$I_{R_2} + I_0 = I_{R_1}$ (1), where $I_{R_2} = \frac{u_1 - u_3}{R_2}$, $I_0 = I_{S_1}$, $I_{R_1} = \frac{u_3}{R_1}$

$\frac{u_1 - u_3}{R_2} + I_{S_1} = \frac{u_3}{R_1}$ $\qquad V_S = u_1$ $\qquad \frac{V_S - u_3}{R_2} + I_{S_1} = \frac{u_3}{R_1}$

$u_2 = \frac{V_S R_4 - I_{S_1} R_3 R_4}{R_3 + R_4}$

$u_3 = \frac{V_S R_1 + I_{S_1} R_1 R_2}{R_1 + R_2}$

$\frac{V_S}{R_2} - \frac{u_3}{R_2} + I_{S_1} = \frac{u_3}{R_1}$ $\qquad$ Solve for node potential u_3

$\frac{V_S}{R_2} + I_{S_1} = \frac{u_3}{R_2} + \frac{u_3}{R_1}$

$\frac{V_S}{R_2} + I_{S_1} = \frac{(R_1 + R_2)\,u_3}{R_1 R_2}$ $\qquad$ Multiply both sides by $\frac{R_1 R_2}{R_1 + R_2}$

$\frac{V_S \cdot R_1 R_2}{R_2 (R_1 + R_2)} + I_{S_1} \cdot \frac{R_1 R_2}{R_1 + R_2} = u_3$

$\frac{V_S R_1 + I_{S_1} R_1 R_2}{R_1 + R_2} = u_3$

$I_{R_3} - I_0 - I_{R_4} = 0$ (3)

$\frac{u_1 - u_2}{R_3} - I_{S_1} - \frac{u_2}{R_4} = 0$

$I_{R_2} + I_{V_S} + I_{R_3} = 0$

$\frac{u_1 - u_3}{R_2} + I_{V_S} + \frac{u_1 - u_2}{R_3} = 0$

Since $V_S = u_1 - 0$

$\frac{V_S - u_2}{R_3} - I_{S_1} - \frac{u_2}{R_4} = 0$

$\frac{V_S}{R_3} - \frac{u_2}{R_3} - I_{S_1} - \frac{u_2}{R_4} = 0$ $\qquad$ Solve for node potential u_2

$\frac{V_S}{R_3} - I_{S_1} = \frac{u_2}{R_4} + \frac{u_2}{R_3}$

$\frac{V_S}{R_3} - I_{S_1} = \frac{(R_3 + R_4)\,u_2}{R_3 R_4}$ $\qquad$ Multiply both sides by $\frac{R_3 R_4}{R_3 + R_4}$

$\frac{V_S (R_3 R_4)}{R_3 (R_3 + R_4)} - I_{S_1}\left(\frac{R_3 R_4}{R_3 + R_4}\right) = u_2$

$\frac{V_S R_4 - I_{S_1} R_3 R_4}{R_3 + R_4} = u_2$

b)

$I_{V_{S_1}}$	I_0	I_{R_1}	I_{R_2}	I_{R_3}	I_{R_4}	u_1	u_2	u_3		$\vec{x}$		$\vec{b}$
0	1	-1	1	0	0	0	0	0		$I_{V_{S_1}}$		0
1	0	0	1	1	0	0	0	0		I_0		0
0	-1	0	0	1	-1	0	0	0		I_{R_1}		0
0	1	0	0	0	0	0	0	0		I_{R_2}	=	I_{S_1}
0	0	$-R_1$	0	0	0	0	0	1		I_{R_3}		0
0	0	0	$-R_2$	0	0	1	0	-1		I_{R_4}		0
0	0	0	0	0	0	1	0	0		u_1		V_S
0	0	0	0	$-R_3$	0	1	-1	0		u_2		0
0	0	0	0	0	$-R_4$	0	1	0		u_3		0

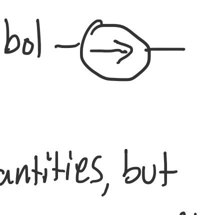
c) $I_{R_2} = \frac{u_1 - u_3}{R_2}$ $\qquad u_1 = V_S$

$I_{R_2} = \frac{V_S - u_3}{R_2}$ where $u_3 = \frac{V_S R_1 + I_{S_1} R_1 R_2}{R_1 + R_2}$ from part b

Questions:

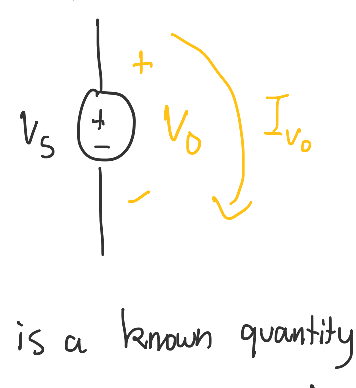
1. Is I_{S_1} a known value?

Yes! I_{S_1} is the current delivered by the current source

A current source is denoted by the symbol 

Both V_S and I_{S_1} are considered known quantities, but the element voltages and currents are unknown quantities (element current for the current source in problem 3 is I_0 which was shown to be equal to I_{S_1}).

For example,



V_S is a known quantity, but V_0, the element voltage for the voltage source, is something we defined so it is considered an "unknown" (in this case $V_S = V_0$).