EECS 16A Designing Information Devices and Systems I
Spring 2021 Midterm 2 Instructions

Good luck for the midterm! You've studied hard and we are rooting for you to do well! Please read these instructions and the proctoring guidelines before the exam.

Our advice to you: if you can't solve a particular problem, move on to another, or state and solve a simpler one that captures at least some of its essence. You will perhaps find yourself on a path to the solution. **Good luck! We believe in you.**

Format & How to Submit Answers

There are 8 problems (2 introductory questions, and 6 exam questions with subparts) of varying numbers of points on the exam. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can. If you are having trouble with one problem, there may be easier points available later in the exam!

Complete your exam using either the template provided or appropriately created sheets of paper. Either way, you should submit your answers to the Gradescope assignment that is marked Midterm 2 for your specific exam group. Make sure you submit your assignment to the correct Gradescope assignment. You MUST select pages for each question. We cannot grade your exam if you do not select pages for each question. If you are having technical difficulties submitting your exam, you can email your answers to eecs16a@berkeley.edu.

In general, show all your work legibly to receive full credit; we cannot grade anything that we cannot read. For some problems, we may try to award partial credit for substantial progress on a problem, and showing your work clearly and legibly will help us do that.

Timing & Academic Honesty

You are expected to follow the rules provided in the Exam Proctoring Guidelines (`https://docs.google.com/document/d/10pnWwxyZ40nlpbCM4aOYTxXOjc36sIQaMx9m8zyaR8w/edit?usp=sharing`). The exam will be available to you at the link sent to you by email. The exam will start at 7pm Pacific Time, Monday, April 12th 2021, unless you have an exam accommodation. If you experience technical difficulties and cannot access your exam, let us know by making a private post on Piazza and we will try to help.

You have 120 minutes for the exam, with 40 extra minutes for scanning and submitting to Gradescope. Unless you have an accommodation, you must submit your exam by 9:40pm. Late submissions will be penalized exponentially. An exam that is submitted N minutes after the end of the submission period will lose 2^N points. This means that if you are 1 minute late you will lose 2 points; if you are 5 minutes late you will lose 32 points and so on.

This is a closed-note, closed-book, closed-internet, and closed-collaboration exam. Calculators are not allowed. You may consult two handwritten 8.5" by 11" cheat sheets. Do not attempt to cheat in any way. We have a zero tolerance policy for violations of the Berkeley Honor Code.

EECS 16A Designing Information Devices and Systems I
Spring 2021 Midterm 2

1. **HONOR CODE**
   If you have not already done so, please copy the following statements into the box provided for the honor code on your answer sheet, and sign your name.

   I will respect my classmates and the integrity of this exam by following this honor code. I affirm:

   - *I have read the instructions for this exam. I understand them and will follow them.*
   - *All of the work submitted here is my original work.*
   - *I did not reference any sources other than my two reference cheat sheets.*
   - *I did not collaborate with any other human being on this exam.*

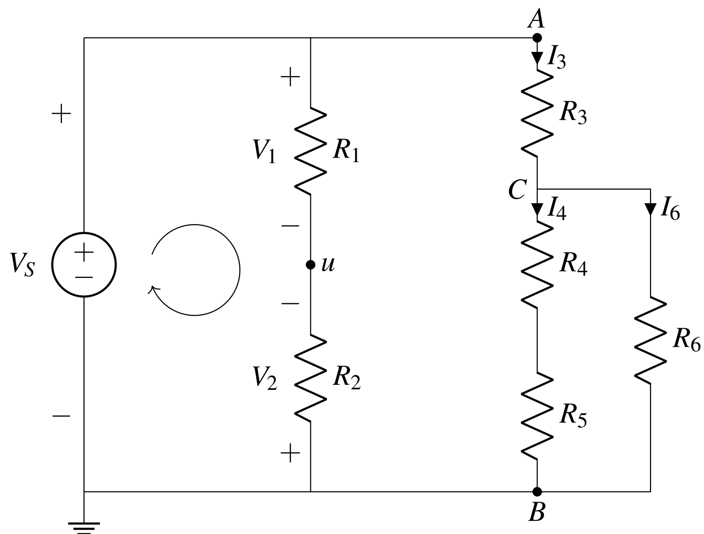
2. (a) **What other courses are you taking this semester?** *All answers will be awarded full credit; you can be brief.* **(2 points)**

   (b) **What has been your favorite part of 16A so far?** *All answers will be awarded full credit; you can be brief.* **(2 points)**

3. Circuit Analysis (18 points)

For the circuit in the following diagram, answer parts (a) – (e).

