

Welcome to EECS 16A!

Designing Information Devices and Systems I

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Fall 2021

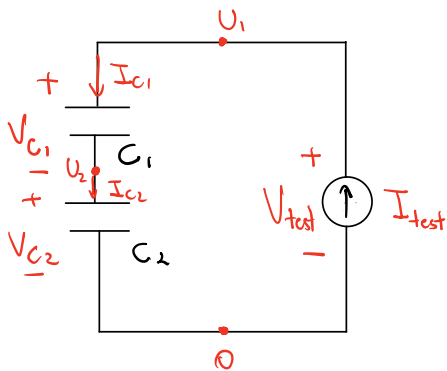
Module 2
Lecture 8

Capacitance Modeling and Comparators
(Note 17)



Example 2 :

"Capacitors in series"



KCL : $I_{C1} = I_{C2} = I_{test}$

Elements :

$$I_{C2} = C_2 \frac{dV_{C2}}{dt}$$

$$I_{C1} = C_1 \frac{dV_{C1}}{dt}$$

Voltage Def :

$$V_{C2} = U_2 - 0$$

$$V_{C1} = U_1 - U_2$$

$$V_{test} = U_1 - 0$$

For V_{C2} :

$$I_{C2} = C_2 \frac{dV_{C2}}{dt}$$

$$I_{test} = C_2 \frac{dU_2}{dt} \equiv \frac{dU_2}{dt} = \frac{I_{test}}{C_2}$$

For V_{C1} :

$$I_{C1} = C_1 \frac{dV_{C1}}{dt}$$

$$\frac{dV_{C1}}{dt} = \frac{I_{C1}}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1}$$

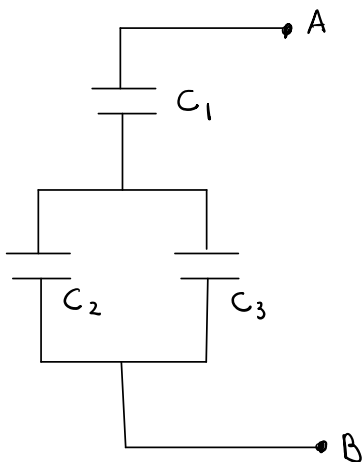
$$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt} = I_{test} \left(\frac{1}{C_2} + \frac{1}{C_1} \right)$$

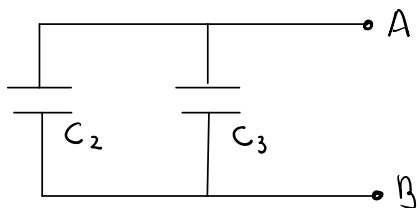
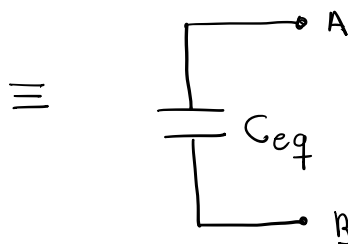
$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \parallel C_2$$

$$C_{eq} = C_1 \parallel C_2 \quad (\parallel - \text{parallel mathematical operator})$$

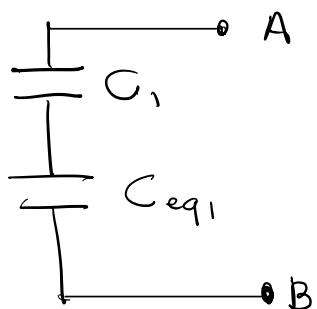
Example 3



$$C_{eq} = C_1 \parallel (C_2 + C_3)$$

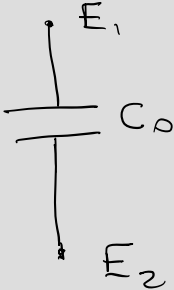
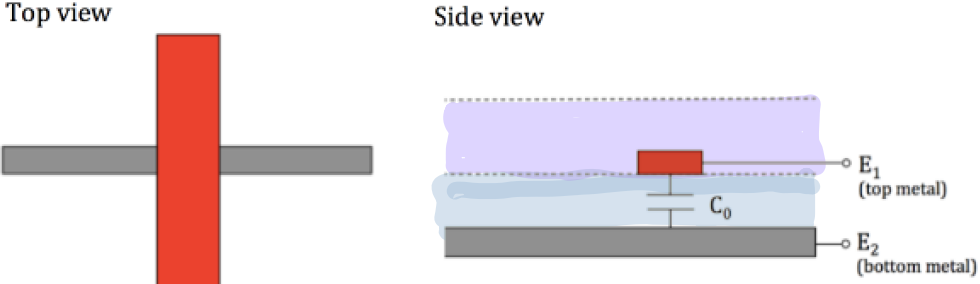


