EECS 16A   Designing Information Devices and Systems I
Fall 2021   Homework 9

This homework is due Friday, October 29, 2021 at 23:59.
Self-grades are due Monday, November 01, 2021 at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw9.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

For this homework, please read Notes 16, 17 (17.1 - 17.2, specifically) and 17B. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them. Sections 17.1 - 17.2 in Note 17 will provide an overview of the capactive touchscreen and how to measure capacitance. Note 17B will provide a walkthrough of the charge-sharing algorithm.

   (a) Describe the key ideas behind how a capacitor works. How are capacitor equivalences calculated? Compare this with how we calculate resistor equivalences.

   (b) Consider the capacitive touchscreen. Describe how it works, and compare and contrast it to the resistive touchscreens we have seen in previous lectures and homeworks.

   (c) In the charge sharing algorithm, what property of charge is applied in connecting phase 1 calculations to phase 2 calculations?

2. **Thévenin and Norton Equivalent Circuits** Find the Thévenin and Norton equivalent circuits seen from terminals A and B.
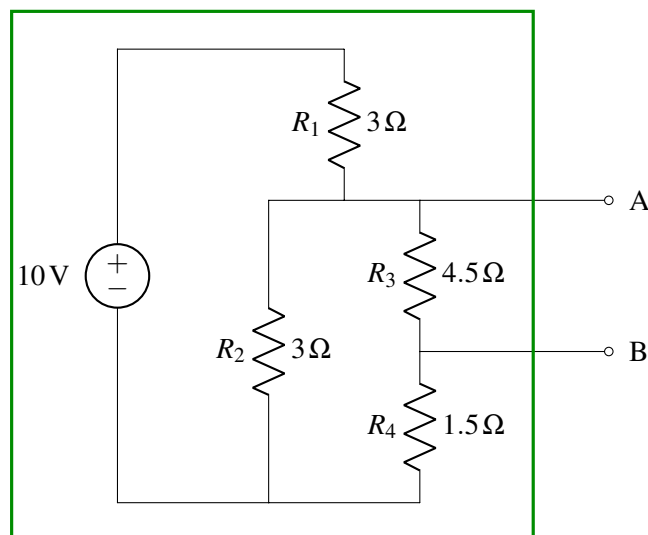
3. Capacitive Touchscreen

The model for a capacitive touchscreen can be seen in Figure 1. See Table 1 for values of the dimensions. The green area represents the contact area of the finger with the top insulator. It has dimensions $w_2 \times d_1$, where w_2 is the horizontal width of the finger contact area and d_1 is the depth (into the page) of the finger contact area. The top metal (red area) has dimensions $w_1 \times d_1$. The bottom metal (grey area) has dimensions $w \times d_2$, where w is larger than both w_1 and w_2. The vertical distance between the top metal (red) and bottom plate (grey) is t_1, and the vertical distance between the finger (green) and the bottom plate (grey) is t_2.
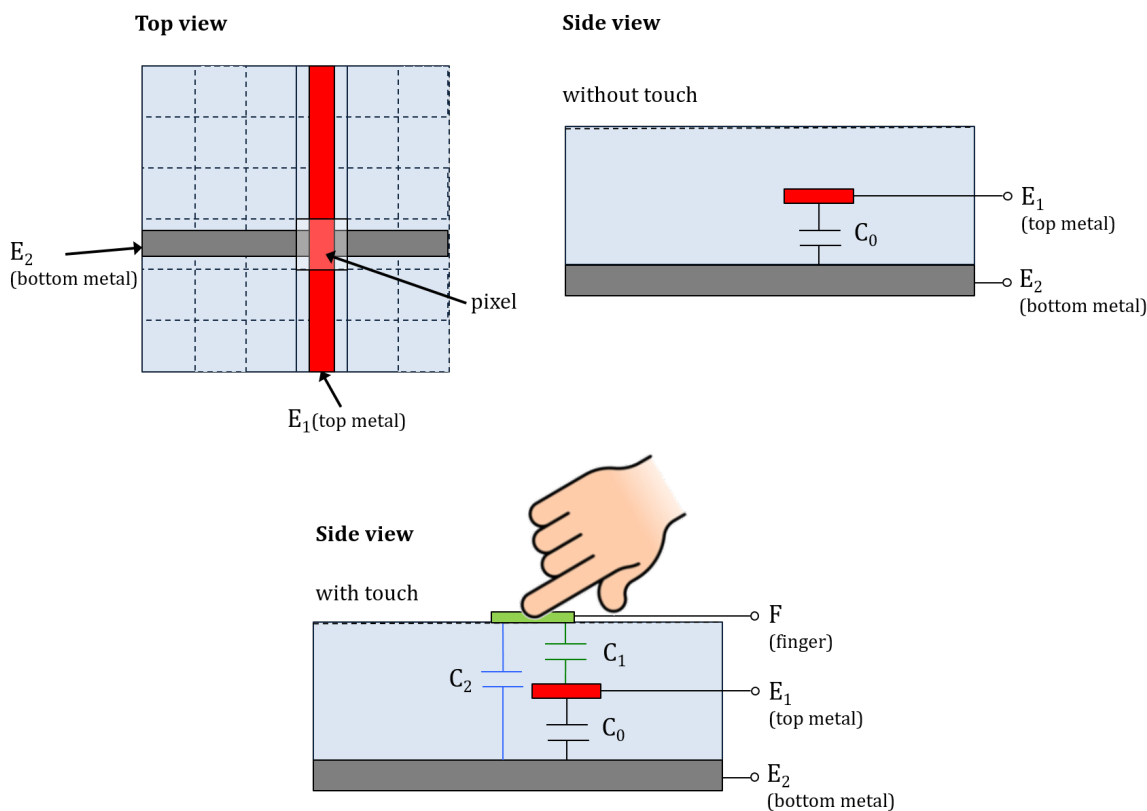

