EECS 16A Designing Information Devices and Systems I
Fall 2021 Homework 10

This homework is due November 5, 2021, at 23:59.
Self-grades are due November 8, 2021, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw10.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

No submission is required for this problem, however we ask that you read and understand the notes.

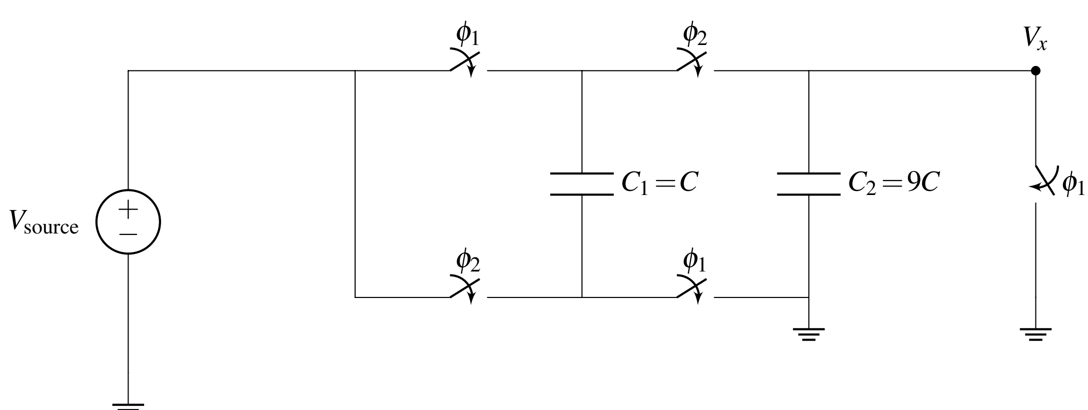
For this homework, please read Note 17A to learn about comparators and op-amps, and Note 17B to learn about charge sharing. You are always encouraged to read beyond this as well.

(a) If the op-amp supply voltages are $V_{DD} = 5\,\text{V}$ and $V_{SS} = 0\,\text{V}$, then what is the minimum/maximum value of V_{out}?

(b) What is the purpose of a comparator? How can we use a comparator circuit to detect a touch for a capacitive touchscreen?

2. **Charge Sharing**

Consider the following circuit:



In the first phase, all of the switches labeled ϕ_1 will be closed and all switches labeled ϕ_2 will be open. In the second phase, all switches labeled ϕ_1 are opened and all switches labeled ϕ_2 are closed.

(a) Draw the polarity of the voltage (using + and − signs) across the two capacitors C_1 and C_2. (It doesn't matter which terminal you label + or −; just remember to keep these consistent through phase 1 and 2!)

(b) Draw the circuit in the first phase and in the second phase. Keep your polarity from part (a) in mind.

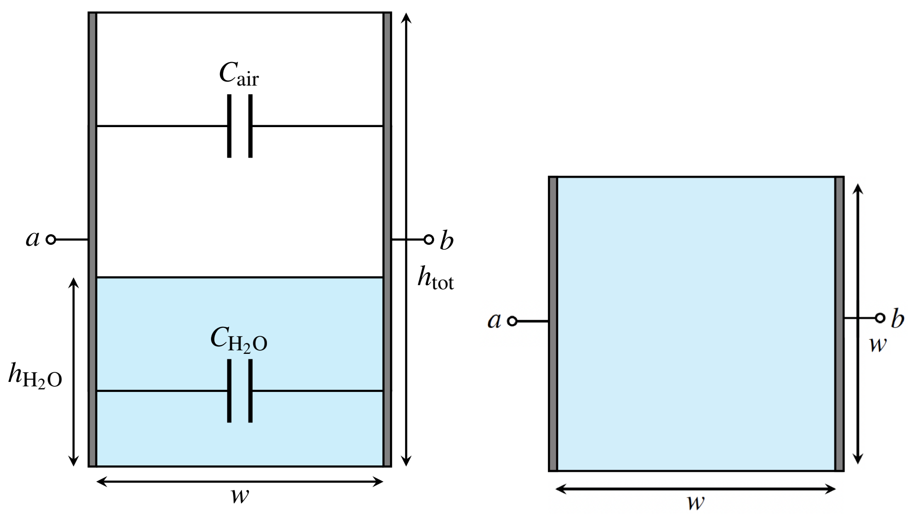
(c) Find the voltage across and the charge on C_1 and C_2 in phase 1. Be sure to keep the polarities of the voltages the same!

(d) Now, in the second phase, find the voltage V_x.

(e) If the capacitor C_2 did not exist (i.e. had a capacitance of 0 F), what would the voltage V_x be?