$$\Rightarrow C_{eq1} = C_2 + C_3$$



$$C_{eq} = C_1 \parallel C_{eq1}$$

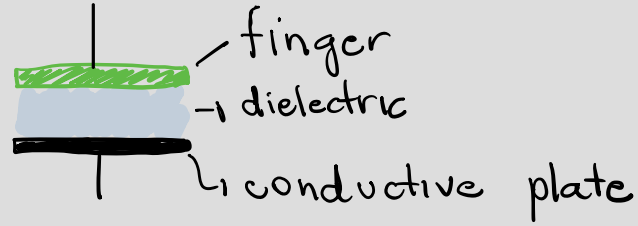
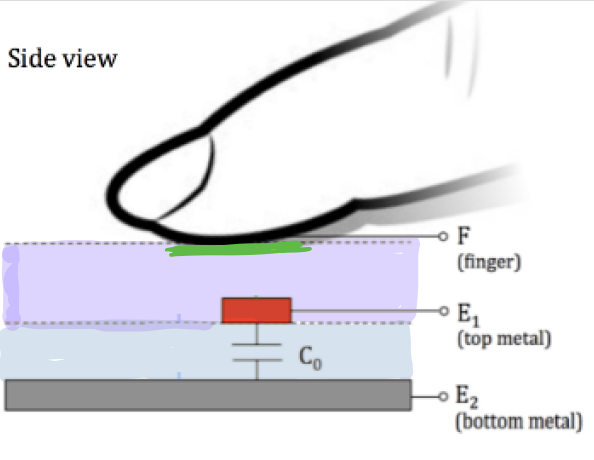
Capacitive Touchscreen – Model without touch



$$C_0 = \epsilon \cdot \frac{A}{d}$$

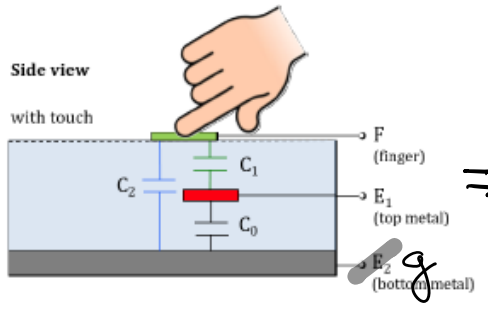
Capacitive Touchscreen – Model with touch

When there is a touch, it makes a capacitor!

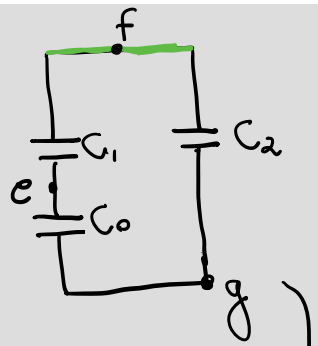


Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture

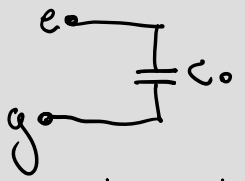


⇒ circuit model

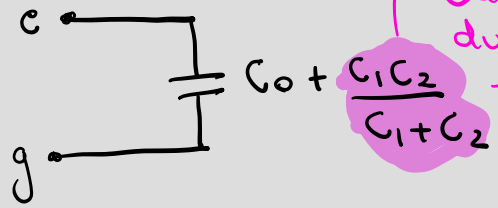


We only have access to nodes e and g, not f

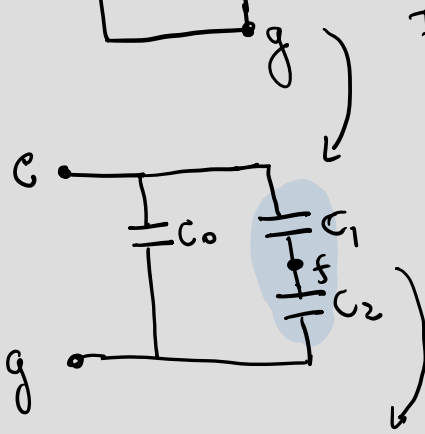
when no touch:



with touch:

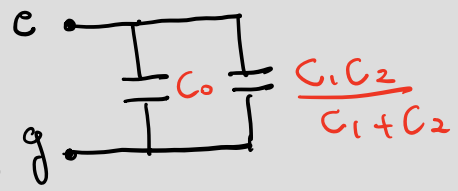


Extra Capacitance due to touch!



