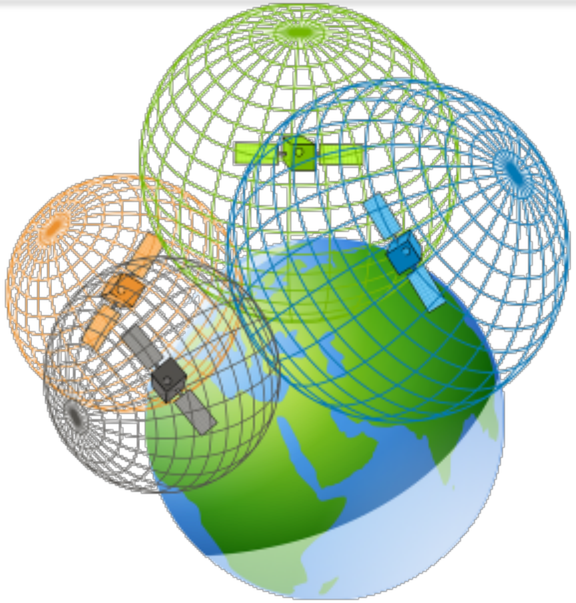

Welcome to EECS 16A!

Designing Information Devices and Systems I

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Fall 2021

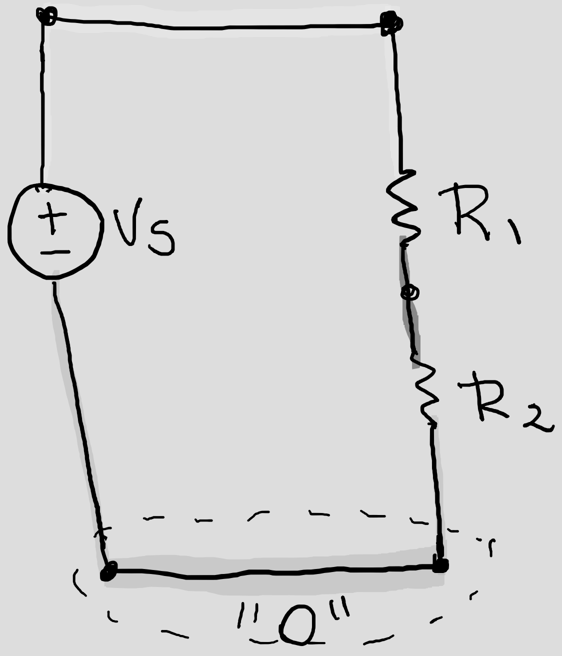
Module 2
Lecture 2
Introduction to Modeling with Circuit Elements
(Note 12)

Node Voltage Analysis – Voltage Divider

→ make circuit analysis faster

(Operators)

✓ Step 1 – Pick a node and label it as ground

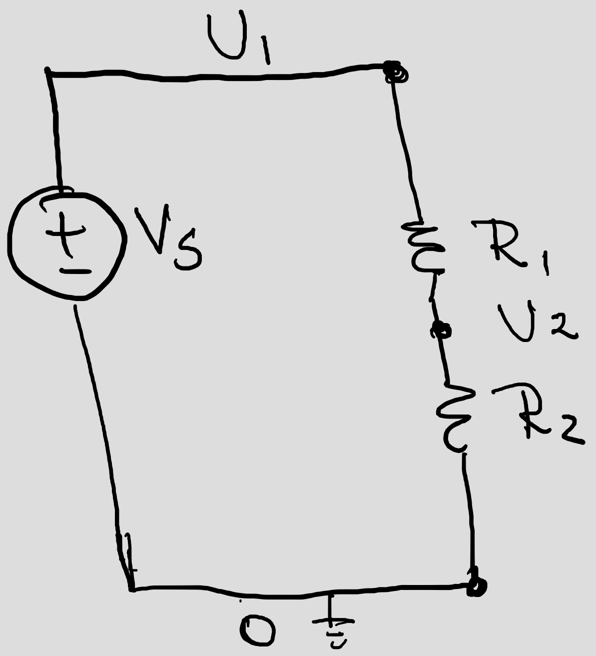


R2 has 2 junctions

Potential @ wire the same

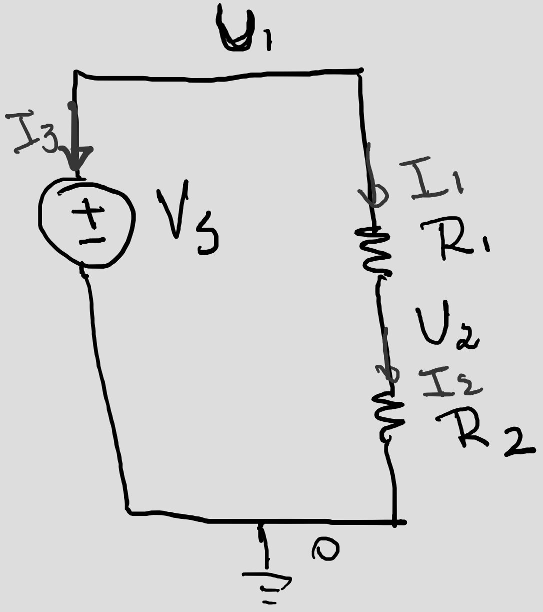
Node Voltage Analysis – Voltage Divider

Step 2 – Label all remaining nodes as some potential U_i.