You should **not** change the labels that are already given in the diagram. If you add any additional labels for your analysis, you should show your labels in the answer sheet for the corresponding part(s).



(a) (3 points) Redraw the circuit diagram in your answer sheet. Following the passive sign convention, **label** (i) the current I_S through the voltage source V_S, (ii) the current I_1 through the resistor R_1, and (iii) the voltage V_3 across the resistor R_3.

(b) (2 points) **Write the KVL expression** for the loop drawn in the circuit diagram in terms of voltages V_S, V_1, and V_2.

(c) (2 points) **Write the KCL expression** at node C in terms of currents I_3, I_4, and I_6 as labeled in the circuit diagram.

(d) (5 points) Given $V_S = 5$ V, $R_1 = 1$ kΩ, $R_2 = 4$ kΩ, $R_3 = 2.5$ kΩ, $R_4 = 1$ kΩ, $R_5 = 4$ kΩ, $R_6 = 5$ kΩ, **solve for the values of the element voltages V_1, V_2, and the node voltage u.** Show your work. You can use any circuit analysis techniques you have learned in this course.

(e) (6 points) Given $V_S = 5$ V, $R_1 = 1$ kΩ, $R_2 = 4$ kΩ, $R_3 = 2.5$ kΩ, $R_4 = 1$ kΩ, $R_5 = 4$ kΩ, $R_6 = 5$ kΩ.

i. **If we combine R_3, R_4, R_5, and R_6 as an equivalent resistor R_{eq} connecting between the nodes A and B, what is the value of R_{eq}?**

ii. **What are the values of the current I_3 and the power dissipated by R_3?** Show your work.

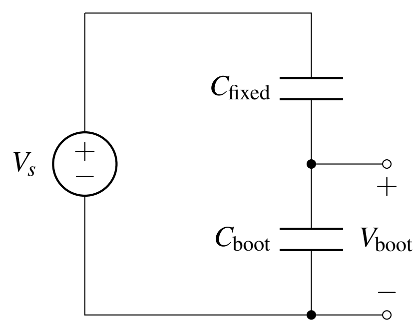
4. Capacitive TouchSki (12 points)

One of your friendly lab TAs is preparing to go skiing for the first time! As excited as she is, she's very worried about losing her balance. To ease her mind, she decides to apply what she knows about capacitors to create a circuit that will indicate if there is excessive force applied to either ski.

In this question, we will examine a force-sensing circuit for a single ski.

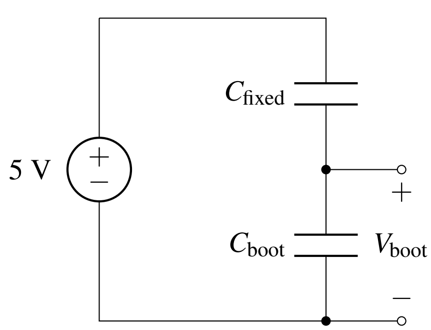
(a) (3 points) To create a sufficiently large capacitance, your TA affixes conductive plates to both the bottom of the boot and the top of the ski, with a thin insulating layer with permittivity ε in between. The boot has area A_{boot} and overlaps completely with the ski. Measurements show that the thickness t of the insulating layer varies with the force F as $t = \frac{1}{kF}$, where k is some constant. **Write the capacitance C_{boot} as a function of the force applied, the area of the boot, and constants.**

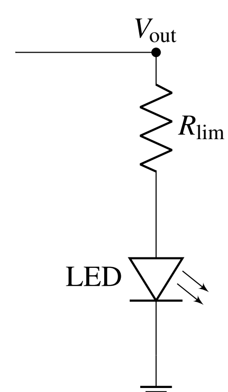
(b) (4 points) In order to measure the capacitance C_{boot}, you are given the following circuit. Assume the capacitors have no initial charge before connected to the voltage source.