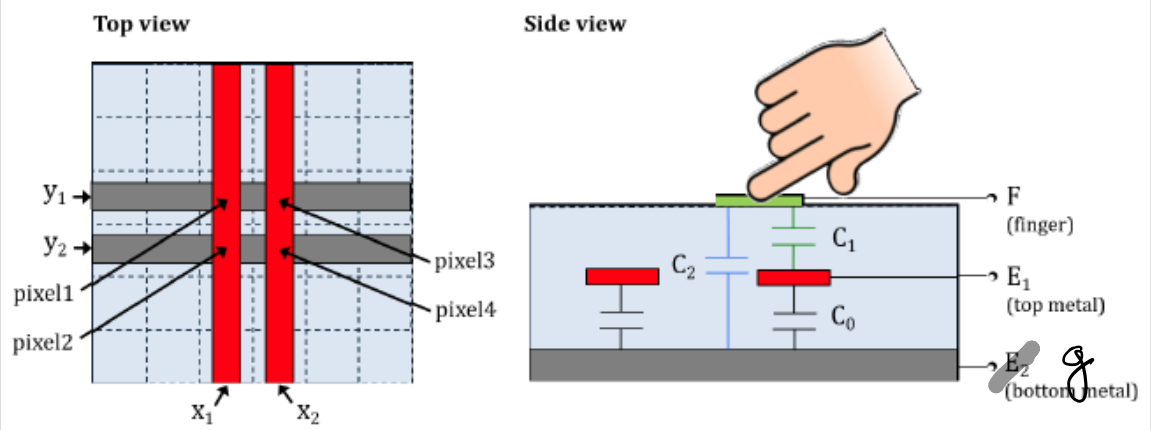
Redraw to focus on terminals (nodes) e and g

Equivalent capacitance for C₁ in series with C₂



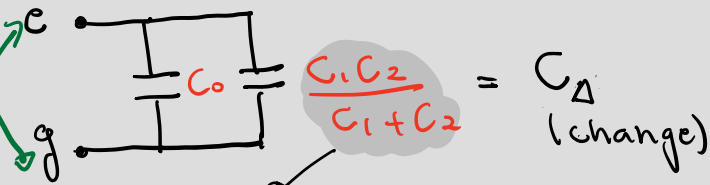
⇒ Equiv. Capacitance for C₀ in parallel to $\frac{C_1 C_2}{C_1 + C_2}$

2D View – How do we measure Capacitance?



Problem: We don't have a capacitance meter!

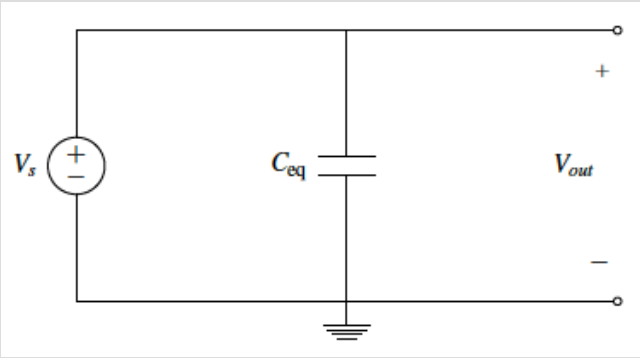
We want to measure capacitance here



This capacitor goes away with no touch

We will try ideas to get to a final model.

Measuring Capacitance Models – Attempt #1

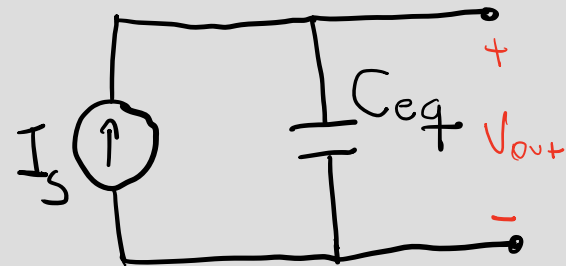


If there is touch: $V_c = V_s$

If there is no touch: $V_c = V_s$

V_{out} does not change!