Figure 1: Model of capacitive touchscreen.

Table 1: Touchscreen Dimension Values

d_1	10 mm
d_2	1 mm
t_1	2 mm
t_2	4 mm
w_1	1 mm
w_2	2 mm

(a) Draw the equivalent circuit of the touchscreen that contains the nodes F, E_1, and E_2 when: (i) there no finger present; and (ii) when there is a finger present. Express the capacitance values in terms of C_0, C_{F-E1}, and C_{F-E2}.

Hint: Note that node F represents the finger. When there is no touch node F would be non-existent.
Hint: Treat E_1 as the "top node", E_2 as the "bottom node", and the finger F as an intermediate node when present.

(b) What are the values of C_0, C_{E1-F}, and C_{F-E2}? Assume that the insulating material has a permittivity of $\varepsilon = 4.43 \times 10^{-11} F/m$ and that the thickness of the metal layers is small compared to t_1 (so you can ignore the thickness of the metal layers). Also assume that the right edge of the top metal (red area) in the diagram is aligned with the right edge of the finger (green area) in the diagram.

(c) What is the difference in effective capacitance between the two metal plates (nodes E_1 and E_2) when a finger is present?

(d) What are the advantages of capacitive touchscreen over resistive touchscreen?
Hint: Can we do multi-fingers detections using capacitive touchscreen? What about resistive touchscreen?

4. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from the ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how maglev trains use capacitors to stay elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, we put two parallel strips of metal (T_1, T_2) along the bottom of the train and we have one solid piece of metal (M) on the ground below the train (perhaps as part of the track).

Side View   Top View
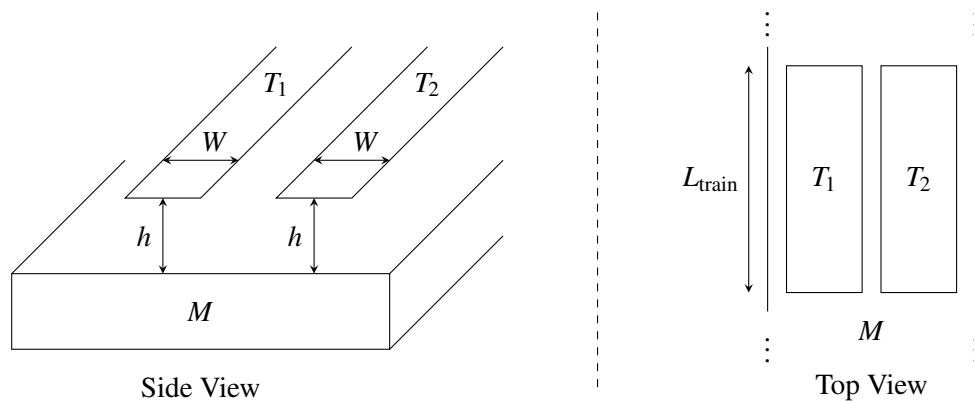
Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., we can use the simple equations developed in lecture to model the capacitance), as a function of L_{train} (the length of the train), W (the width of T_1 and T_2), and h (the height of the train off of the track), what is the capacitance between T_1 and M? What is the capacitance between T_2 and M?

(b) Any circuit on the train can only make direct contact at T_1 and T_2. Thus, you can only measure the equivalent capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance between T_1 and T_2.

(d) We want to build a circuit that creates a voltage waveform with an amplitude that changes based on the height of the train. Your colleague recommends you start with the circuit as shown below, where I_s

is a periodic current source, and C_{eq} is the equivalent capacitance between T_1 and T_2. The graph below shows I_s, a square wave with period τ and amplitude I_1, as a function of time.