3. It's finally raining!

A lettuce farmer in Salinas Valley has grown tired of weather.com's imprecise rain measurements. Therefore, they decided to take matters into their own hands by building a rain sensor. They placed a square tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.



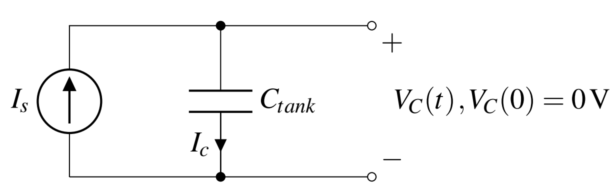
Tank side view (left) and top view (right).

The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) What is the capacitance between terminals a and b when the tank is full? What about when it is empty? *Note:* the permittivity of air is ε, and the permittivity of rainwater is 81ε.

(b) Suppose the height of the water in the tank is h_{H_2O}. Model the tank as a pair of capacitors in parallel, where one capacitor has a dielectric of air, and one capacitor has a dielectric of water. Find the total capacitance between the two plates using equivalence. Call this capacitance C_{tank}.

(c) After building this capacitor, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends the following:



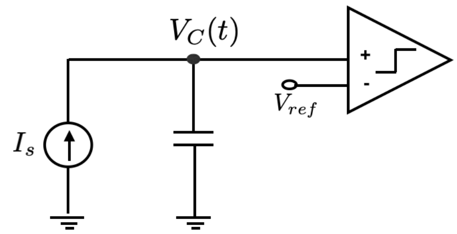
In this circuit, C_{tank} is the total tank capacitance that you calculated earlier. I_s is a known current supplied by a current source.

The suggestion is to measure V_C for a brief interval of time, and then use the difference to determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank}, i.e. V_C, is initialized to $0\,\text{V}$, i.e. $V_C(0) = 0$.

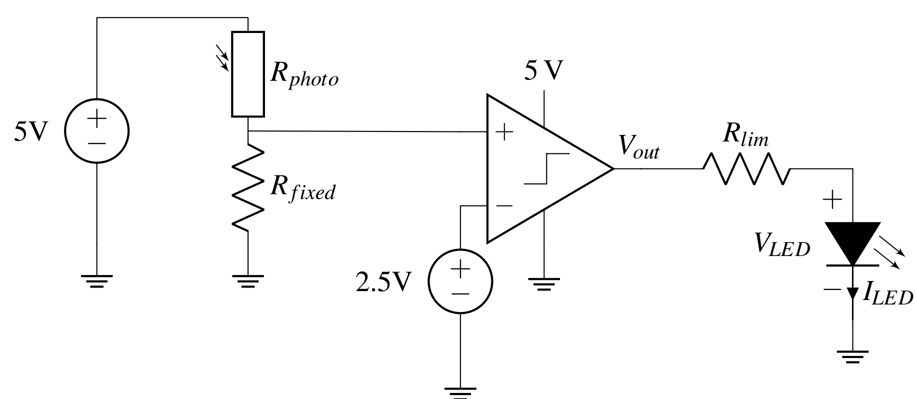
(d) Using the equation you derived for $V_C(t)$, describe how you can use this circuit to determine C_{tank} and $h_{\text{H}_2\text{O}}$.

(e) However, after spending some time thinking about different ways of measuring this capacitance you came up with a better idea. You decided to use the circuit proposed in part (c) along with a comparator, as shown in the figure below. What you are basically interested in, is the time T_1 needed for V_c to reach V_{ref}. In order to measure time you use a timer. When voltage V_c becomes larger than V_{ref}, the comparator flips its value and you stop the timer. How would you measure in that case the value of the capacitance?



4. LED Alarm Circuit

One day, you come back to your dorm to find that your favorite candy has been stolen. Determined to catch the perpetrator red-handed, you decide to put the candy inside a kitchen drawer. Using the following circuit design, you would like to turn on a light-emitting diode (LED) "alarm" if the kitchen drawer is opened.



Note R_{photo} is a photoresistor, which acts like a typical resistor but changes resistance based on the amount of light it is exposed to. This photoresistor is located inside the kitchen drawer, so we can tell when the drawer is opened or closed.

V_{LED} indicates the voltage across the LED; we will guide you through the IV behavior of this element later in the problem. The LED is located in your room (and connected to a long wire going to the kitchen), so that you can remotely tell when the kitchen drawer has been opened.

(a) What is V_+, the voltage at the positive voltage input of the comparator? Your answer should be written in terms of R_{photo} and R_{fixed}.