Bad idea! ✗



Very hard to make current sources! ✗

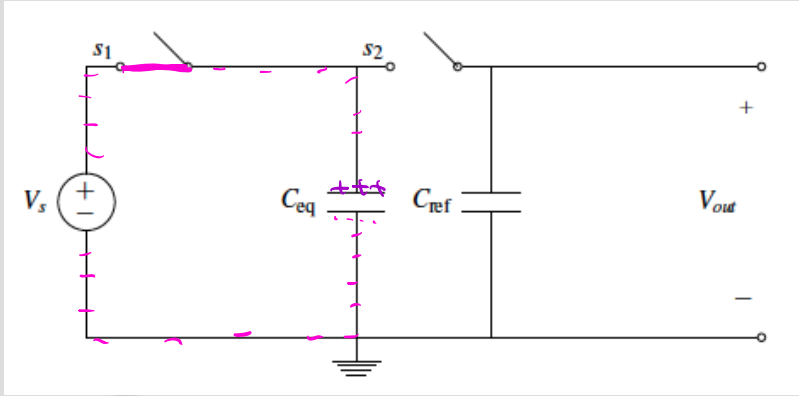
Assume starts out discharged:

$$V_{out}(t=0) = 0$$

$$I_s = C_{eq} \frac{dV_{out}(t)}{dt} \rightarrow V_{out}(t) = \int_0^t \frac{I_s}{C_{eq}} dt$$

$$V_{out} = \frac{I_s t}{C_{eq}} \Rightarrow C_{eq} = \frac{I_s}{\frac{dV_c(t)}{dt}}$$

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor



- 1st – Close both switches
We want to charge C_{ref} and measure V_{out} as C_{ref} discharges.

IS both closed – nothing happens.!. Attempt #1

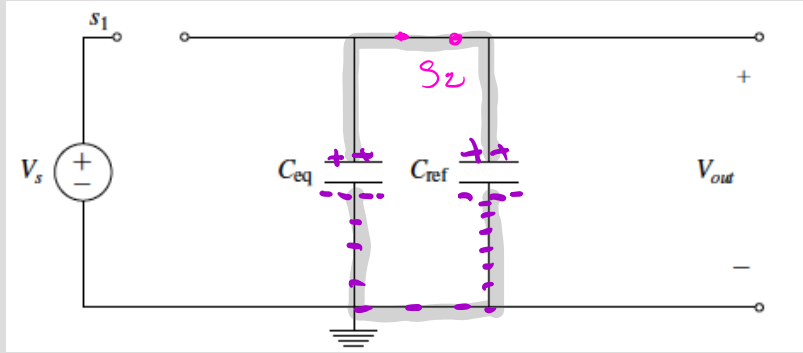
Phase 1: Close S_1 ; Open S_2

C_{eq} is charging

$q = C_{eq} \cdot V_s$ accumulates on capacitor plates.

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Phase 2: close S_2 , open S_1



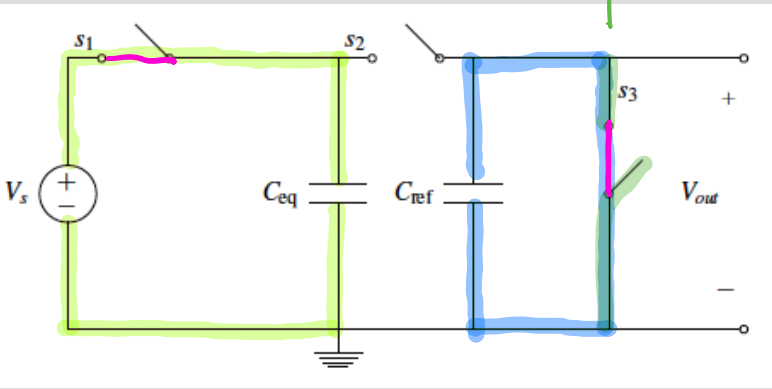
Charge will split between C_{eq} and C_{ref}

"charge sharing"

So close! But we don't know initial C_{ref} .

- There is a path for charge to move.
- C_{eq} can provide the energy needed for current.

Measuring Capacitance Models – Attempt #3 – known initial condition



Use S_3 to discharge C_{ref} so we know $C_{ref} = 0$

Phase 1: S_3 closed, S_1 closed, S_2 open

C_{ref} discharges $V_{out} \rightarrow 0$

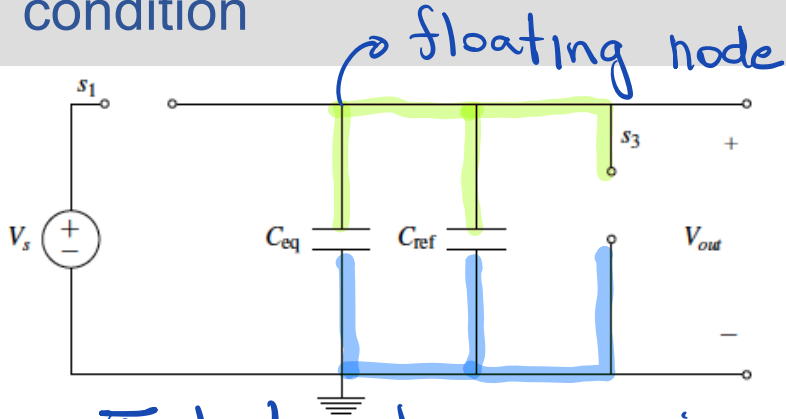
$$q = C_{ref} \cdot V_{out} = 0 \quad \checkmark$$