Node Voltage Analysis – Voltage Divider

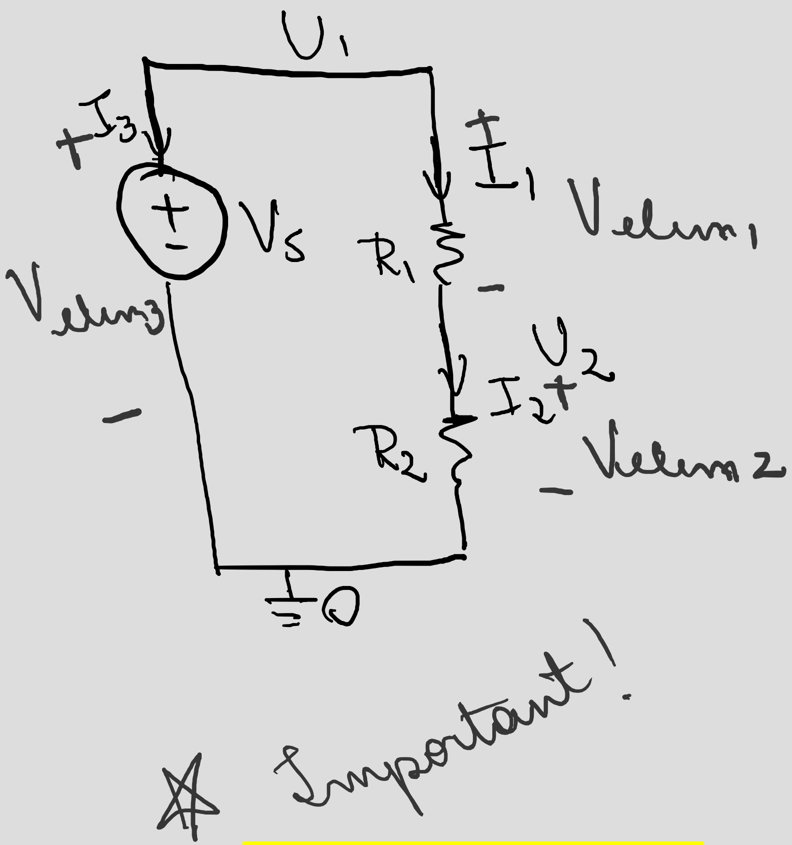
Step 3 – Label the current through every non-wire element in the circuit with I_n.



→ Next step requires some thinking....

Node Voltage Analysis – Voltage Divider

Step 4 – Add +/- labels on each non-wire element, following the passive sign convention.

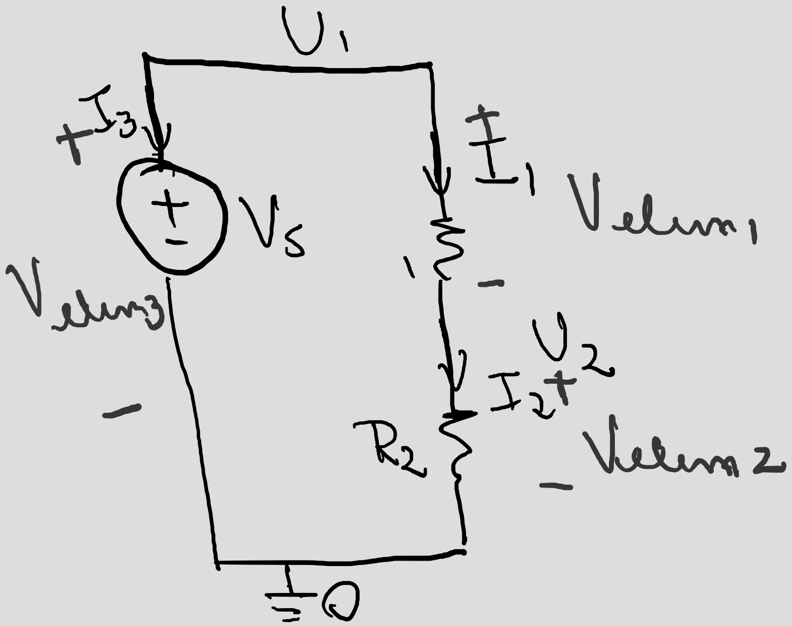


Passive sign convention: the current enters at the positive terminal and exits are the negative terminal.

Node Voltage Analysis – Voltage Divider

Step 5 – Set up the relationship A $\vec{x} = \vec{b}$ *where* $\vec{x}$ consists of the unknown currents and potentials.

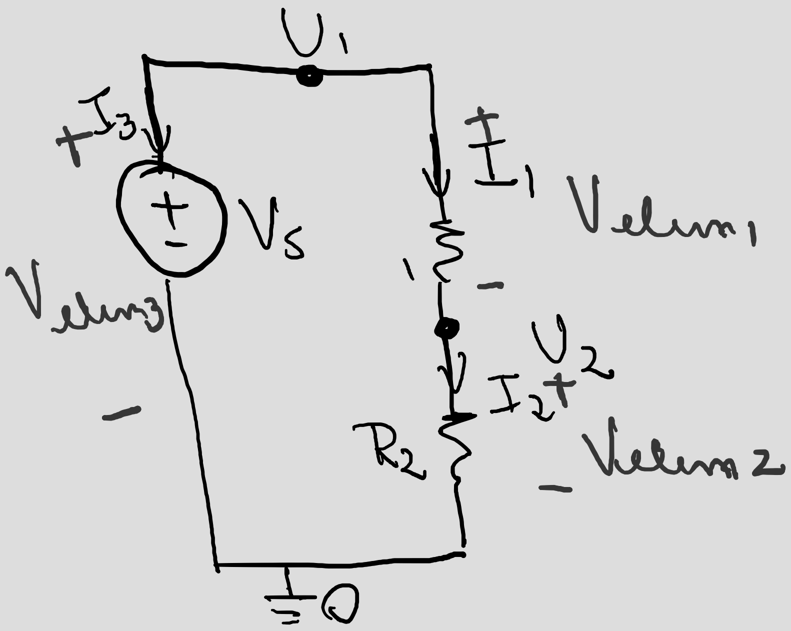
* Identify the unknowns



$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix}$$

Node Voltage Analysis – Voltage Divider

Step 6 – Use KCL to fill in as many linearly independent rows in A and $\vec{b}$ *as possible* .