(b) We now want to choose a value for R_{fixed}. From the photoresistor's datasheet, we see the resistance in "light" conditions (i.e. drawer open) is $1\,\mathrm{k\Omega}$. In "dark" conditions (i.e. drawer closed), the resistance is $10\,\mathrm{k\Omega}$.

To ensure the comparator detects the light condition with more tolerance, we decide to design R_{fixed} so that V_+ is $3\,\mathrm{V}$ under the "light" condition. Solve for the value of R_{fixed} to meet this specification.

(c) Write down V_{out} with any conditions in terms of V_+. For simplicity, consider the case when $V_+ \neq V_-$ and assume the comparator is ideal.

(d) Using your answers to the previous parts, write down V_{out} with the conditions on its output in terms of R_{photo}. You can substitute the value of R_{fixed} you found in part (b). As before, you can assume that $V_+ \neq V_-$ and the comparator is ideal.

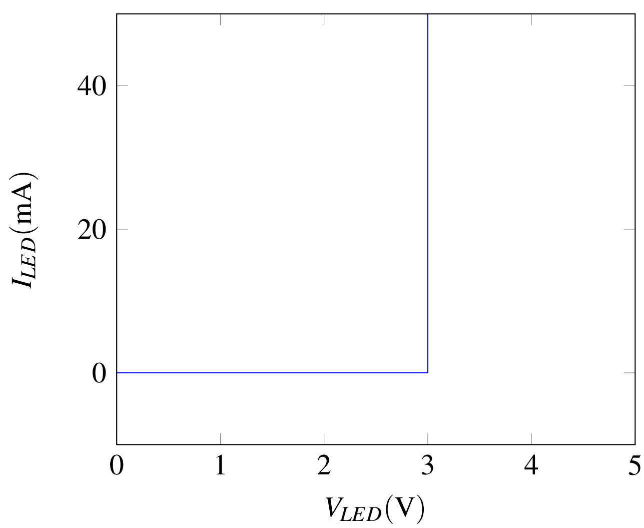
(e) From the design steps in the previous parts, we have designed a circuit that outputs non-zero voltage when the photoresistor is exposed to light (i.e. kitchen drawer open). We now want to design the LED portion of the circuit, so we get a visual alarm when the drawer is open.

From the LED's datasheet, the forward voltage, V_F is $3\,\mathrm{V}$. Essentially, if V_{LED} is less than this voltage, the LED won't light up and I_{LED} will be $0\,\mathrm{A}$.

Here is an idealized IV curve of this LED. The LED behaves in one of the following two modes:

i. If the voltage across the LED is less than $V_F = 3\,\mathrm{V}$ or if $I_{LED} < 0\,\mathrm{A}$, then the LED acts like an open circuit.

ii. If the voltage across the LED is $V_F = 3\,\mathrm{V}$, then the LED acts like a voltage source, except that it only allows positive current flow (i.e. only in the direction of current marked on the circuit diagram).



To avoid exceeding the power rating of the LED (and having it burn out), the recommended value for I_{LED} is $20\,\mathrm{mA}$.

Find the value of the current-limiting resistor, R_{lim}, such that when the photoresistor is in the "light" condition, $I_{LED} = 20\,\mathrm{mA}$.

5. Op-Amp in Negative Feedback

In this question, we analyze op-amp circuits that have finite op-amp gain A. We replace the op-amp with its circuit model with parameterized gain and observe the gain's effect on terminal and output voltages as the gain approaches infinity. Figure 1 shows the equivalent model of the op-amp. **Note here that $V_{SS} = -V_{DD}$.**

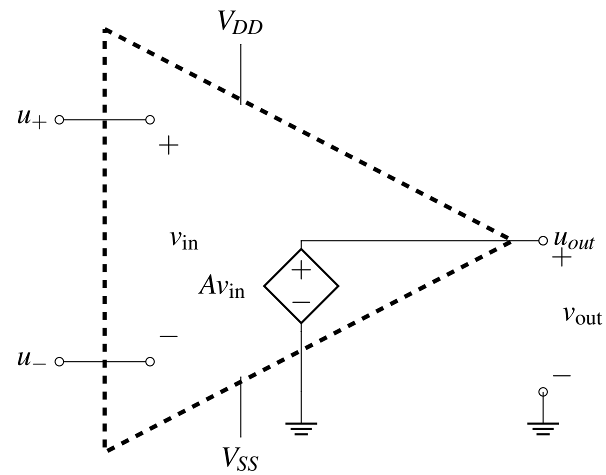


Figure 1: Op-amp model

For parts (a) - (e) only, assume that the op-amp is ideal (i.e., $A \to \infty$). We will consider the case of finite gain A in parts (f) - (h).