What is the voltage V_{boot} in terms of V_s, C_{boot}, and C_{fixed}?

(c) (5 points) Now, you'd like to control an LED based on the force applied to the ski. Redraw the following circuit in your answer sheet, **complete the circuit so it sets $V_{\text{out}} = 5\,\text{V}$ when $V_{\text{boot}} < 2.5\,\text{V}$, and $V_{\text{out}} = 0\,\text{V}$ when $V_{\text{boot}} > 2.5\,\text{V}$** (you don't need to consider the special case when $V_{\text{boot}} = 2.5\,\text{V}$). You may use one comparator and up to two additional voltage sources.





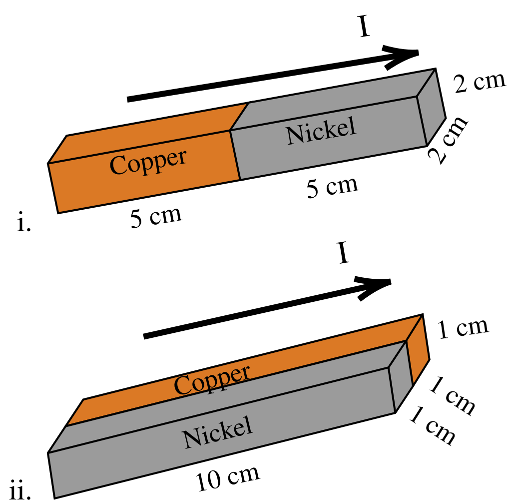
5. Resistive Temperature Sensor (17 points)

Oh no! Predictably, your lab TA gets hurt on the first day of her ski trip and is instructed to ice her injury regularly. However, she's finding that the ice packs are often too cold or too warm and needs a way to track their temperature.

Fortunately, she remembers from 16A that the resistance of many resistors is dependent on temperature! Using this information, you decide to help her build a temperature-sensing device.
Note: in this problem, temperature T is measured in the unit of Celsius.

(a) (6 points) You have different types of resistive bars available in your lab. **For each of the following two resistive bars, express the total resistance in terms of the given quantities and dimensions.**

$$\text{resistivity of copper } = \rho_{C_o}(1+\alpha T)\ \Omega\cdot\text{cm}$$
$$\text{resistivity of nickel } = \rho_{N_o}(1+\beta T)\ \Omega\cdot\text{cm}$$



(b) (5 points) You have the following circuit that has a temperature dependent resistive bar R_T and a resistor with fixed resistance R_f. **For this part only**, assume $V_s = 5$ V, $R_f = 4$ kΩ, and R_T has resistivity $\rho_T = 100(1+0.01T)\ \Omega \cdot \text{cm}$ and cross-sectional area $A = 1\ \text{cm}^2$.

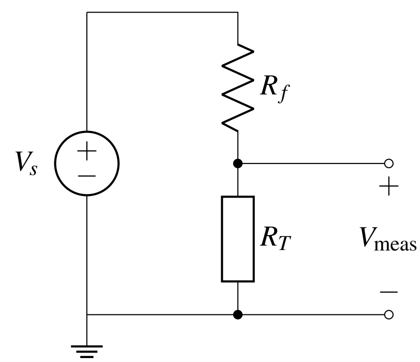


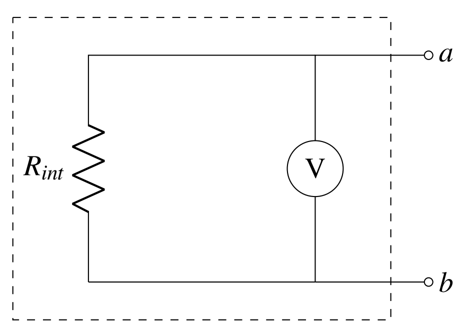
Figure 5.1: Circuit diagram for parts (b) and (c).