$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} \xrightarrow{\text{KCL}}$$

$$I_1 + I_3 = 0 \quad (1)$$

$$I_1 = I_2$$

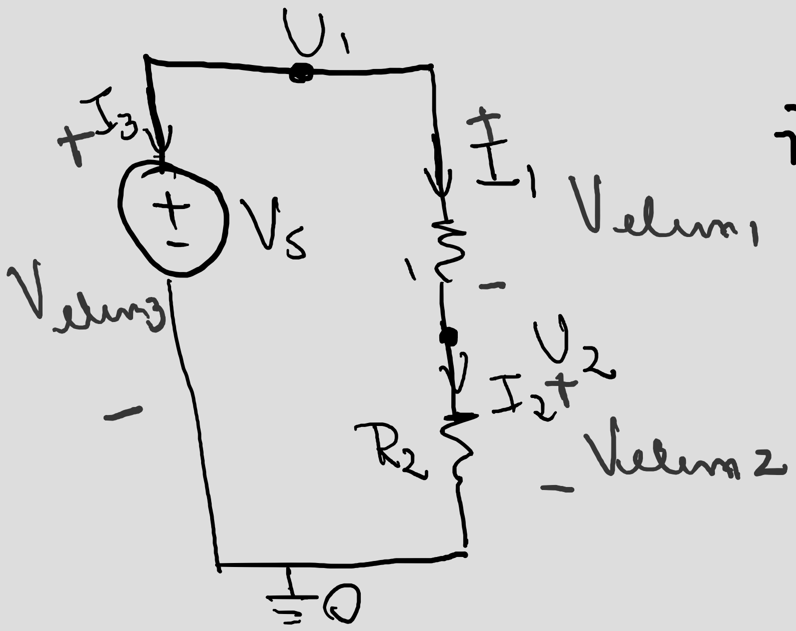
$$-I_1 + I_2 = 0 \quad (2)$$

I coming out ⊕ ⇒ convention

KCL: the current flowing into a junction must equal the current flowing out of that junction.

Node Voltage Analysis – Voltage Divider

Step 7 – Use the IV relationships of each of the non-wire elements to fill in the remaining rows of A and $\vec{b}$.



$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}$$

→ KCL

$I_1 + I_3 = 0$ (1)

$-I_1 + I_2 = 0$ (2)

Voltage Potential | Element IV

$V_{el_1} = U_1 - U_2$ | $V_{el_1} = R_1 I_1$

$V_{el_2} = U_2 - 0 = U_2$ | $V_{el_2} = R_2 I_2$

$V_{el_3} = V_1 - 0 = V_1$ | $V_{el_3} = V_S$

$R_1 I_1 = U_1 - U_2 \Rightarrow R_1 I_1 - U_1 + U_2 = 0$ (3)

$U_2 = R_2 I_2 \Rightarrow R_2 I_2 - U_2 = 0$ (4)

$U_1 = V_S$ (5)

Node Voltage Analysis – Voltage Divider

Step 8 – Solve the system of equations to determine values of unknown variables.

$$I_1 + I_3 = 0 \quad (1)$$
$$-I_1 + I_2 = 0 \quad (2)$$
$$R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$
$$R_2 I_2 - U_2 = 0 \quad (4)$$
$$V_1 = V_S \quad (5)$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_S \end{bmatrix}$$

$$I_1 = \frac{V_S}{R_1 + R_2}, \quad I_2 = \frac{V_S}{R_1 + R_2}, \quad I_3 = -\frac{V_S}{R_1 + R_2}$$

$$U_1 = V_S, \quad \boxed{U_2 = \underbrace{\frac{R_2}{R_1 + R_2}}_{\alpha < 1} \cdot V_S} = R_2 \cdot \underbrace{\frac{V_S}{R_1 + R_2}}_{I_2}$$

α is an operator!

Electrical Quantities – Some Physics…

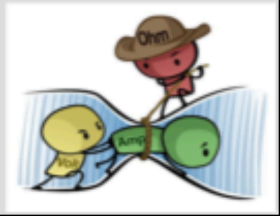
* Charge – can be either positive or negative; basic element of electrical flow.
  UNIT : Coulomb [C]

* Current – net amount of charge that passes through some cross-section area over some period of time.

$$[A] \rightarrow I = \frac{dQ}{dt} \quad \frac{[C]}{[s]}$$

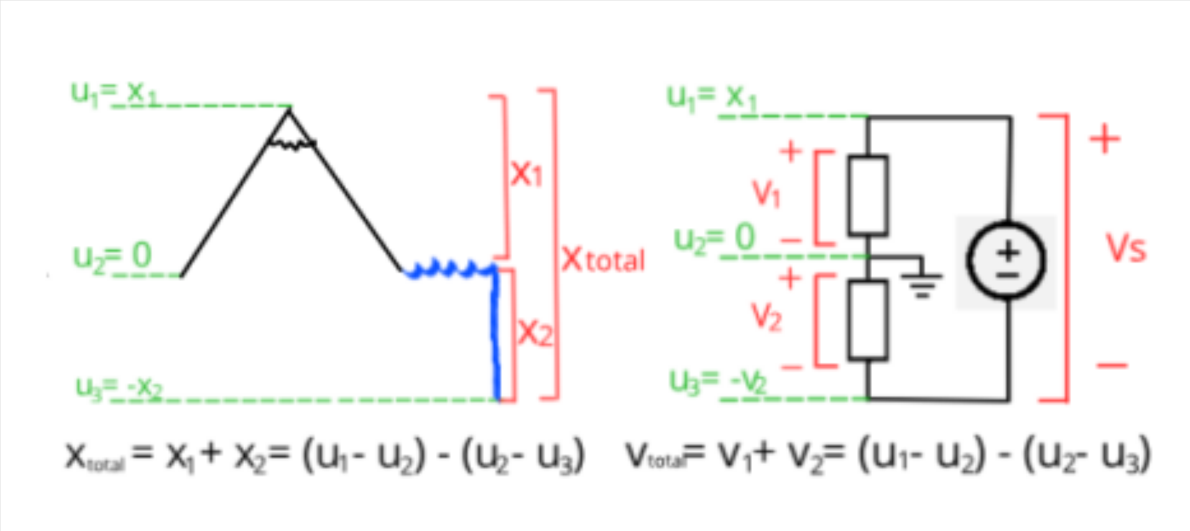
Quantities	Analytical Symbol	Units
Current	I	Amperes [A]
Voltage	V	Volts [V]
Resistance	R	Ohms [Ω]