C_{eq} charges

$$q = C_{eq} \cdot V_s$$

Phase 2: S_1 open,
 S_2 closed, S_3 open
 C_{eq} - charged

Measuring Capacitance Models – Attempt #3 – known initial condition



Voltage across C_{eq} : V_{out}
 Voltage across C_{ref} : V_{out}
 Charge in C_{eq} : $q_1 = C_{eq} \cdot V_{out}$
 charge in C_{ref} : $q_2 = C_{ref} \cdot V_{out}$

Total charge is conserved!

$$q(\text{phase 1}) = q(\text{phase 2})$$

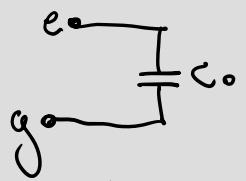
$$C_{eq} \cdot V_s = C_{eq} V_{out} + C_{ref} \cdot V_{out}$$

$$V_{out} = \frac{C_{eq} V_s}{C_e + C_r}$$

\Rightarrow V_{out} changes when C_e chan !!!

Effect of touch on total capacitance

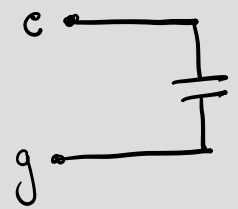
when no touch:



\Rightarrow

$$V_{OUT} = \frac{C_0}{C_0 + C_{ref}} \cdot V_S$$

with touch:



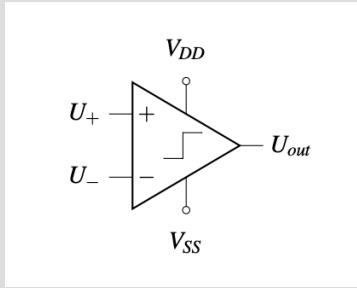
Extra Capacitance due to touch!
 $C_0 + \frac{C_1 C_2}{C_1 + C_2}$
 C_Δ

\Rightarrow

$$V_{OUT} = \frac{(C_0 + C_\Delta)}{C_0 + C_\Delta + C_{ref}} \cdot V_S$$



How can we go from voltage measurement to binary
answer: touch or no touch?



- We need to choose a Voltage that we call : Threshold Voltage (V_{th})

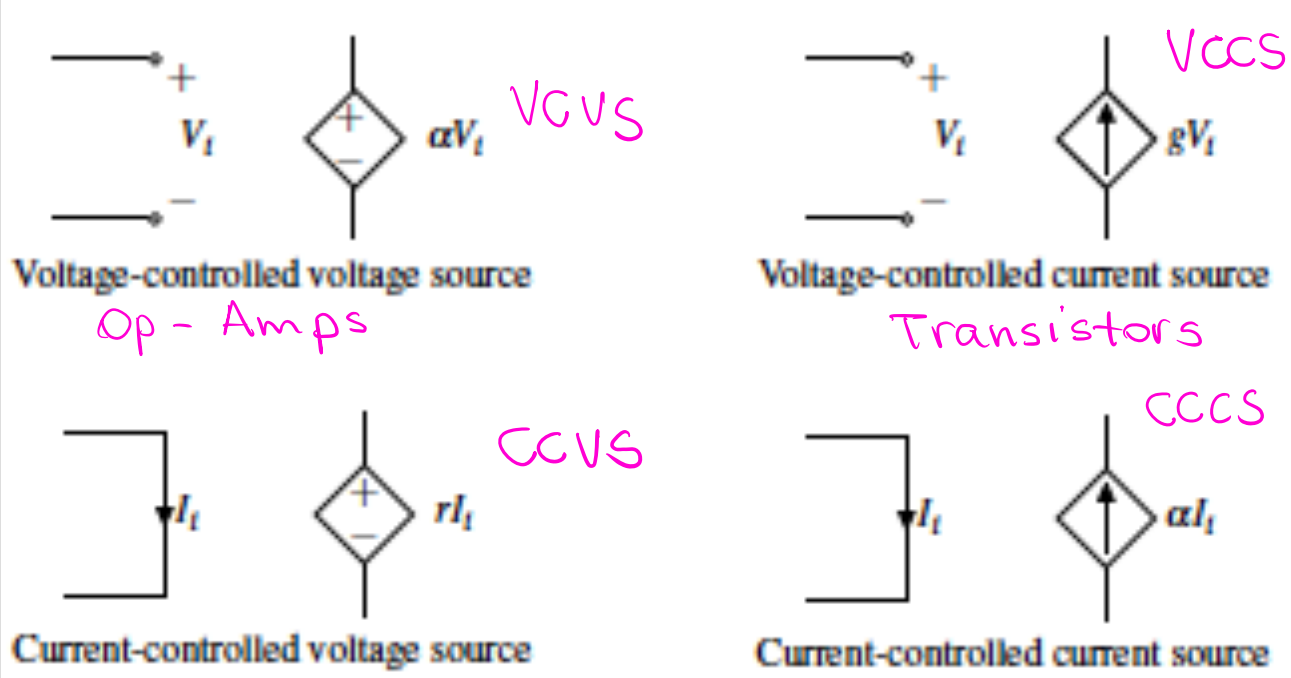
- Above $V_{th} \therefore 1$ (touch)

