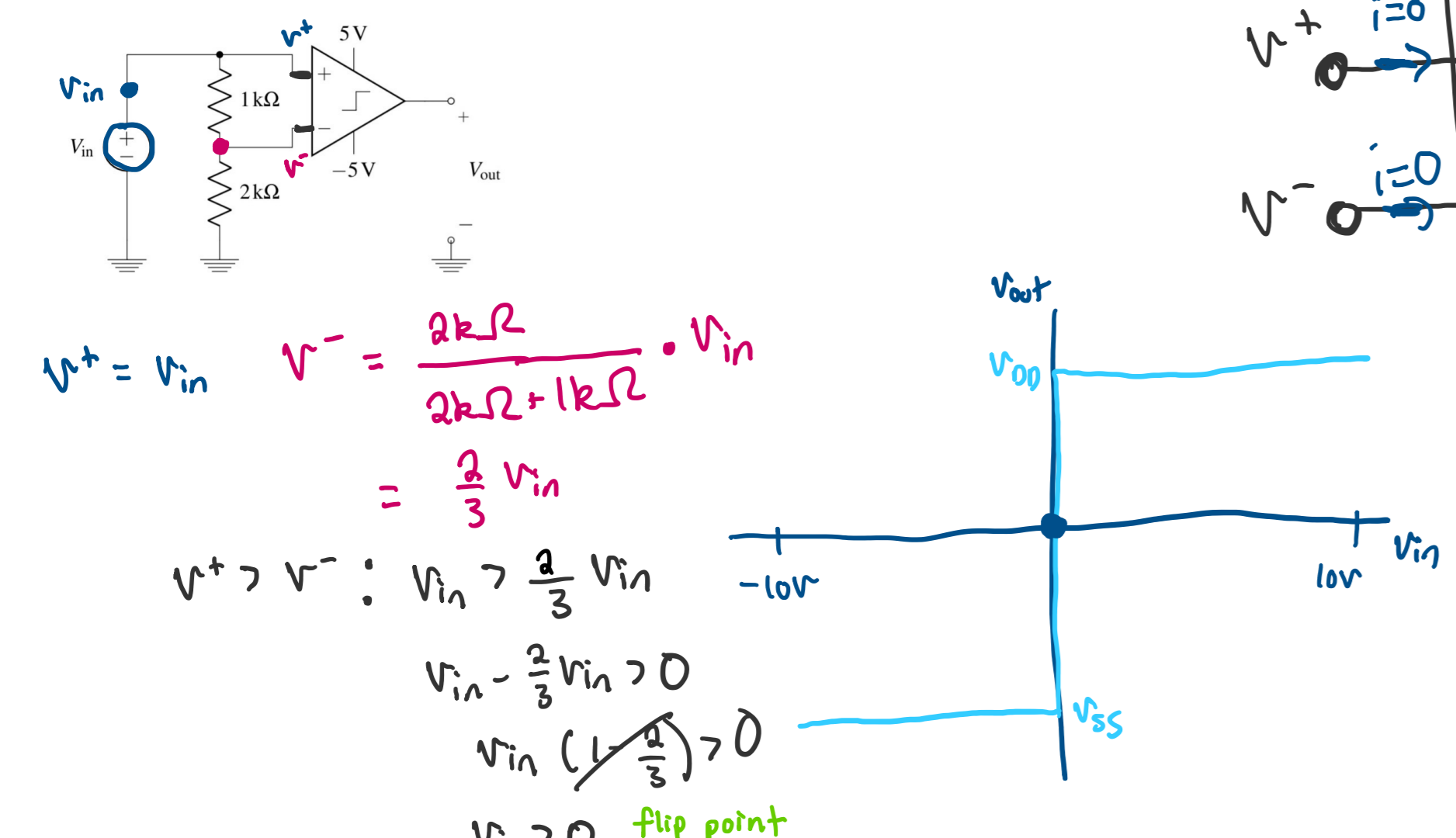
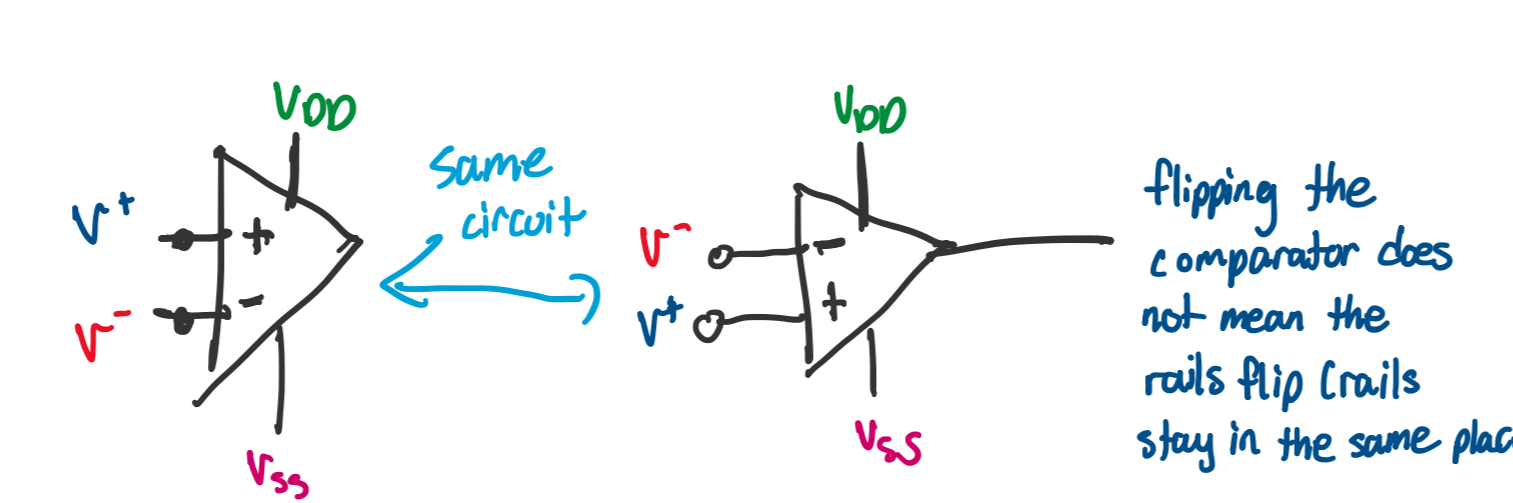
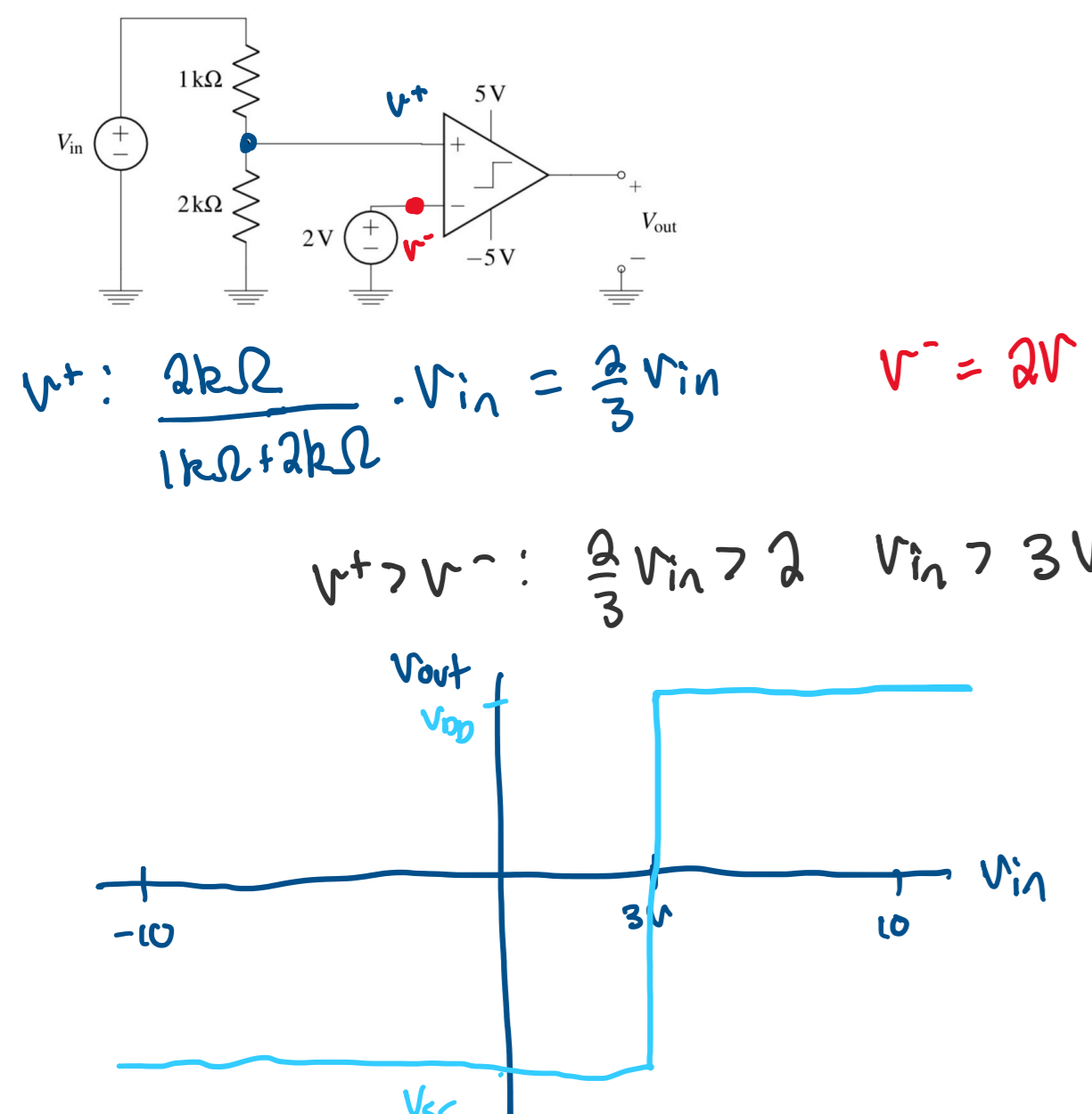