I ⇒ flows through an element
V ⇒ applied across an element
R ⇒ opposition to current flow



* Important:
ALWAYS DO UNIT CHECKS!

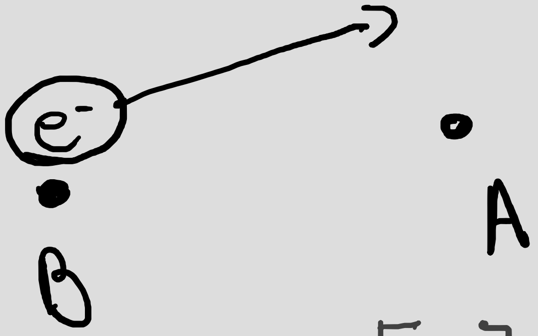
Electrical Quantities – Some Physics…



Note 11

- Voltage: represents

the energy to move a positive unit of charge from one point to the other.



V_{AB}

$[V] = \frac{[J]}{[C]}$ ([J]: energy)

Quantities	Analytical Symbol	Units
Current	I	Amperes [A]
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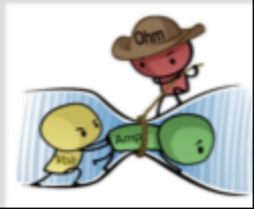

Resistance, Resistivity, Conductivity – Physics and Materials...

Material	Electrical characteristics	
	Electrical Resistivity (Ω x cm)	Electrical Conductivity (Ω^{-1} x cm^{-1})
Cu	0.034×10^{-5}	29×10^{5}
Fe	32.54×10^{-5}	0.031×10^{5}
Ag	0.36×10^{-5}	2.8×10^{5}
Al	0.03×10^{-5}	33.3×10^{5}
Ni	0.046×10^{-5}	21.7×10^{5}
Cu-Fe	33.37×10^{-5}	0.030×10^{5}
Cu-Ag	2.71×10^{-5}	0.37×10^{5}
Al-Ni	0.564×10^{-5}	1.77×10^{5}

* Resistance: real materials/metals/ conductors require a certain amount of energy to allow charge to flow through.

$$V = I \cdot R$$

$$[V] = [A] \cdot [\Omega]$$

Resistance, Resistivity, Conductivity – Physics and Materials…

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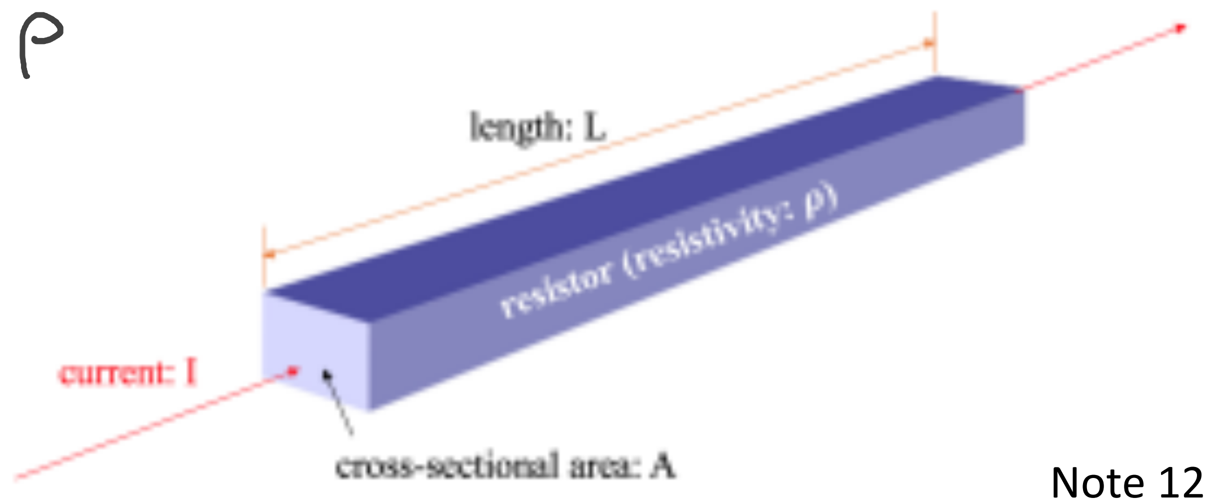
$\frac{1}{\rho}$