- Below $V_{th} \therefore 0$ (no-touch)

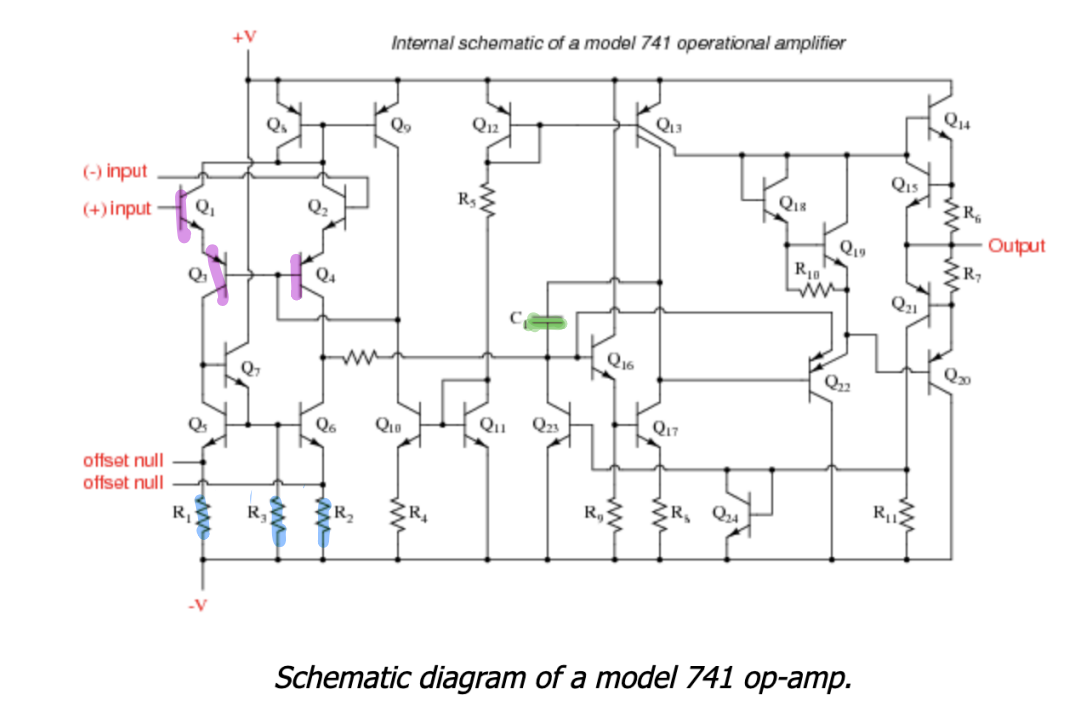
We need to compare Voltages to determine if 1 or 0

How can we go from voltage measurement to binary answer: touch or no touch?

- New tools are needed – new circuit elements

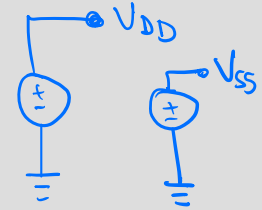
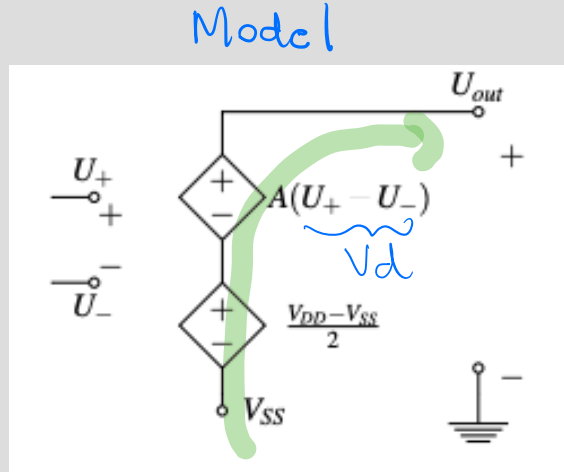
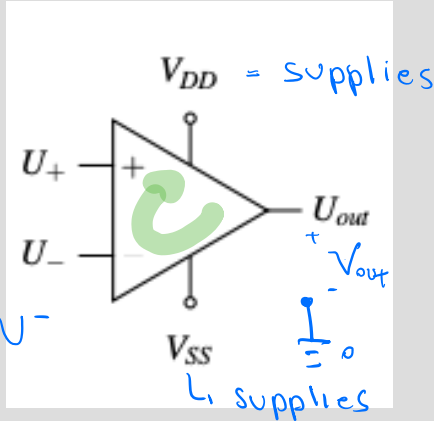