Feedback form: [tinyurl.com/anshal6aFeedback](https://tinyurl.com/anshal6aFeedback)

1. Comparators  
 For each of the circuits shown below, find  $V_{out}$  as a function of  $V_{in}$  for part (a) and from  $V_{in}$  to  $V_{out}$  for part (b).



$$V_{out} = \begin{cases} V_{DD} & \text{if } V^+ > V^- \\ V_{SS} & \text{if } V^+ < V^- \end{cases}$$

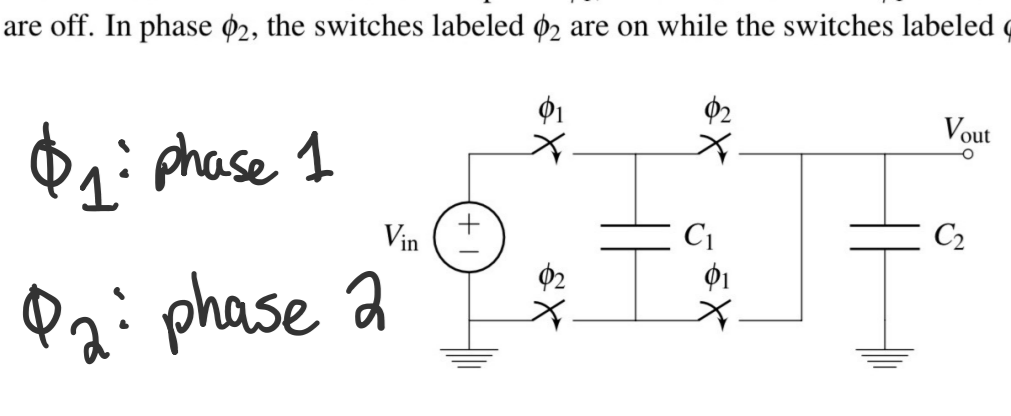


2. Charge Sharing

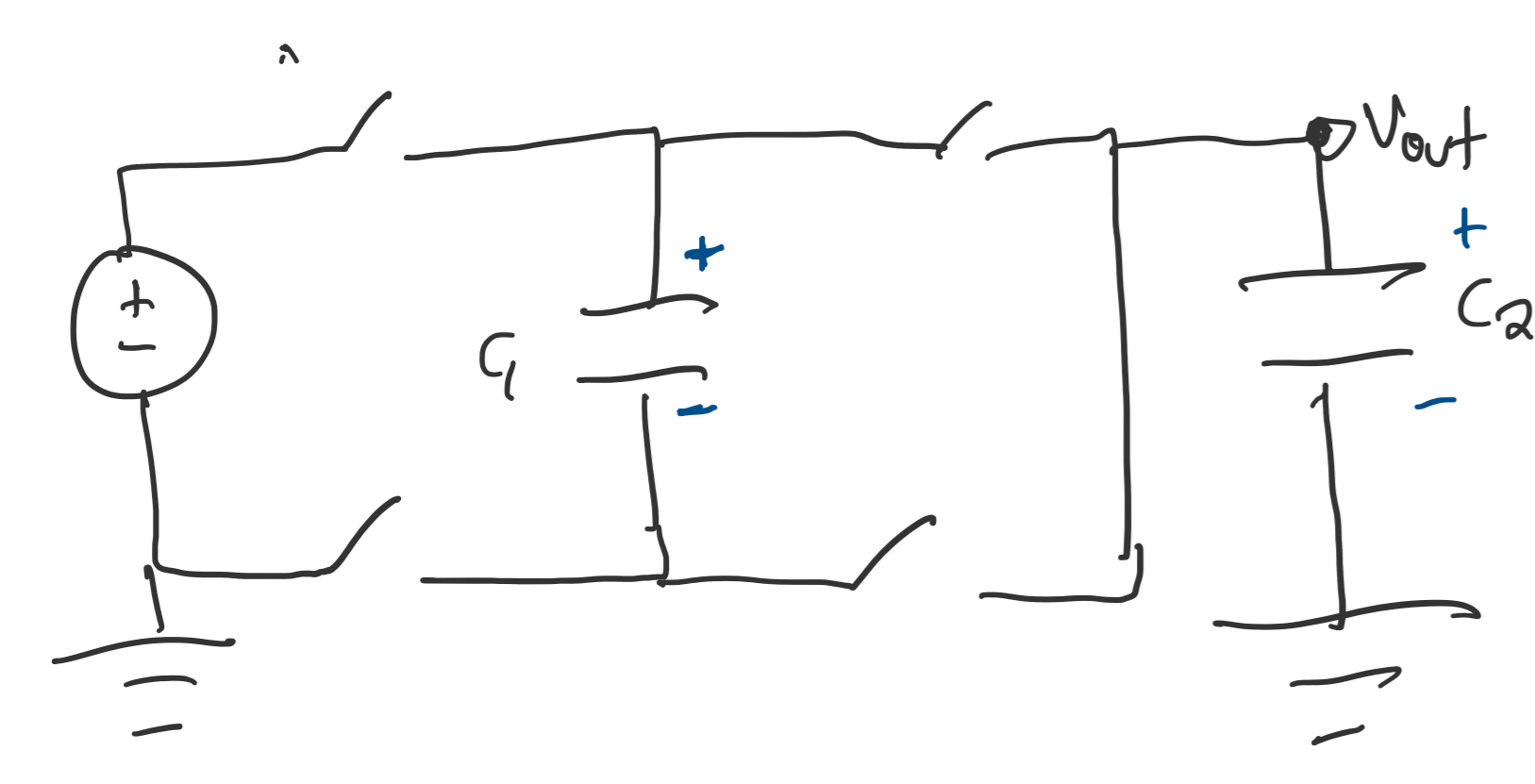
**Before the charge sharing:**  
 Step 1: Label the voltages across all the capacitors. Choose whichever direction (polarity) you want for each capacitor - this means you can mark any one of the plates with the "+" sign, and then you can mark the other plate with the "-" sign. Just make sure you stay consistent with this polarity across phases.  
 Step 2: Draw the equivalent circuit during both phases (Phase 1:  $\phi_1$  closed,  $\phi_2$  open; Phase 2:  $\phi_1$  open,  $\phi_2$  closed). Also, label all node voltages on the circuit for both phases. No need to try and maintain the same names, since certain nodes of the phase 1 circuit might be merged or split in phase 2.  
 Step 3: Calculate the node voltages during phase 1. This can be done by using one of the previously introduced circuit analysis techniques (most likely KVL will do the job).  
 Step 4: Identify all "floating" nodes in your circuit during phase 2. A floating node is a node out of or into which no charge can flow. You can identify these nodes as the nodes connected only to capacitor plates, on any inputs or compare inputs. These will be the nodes where we apply charge sharing.  
 Step 5: For steps 5-7 we will examine each phase 2 floating node individually. Pick a floating node from the ones you found in step 4 and identify all capacitor plates connected to that node during phase 2. Then, calculate the charge on each of these plates during phase 1.  
 To do so, identify all nodes in your circuit during phase 1. Label all node voltages, and write the voltages across each capacitor as functions of node voltages (step 2 and 3 should help you with that). Do this according to the polarities you have selected. Then the charge is found as  $Q = CV$ , (where  $V$  is the voltage across a capacitor).  
**Caution:** The plate marked with the "-" sign will have  $Q = -CV$  and the plate marked with the "+" sign will have  $Q = CV$  stored onto them.  
 Step 6: Find the total charge on each of the floating nodes during phase 2 as a function of node voltages. Use the same process as in Step 5, but this time using the node voltages during phase 2 to write the voltages across each capacitor. Make sure you kept the polarity same and pay attention to the sign of each plate.  
 Step 7: Equate the total charge calculated in phase 1 (Step 4) to the total charge calculated in phase 2 (Step 6) to charge conservation.  
 Step 8: Repeat steps 5-7 for every floating node. This will give you one equation per floating node (i.e. if you have  $n$  floating nodes you will have  $n$  equations). You can then solve the system of equations to find the node voltages during phase 2 (unknowns). It should have a unique solution!  
 Consider the circuit shown below. In phase  $\phi_1$ , the switches labeled  $\phi_1$  are on while the switches labeled  $\phi_2$  are off. In phase  $\phi_2$ , the switches labeled  $\phi_2$  are on while the switches labeled  $\phi_1$  are off.

Charge Sharing Algorithm:

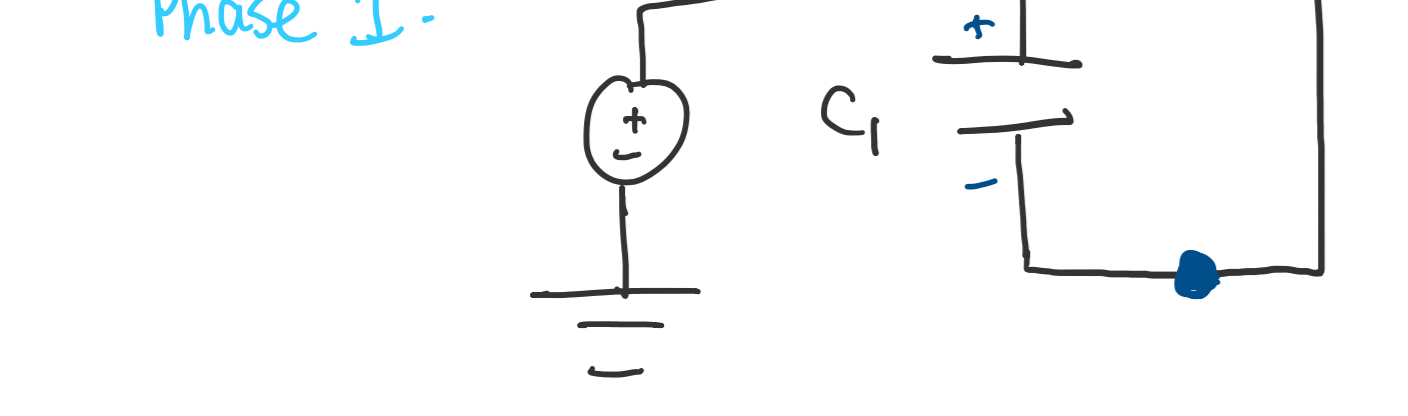
1. Label capacitor voltages and choose polarities to be used for rest of the algorithm
2. Draw circuit for each phase
3. In phase 2, label floating nodes
4. For each floating node, solve for  $Q_{\phi 2}$
5. Solve for  $Q_{\phi 1}$
6. Equate  $Q_{\phi 1} = Q_{\phi 2}$  due to charge conservation
7. Repeat steps 4-6 for each floating node