Note 12

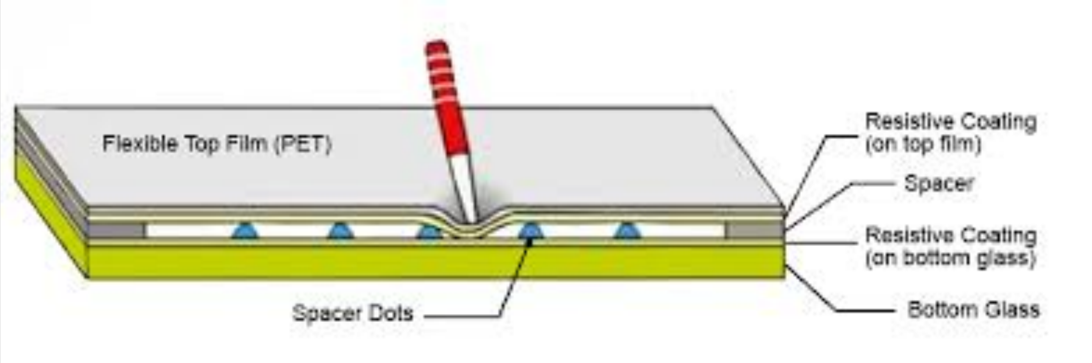
$R = \rho \cdot \frac{L}{A}$

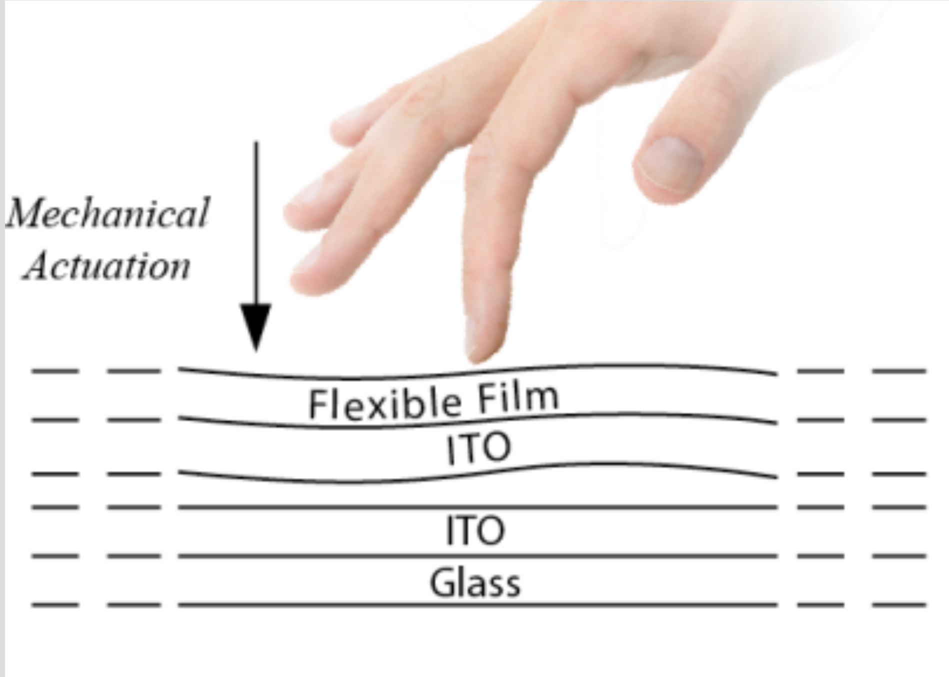
Longer the wire → the more energy is lost.