Find an equation for and draw the voltage $V_{C_{eq}}(t)$ as a function of time. Assume the capacitor C_{eq} is discharged at time $t = 0$, so $V_{C_{eq}}(0) = 0\,\text{V}$.
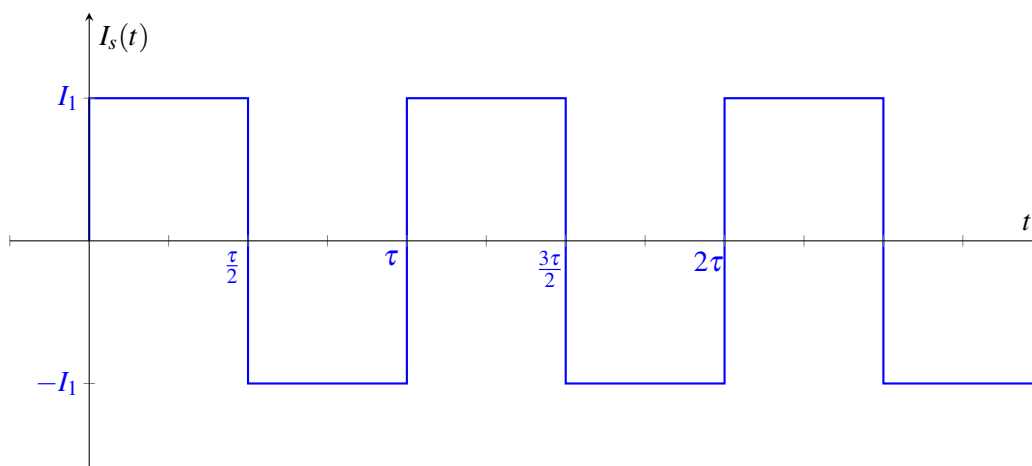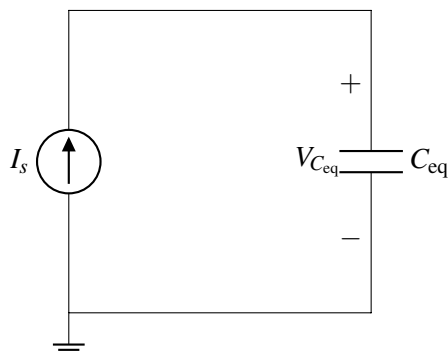
(e) So far we've assumed that the height of the train off of the track is uniform along its entire length, but in practice, this may not be the case. Suggest and sketch a modification to the basic sensor design (i.e., the two strips of metal T_1 and T_2 along the entire bottom of the train) that would allow you to measure the height at the train at 4 different locations.

5. **Circuit with Capacitors**

Task: Find the voltages at nodes u_A and u_B, and currents flowing through all of the capacitors at steady state. Assume that before the voltage source is applied, the capacitors all initially have a charge of 0 Coulombs.

Guide: In general, your strategy to solve circuits with capacitors should be similar to solving resistive circuits. For capacitive circuits we often care about steady state (i.e. what happens to the circuit after a long time and no more changes are happening). If we consider a circuit with capacitors and voltage sources, we will always think about steady state (or the steady state for a phase if we work on a charge sharing problem with switches). When thinking about steady state, you always want to write out the equations for charge that you know, as well as all the KVL type relationships around voltages you know. Then use the key idea that charge is conserved to build out your system of equations. Don't be daunted by the variable names and know that everything just boils down to a system of linear equations.

Here are some principles that are also helpful:

(a) Charge at a node from which charge cannot escape or enter (floating node) is always conserved. — if the sum of charges is 0 on a floating node, the sum of charges on that floating node at steady state will be zero.

(b) The charge Q stored in a capacitor is given by the equation $Q = CV$. That is, the plate that corresponds to the "+" terminal, stores $+Q = +CV$, and the plate that corresponds to the "-" terminal, stores $-Q = -CV$.

(c) If two capacitors are initially uncharged, and then are connected in series, the charges on both capacitors are equal to each other at steady state.

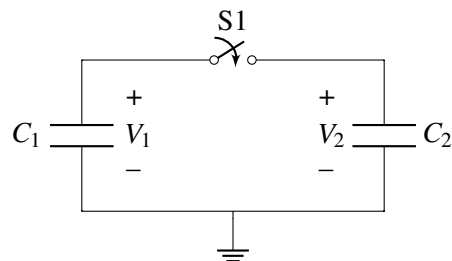(d) The voltage across capacitors in parallel is equal at steady state.
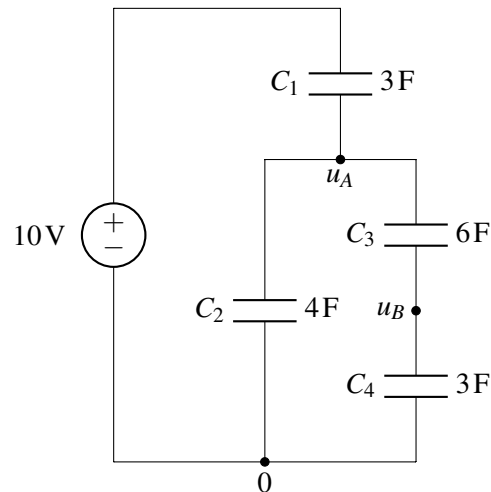
6. Charge Sharing

In the circuit below, switch S_1 is initially open. Capacitor $C_1 = 10^{-3}$F is initially charged to $V_1 = 3$V and capacitor $C_2 = 3 \times 10^{-3}$F is initially charged to $V_2 = 2$V.

Now S_1 is closed. Calculate the new value of V_2.

Hint: Remember that charge is conserved at floating nodes.

7. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.