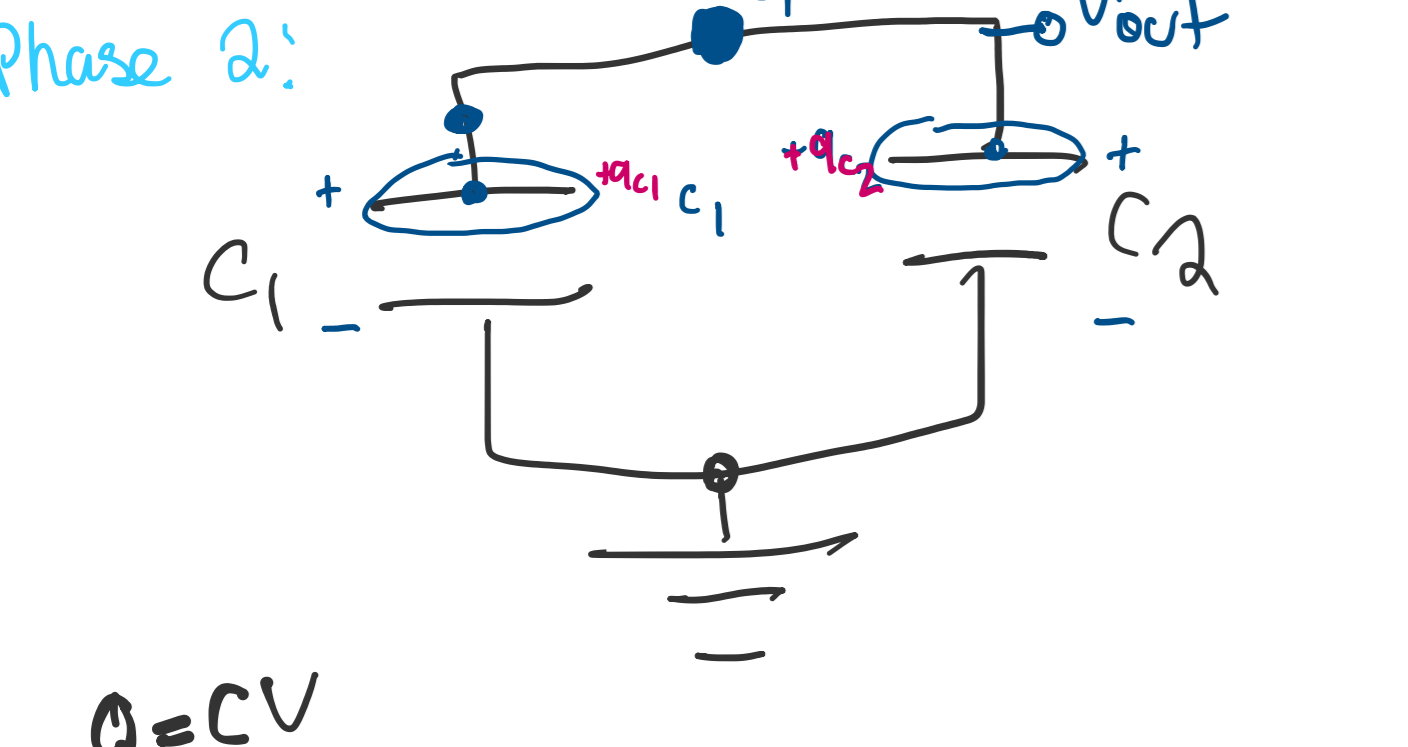
(a) Draw the polarity of the voltage (using + and - signs across the two capacitors  $C_1$  and  $C_2$ ). (It doesn't matter which terminal you label + or -; just remember to keep these consistent through phase 1 and 2)



(b) Redraw the circuit in phase  $\phi_1$  and phase  $\phi_2$ . Keep your polarity from part (a) in mind.



(c) Find  $V_{out}$  in phase  $\phi_2$  as a function of  $V_{in}$ ,  $C_1$ , and  $C_2$ .



Series capacitors:  
 $Q_T = Q_1 = Q_2 = \dots = Q_n$   
 $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$   
 $Q = CV \quad Q_T = \frac{C_1 C_2}{C_1 + C_2} \cdot V_{in} = Q_1$