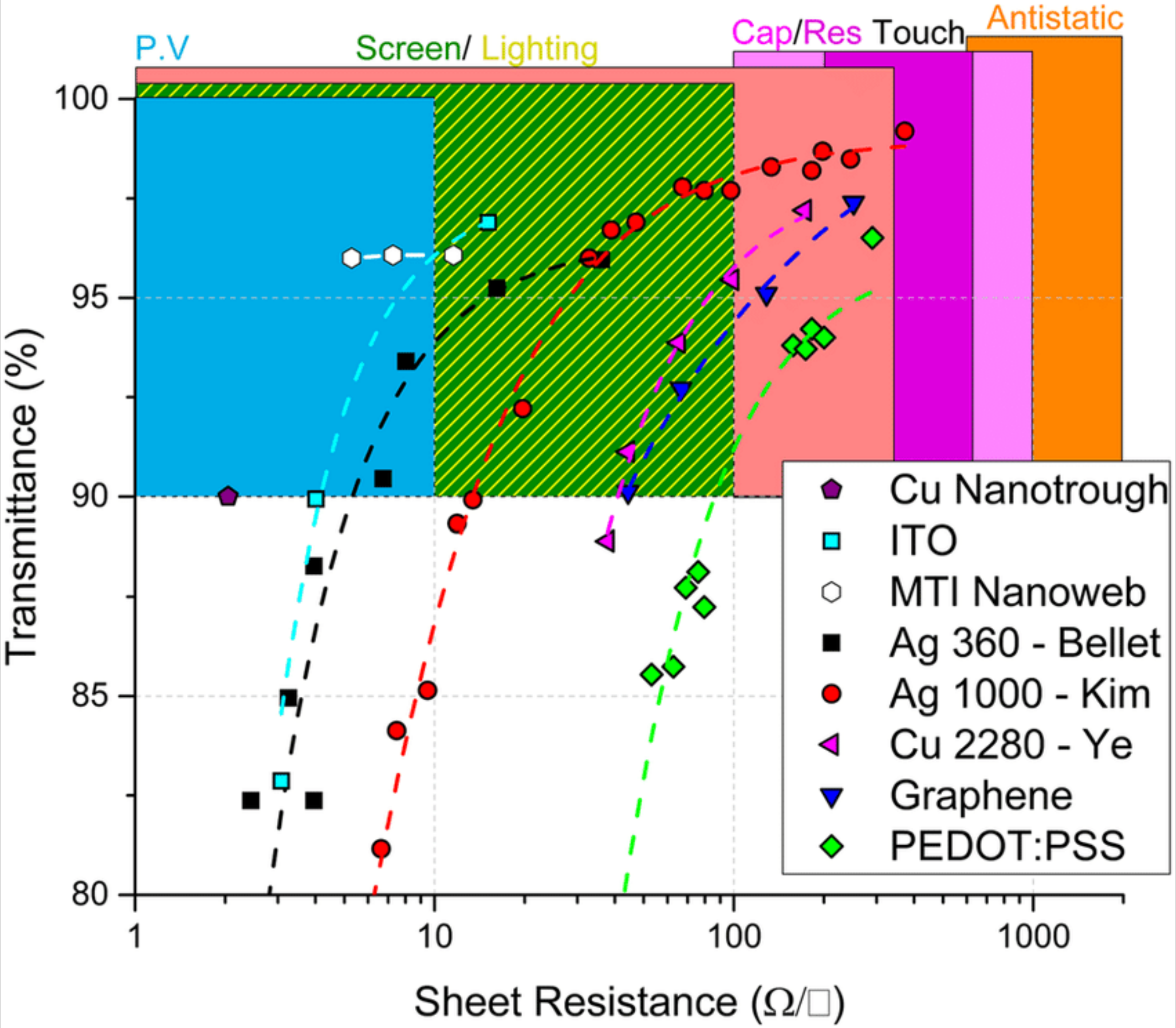
Wide wires → lower resistance

Wire properties depends on materials choice.

Resistive Touch Screen



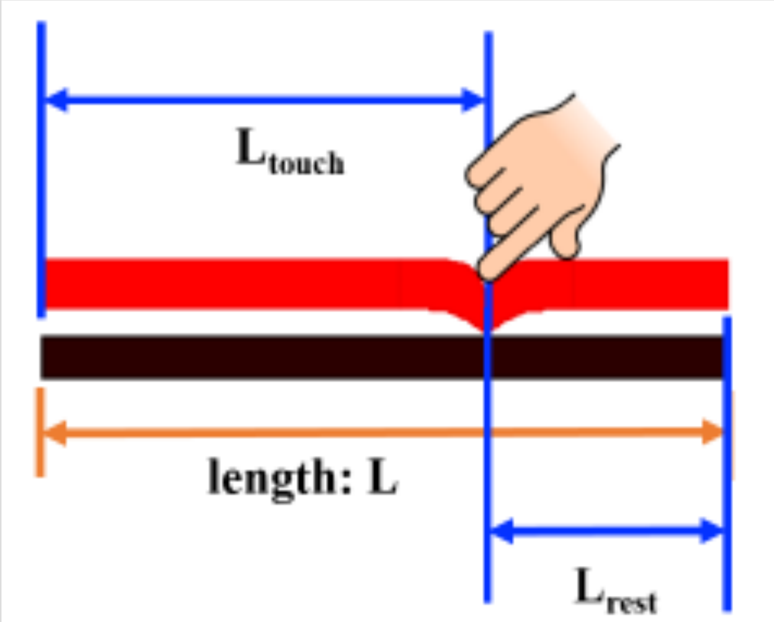




https://doi.org/10.1038/s41598-019-47777-2

Resistive Touch Screen

Problem: To find the location of touch.

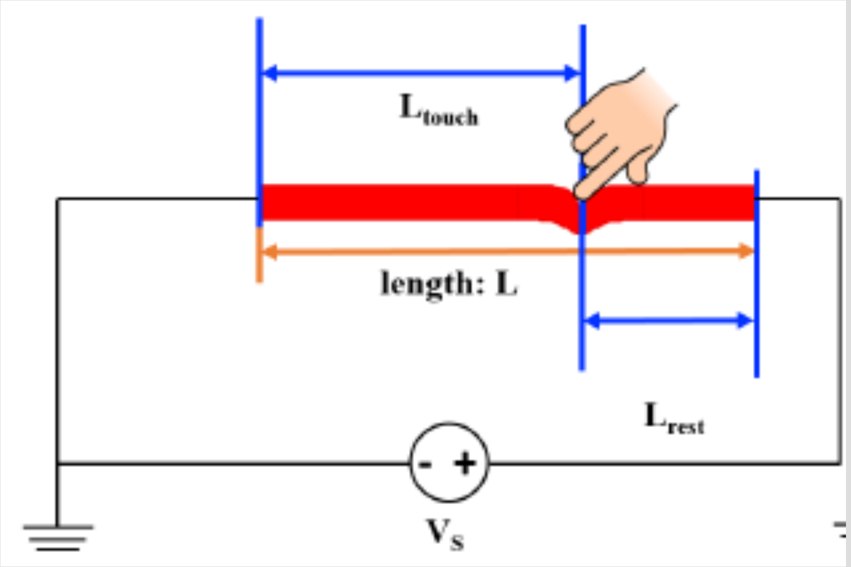


Go from mechanical to electrical quantity!

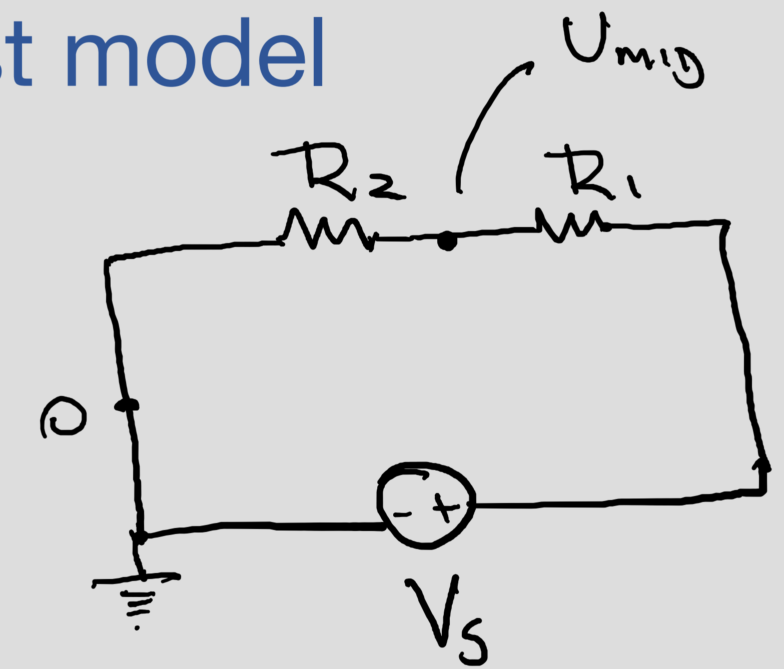
Want to measure $\frac{L_{touch}}{L}$

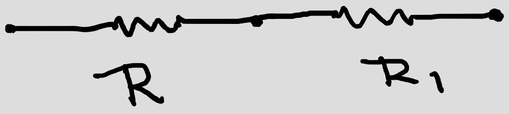
L_{touch} is unknown

Resistive Touch Screen – First model



$\Rightarrow$





$R_1 = \rho \cdot \frac{-}{A}$ (1)