(a) Consider the circuit shown in Figure 2, and again $V_{SS} = -V_{DD}$. What is $u_+ - u_-$?

(b) Find v_x as a function of v_{out}.

(c) What is I_{R_2}, i.e. the current flowing through R_2 as a function of v_s? *Hint: Find the current through R_1 first.*

(d) Find v_{out} as a function of v_s.

(e) What is the current i_L through the load resistor R? Give your answer in terms of v_{out}.

(f) We will now examine what happens when A is not ∞. To understand what happens in this case, first draw an equivalent circuit for Figure 2, **by replacing the ideal op-amp in the non-inverting amplifier in Figure 2 with the op-amp model shown in Figure 1.**

Now, using this setup, calculate v_{out} and v_x in terms of A, v_s, R_1, R_2 and R. Is the magnitude of v_x larger or smaller than the magnitude of v_s? Do these values depend on R? *Hint: Note that the first golden rule still applies, i.e. the currents through the input terminals are zero.*

(g) Using your solution to the previous part, calculate the limits of v_{out} and v_x as $A \to \infty$. You should get the same answer as in part (d) for v_{out}.

(h) **[OPTIONAL, CHALLENGE]** Now you want to make a non-inverting amplifier circuit whose gain is nominally $G_{nom} = \frac{v_{\text{out}}}{v_s} = 1 + \frac{R_2}{R_1} = 4$. However, G_{nom} can only be achieved only if the op-amp is ideal, i.e, if its internal gain $A \to \infty$. But, as with most considerations in the physical world, we must account for nonidealities! In reality, because you will be working with an op-amp with finite gain A, your designed circuit gain may come close to but will never quite reach G_{nom} as a result of the real op-amp's finite internal gain A.

Suppose you would like your real op-amp circuit to have a maximum error of 1% (i.e, a minimum circuit gain of 3.96, i.e. $\frac{v_{\text{out}}}{v_s} \geq 3.96$). Remember that only if your op-amp were ideal, you would have a nominal circuit gain of $G_{nom} = \frac{v_{\text{out}}}{v_s} = 1 + \frac{R_2}{R_1} = 4$.

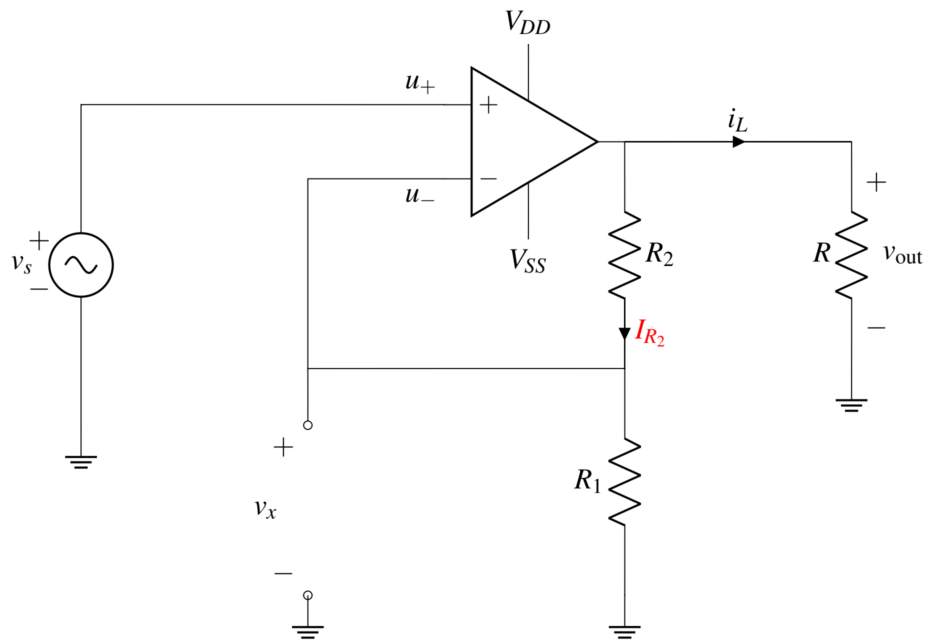


Figure 2: Non-inverting amplifier circuit

What is the minimum required value of A, called A_{min}, to achieve that specification? *Hint: Use your expression of* v_{out} *in part (f) to find an expression for* $G_{min} = \frac{v_{\text{out}}}{v_s}$ *when* $A \not\to \infty$.

6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.