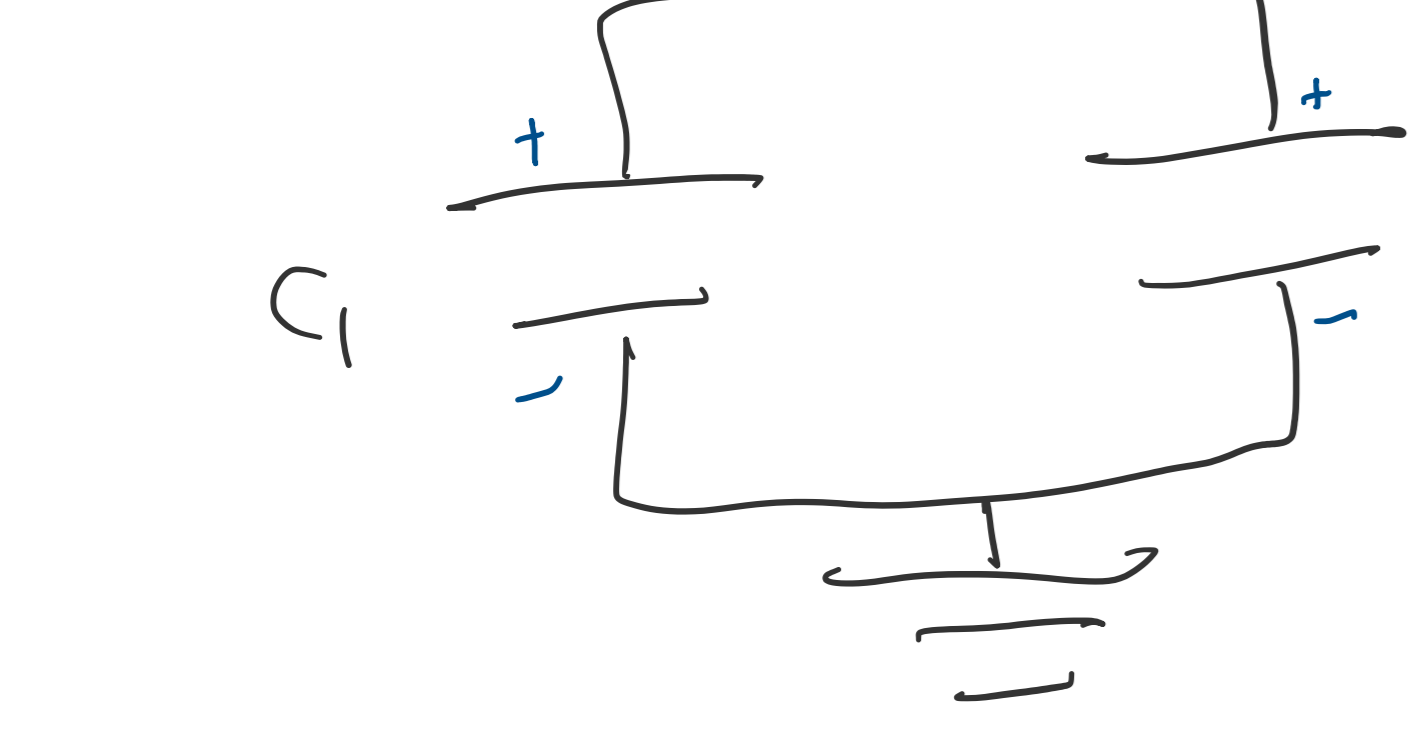
$$Q = CV$$

$$Q_{C1} = C_1 V_{out} + Q_{C2} = Q_{C1} + Q_{C2}$$

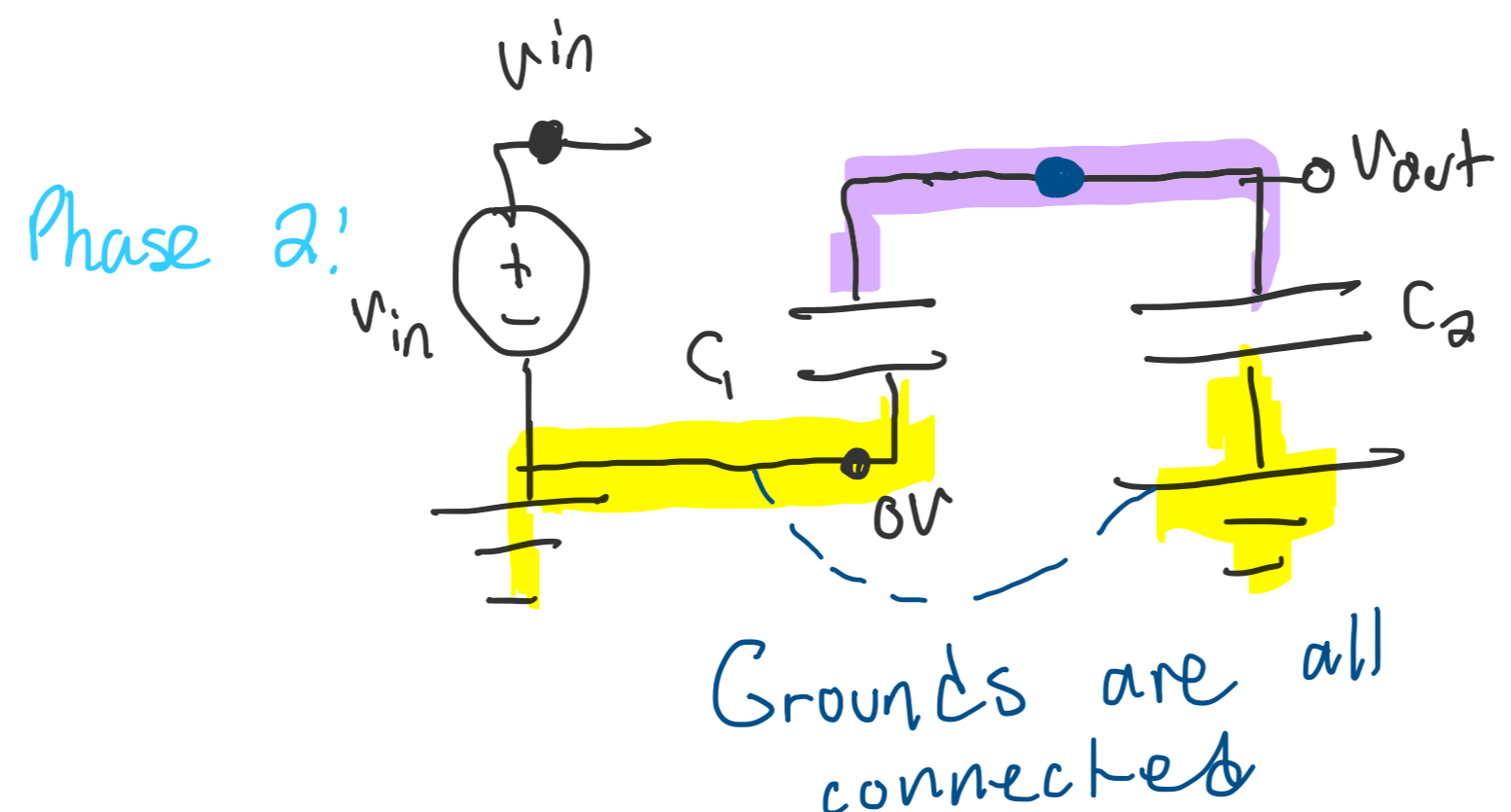
$$Q_{C1} = C_1 V_{out} + C_2 V_{out} = \frac{C_1 C_2}{C_1 + C_2} V_{in} + \frac{C_1 C_2}{C_1 + C_2} V_{in}$$

$$V_{out} = \frac{2 \cdot C_1 C_2 V_{in}}{(C_1 + C_2)^2}$$

(d) How will the charges be distributed in phase  $\phi_2$  if we assume  $C_1 \gg C_2$ ?