An example of an Op-amp circuit diagram



Operational Amplifier

An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.



An op-amp has two input terminals marked (+) and (-) with potentials U_+ and U_- , two power supply terminals called V_{DD} and V_{SS} , and one output terminal with potential U_{out} .

$$V_{out} = V_{SS} + \frac{V_{DD} - V_{SS}}{2} + A \cdot V_d$$

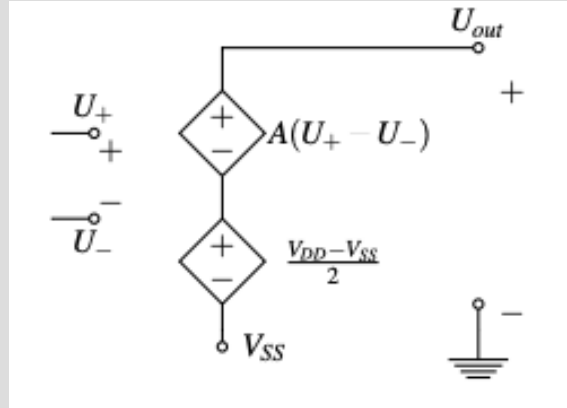
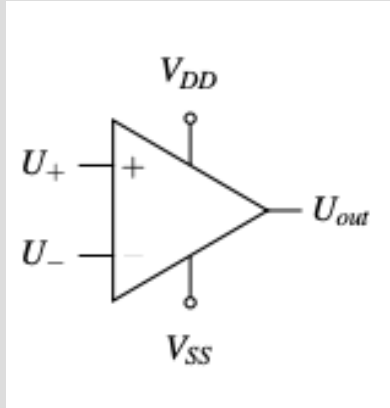
when

$$V_{SS} \leq \frac{V_{DD} - V_{SS}}{2} + A V_d \leq V_{DD}$$

Operational Amplifier

An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.

$$\frac{V_{DD} - V_{SS}}{2} + A U_d \quad V^*$$



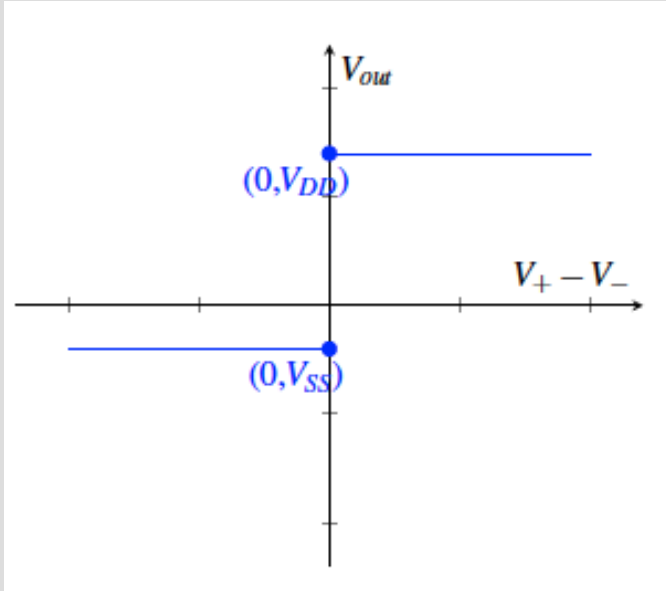
An op-amp has two input terminals marked (+) and (-) with potentials U_+ and U_- , two power supply terminals called V_{DD} and V_{SS} , and one output terminal with potential U_{out} .