$R_2 = \rho \cdot \frac{L_{touch}}{A}$ (2)

$$U_{mid} = \frac{L_{touch}}{L} \cdot V_s$$

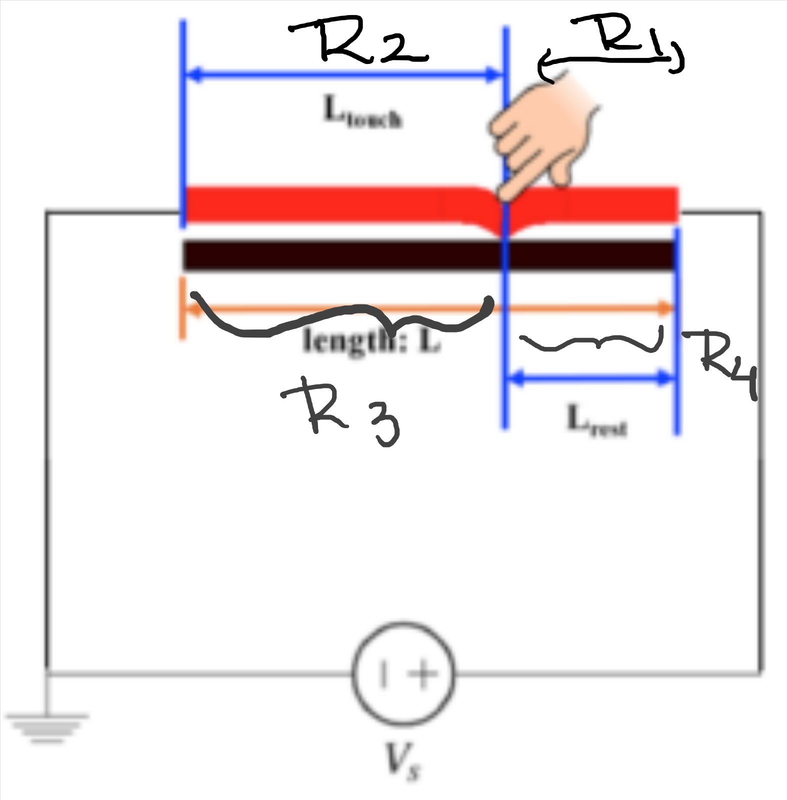
$$U_{MID} = \frac{R_2}{R_2 + R_1} V_s$$ (Voltage Divider) *

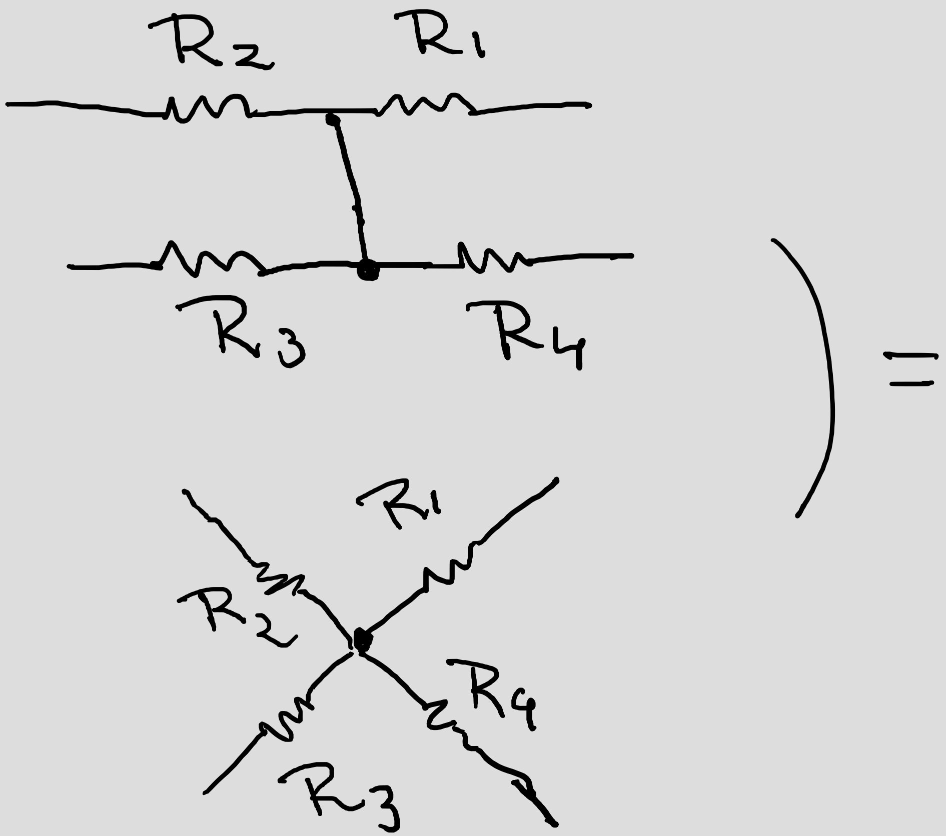
$$U_{mid} = \frac{\rho \frac{L_{touch}}{A}}{\rho \frac{L_{touch}}{A} + \rho \frac{L_{rest}}{A}} \cdot V_s$$

$$= \frac{L_{touch}}{L_{touch} + L_{rest}} \cdot V_s$$

L ← ($L_{touch} + L_{rest}$)