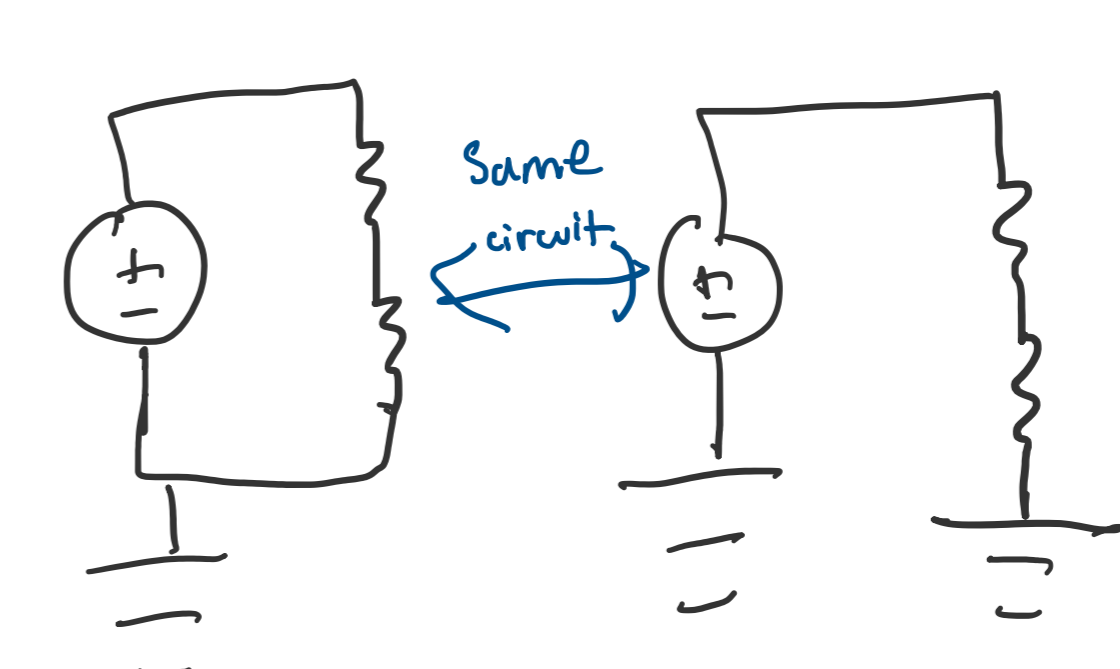


Since  $C_1 \gg C_2$  and  $V_{C1} = V_{C2} = V_{out}$   
 $Q_1 \gg Q_2$

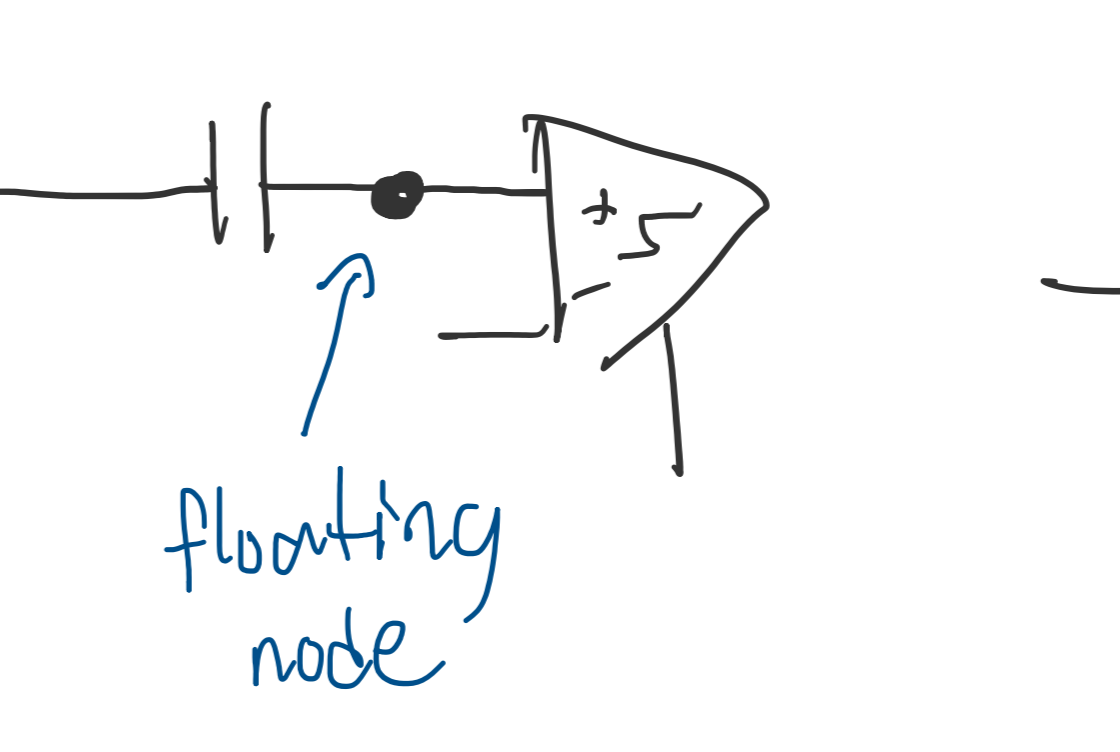
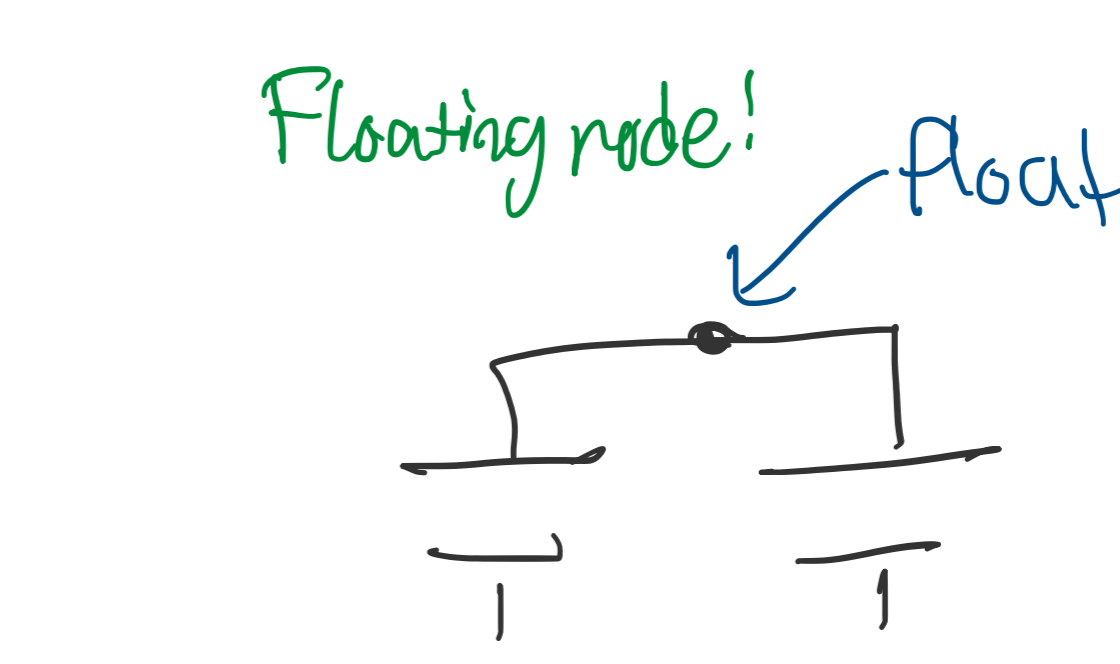


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Since  $C_1 \gg C_2$  and  $V_{C1} = V_{C2} = V_{out}$   
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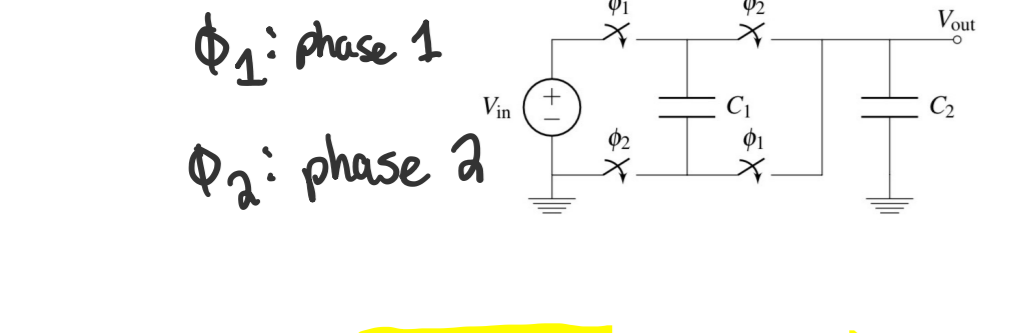
Grounds are all connected



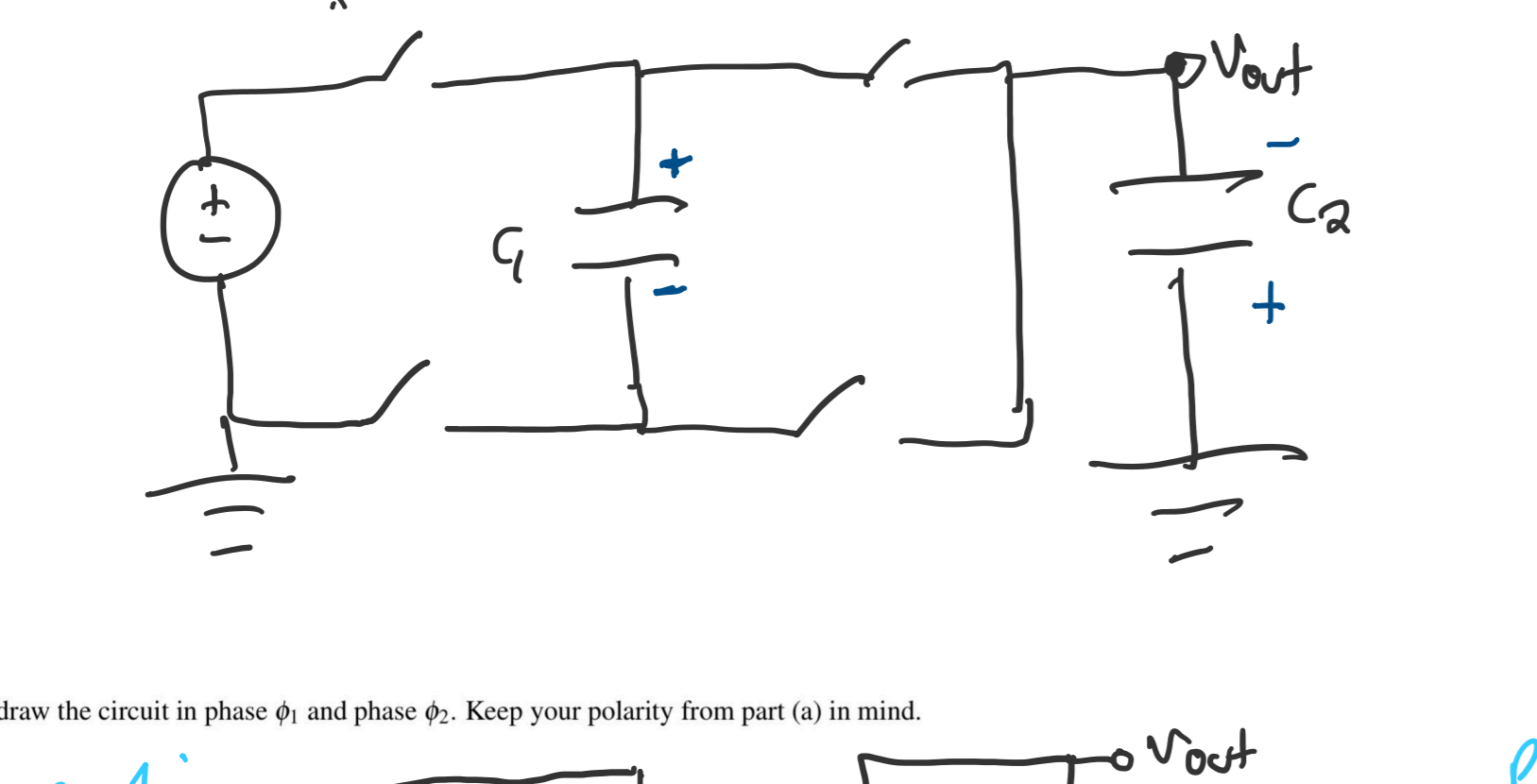
Redo problem 2 w/  $C_2$  polarity flipped