$$U_{out} = V_{DD} \quad \text{if} \quad V^* > V_{DD}$$

$$U_{out} = V_{SS} \quad \text{if} \quad V^* < V_{SS}$$

Can be used to compare Voltage

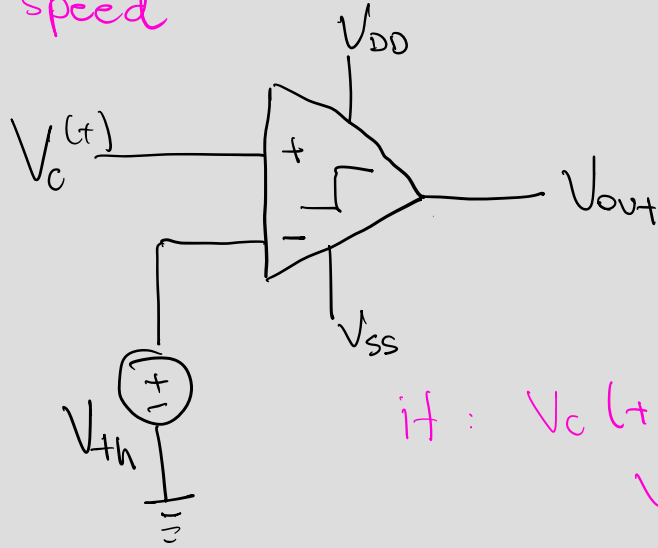
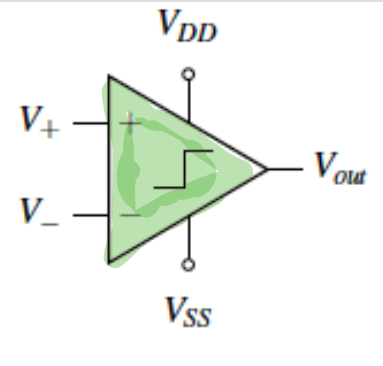
Comparator – optimized for binary output



V_{DD} can be much
higher than V_{SS}
∴ it amplifies the
signal.

Comparator – optimized for binary output

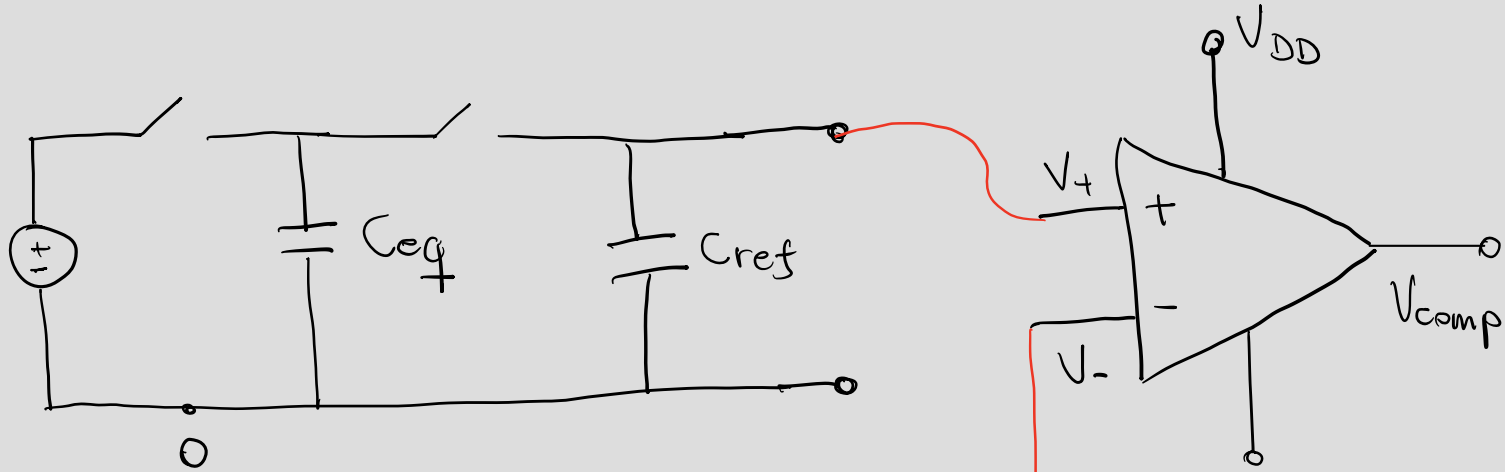
Also optimized for speed



$$\text{if : } V_C(t) > V_{th} \\ V_{out} = V_{DD}$$

$$\text{if : } V_C(t) \leq V_{Th} \\ V_{out} = V_{SS}$$

Back to our Capacitive Touchscreen



$C_{eq} \Rightarrow C_0 + C_A - \text{touch}$
 $C_0 - \text{no touch}$

V_{DD} touch
no touch V_{SS}

$V_{+threshold}$
Should be half way between V_{touch} and $V_{notouch}$