Augmented Matrix Review

→ Setting up Matrix

* line up variables
* pay attention to sign

$$\begin{aligned} 2x &+ 4y &+ 2z &= 8 \\ x &+ y &+ z &= 6 \\ x &- y &- z &= 4 \end{aligned} \quad \rightarrow \quad \left[\begin{array}{ccc|c} 2 & 4 & 2 & 8 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 4 \end{array}\right]$$

→ Manipulation

* Scaling:

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 2 & 4 & 2 & 8 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 4 \end{array}\right] \begin{array}{c} \frac{1}{2} \\ \\ \\ \end{array} \quad \underset{\sim}{\tfrac{1}{2} \times R1} \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 4 \end{array}\right]$$

$$\begin{array}{ll} R1 & (2x + 4y + 2z = 8)\tfrac{1}{2} \\ R2 & x + y + z = 6 \\ R3 & x - y - z = 4 \end{array} \quad \underset{\sim}{\tfrac{1}{2} \times R1} \quad \begin{array}{l} x + 2y + z = 4 \\ x + y + z = 6 \\ x - y - z = 4 \end{array}$$

* Adding Rows:

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 4 \end{array}\right] \quad \underset{\sim}{R1 - R2} \quad \left[\begin{array}{ccc|c} 0 & 1 & 0 & -2 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 4 \end{array}\right]$$

$$\begin{array}{ll} R1 & x + 2y + z = 4 \\ R2 & x + y + z = 6 \\ R3 & x - y - z = 4 \end{array} \quad \underset{\sim}{R1 - R2} \quad \begin{array}{r} x + 2y + z = 4 \\ -(x + y + z = 6) \\ \hline 0 + y + 0 = -2 \end{array}$$

Reduce to Upper Diagonal System

$$\left[\begin{array}{cccc|c} 1 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 1 & * \end{array}\right]$$

($* \equiv$ any #)

this direct equivalence makes solving easy!

Example:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \longrightarrow \begin{aligned} x - y + 2z &= 1 \\ y - z &= 2 \\ z &= 1 \end{aligned}$$

→ Watch for Contradictions:

$$\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & 0 & 0 & 3 \\ * & * & * & * \end{array}\right] \longrightarrow \begin{aligned} 0x + 0y + 0z &= 3 \\ 0 &\neq 3 \end{aligned}$$

→ and Undefined Variables:

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \infty \text{ solutions}$$

$0 = 0$