2. Charge Sharing

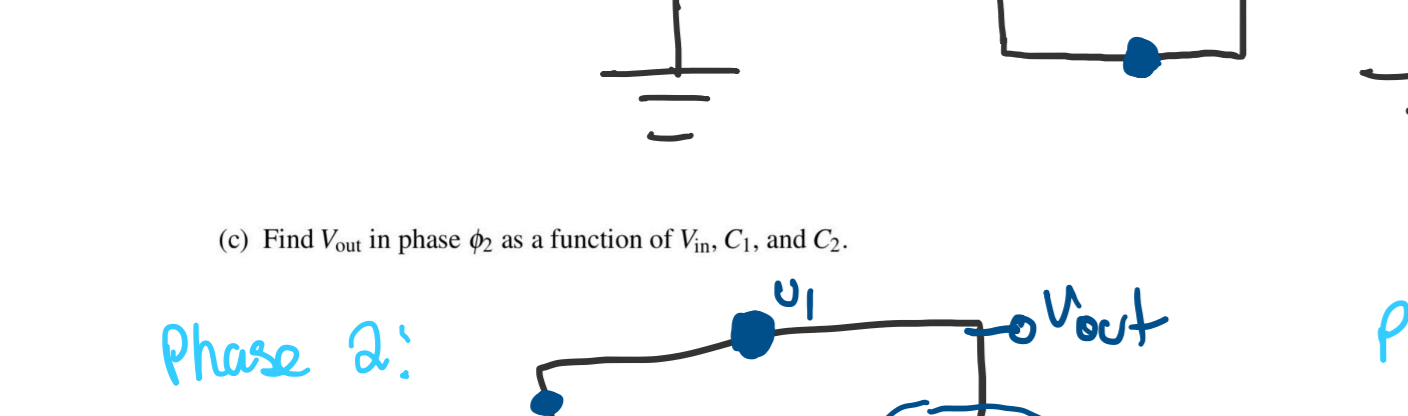
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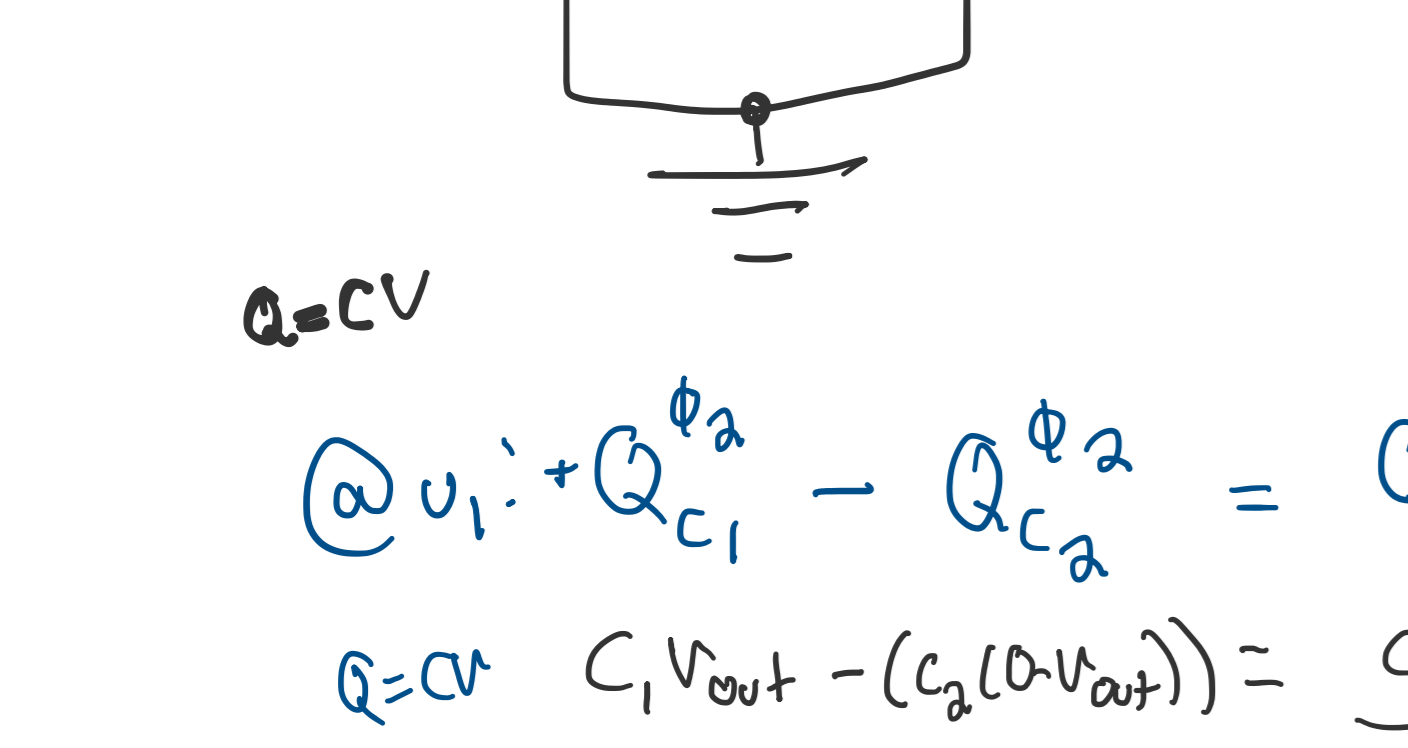
(a) Draw the polarity of the voltage (using + and - signs across the two capacitors  $C_1$  and  $C_2$ ). (It doesn't matter which terminal you label + or -; just remember to keep these consistent through phase 1 and 2)



(b) Redraw the circuit in phase  $\phi_1$  and phase  $\phi_2$ . Keep your polarity from part (a) in mind.