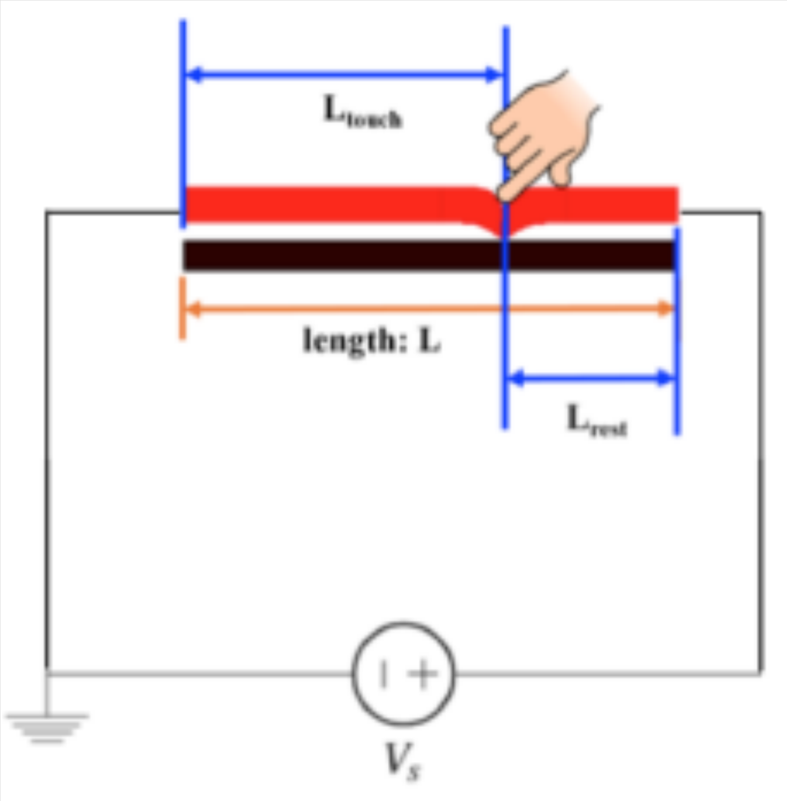
Resistive Touch Screen – More realistic model

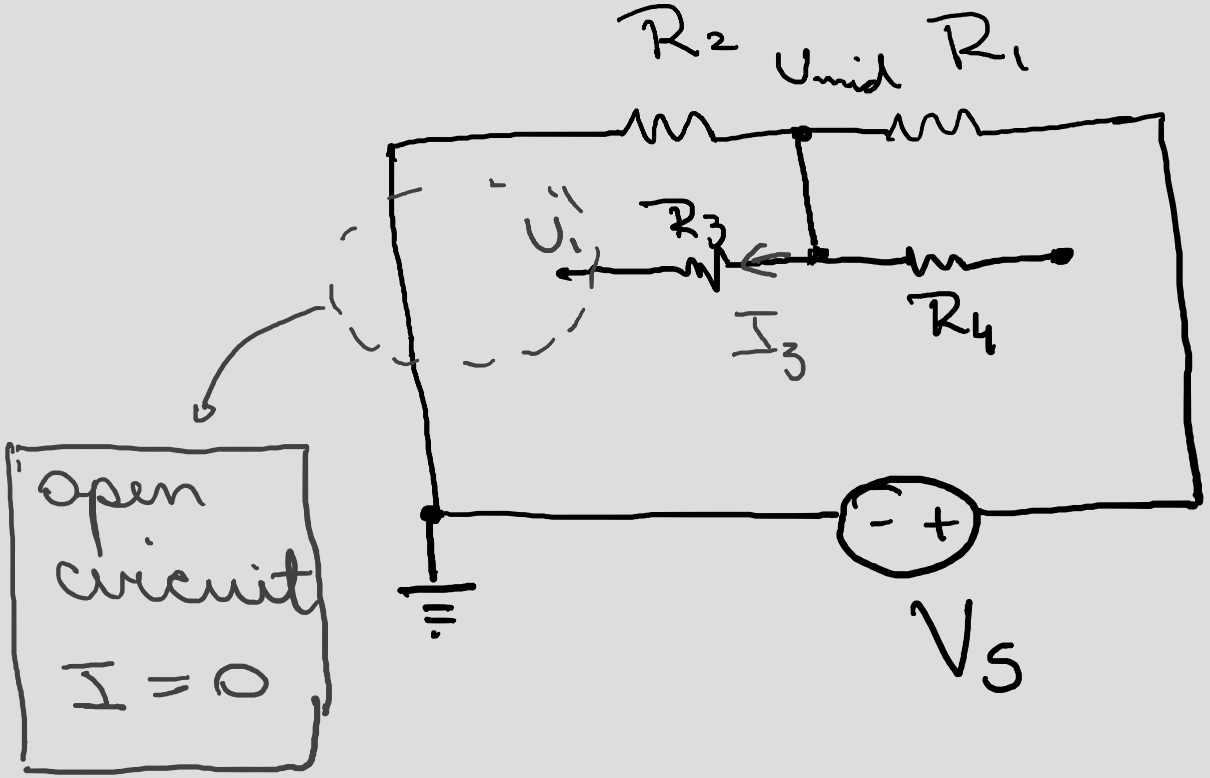




R_1, R_2, R_3 and R_4 are unknown.

Resistive Touch Screen – More realistic model





$I_3 = 0$

$U_1 = 0$

Read out is Voltage!