(c) Find  $V_{out}$  in phase  $\phi_2$  as a function of  $V_{in}$ ,  $C_1$ , and  $C_2$ .



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 $Q = CV \quad Q_T = \frac{C_1 C_2}{C_1 + C_2} \cdot V_{in} = Q_1$

$$Q = CV$$

$$Q_{C1} = C_1 V_{out} - Q_{C2} = Q_{C1} - Q_{C2}$$

$$Q_{C1} = C_1 V_{out} - C_2 (0 - V_{out}) = \frac{C_1 C_2}{C_1 + C_2} V_{in} - \left( \frac{-C_1 C_2}{C_1 + C_2} V_{in} \right)$$

$$C_1 V_{out} + C_2 V_{out} = \frac{C_1 C_2}{C_1 + C_2} V_{in} + \frac{C_1 C_2}{C_1 + C_2} V_{in}$$

$$V_{out} = \frac{2 \cdot C_1 C_2 V_{in}}{(C_1 + C_2)^2}$$

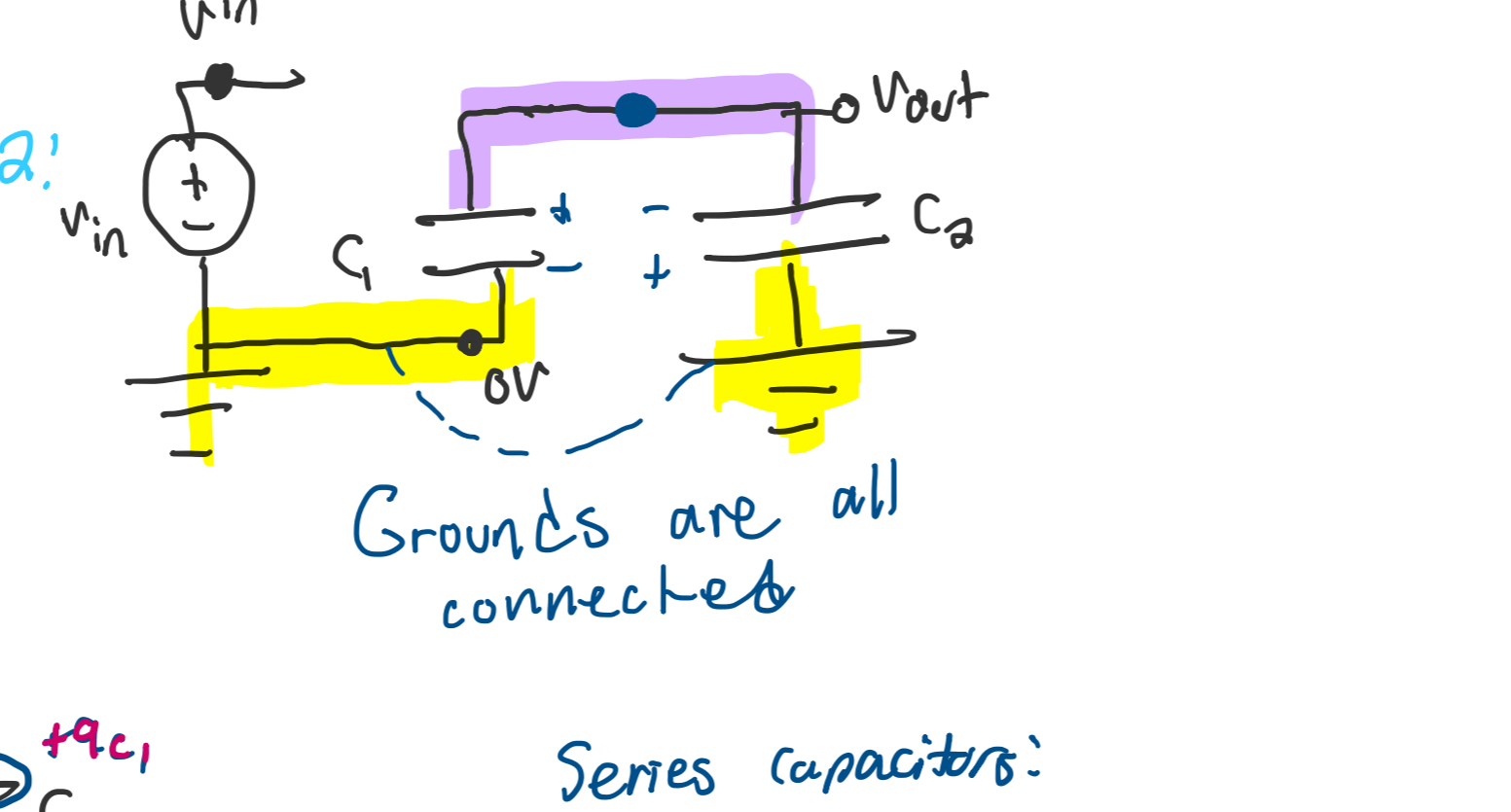
(d) How will the charges be distributed in phase  $\phi_2$  if we assume  $C_1 \gg C_2$ ?



Since  $C_1 \gg C_2$  and  $V_{C1} = V_{C2} = V_{out}$   
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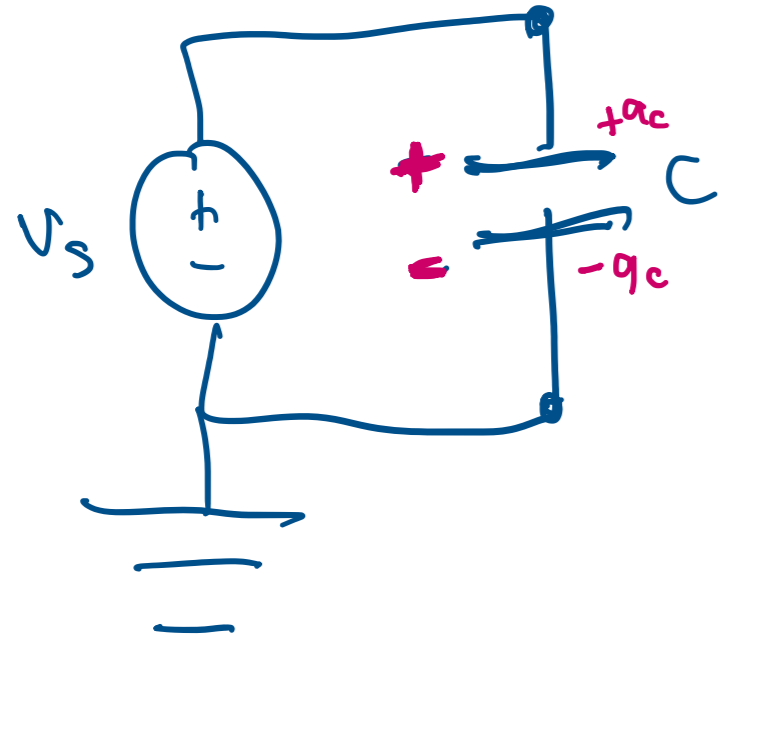


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Since  $C_1 \gg C_2$  and  $V_{C1} = V_{C2} = V_{out}$   
 $Q_1 \gg Q_2$

Questions:

1. How do you know the charges on the plates of the capacitors when it is hooked up to a capacitor?



$Q = CV$   
 $Q_C = C \cdot V_3$   
 charge on capacitor (+qC on positive plate and -qC on negative plate)