You want to be able to measure temperature T within the range $-10°\text{C} \leq T \leq 30°\text{C}$. You also want to limit the current flow through the resistive bar to be no more than 1 mA. **Find the minimum length of the resistive bar R_T such that the current limit is met for *all* temperatures in the specified range.**

(c) (6 points) Next, you are tasked with measuring the voltage across R_T.

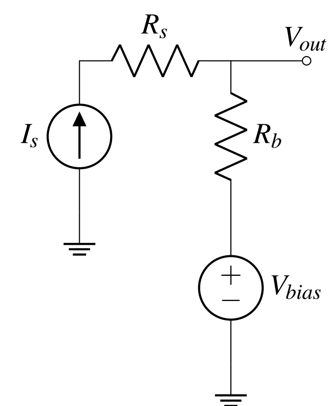
i. **Draw how you would attach an *ideal* voltmeter to the circuit in Figure 5.1, in order to measure the voltage across R_T.**

ii. Instead of an ideal voltmeter, you only have a practical voltmeter that can be modeled as an ideal voltmeter coming with a parallel internal resistance R_{int}, shown below. You connect the practical voltmeter to the same two nodes where you would attach the ideal voltmeter. Assuming R_T = 1000 Ω for this part, **determine the minimum value of R_{int} such that the equivalent resistance across the voltmeter is no less than 99% of R_T.**



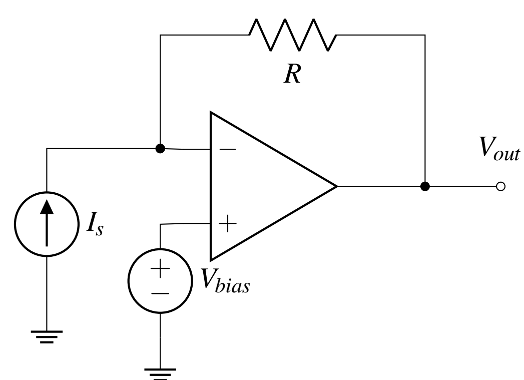
6. Data Conversion Circuits (22 points)

(a) (4 points) Photonic circuits use light to communicate. We still need to convert the light into electricity by a photodiode to process it. We can model the photodiode as a current source I_s. Sometimes it is necessary at the receiver side to adjust the transmitted voltage level V_{out}, and one way to do this is using a voltage source V_{bias}. Consider this simple photonic receiver circuit:



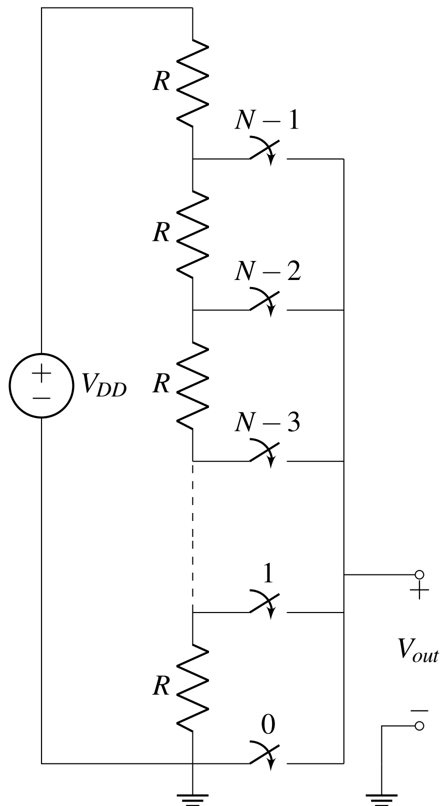
Using *superposition*, solve for the voltage V_{out} in terms of I_s, V_{bias}, R_s, and R_b. Show your work.

(b) (4 points) The previous receiver circuit may have problems with loading. Instead, we may use an op-amp, such as in this circuit:



Calculate the voltage at the output V_{out} in terms of I_s, V_{bias}, and R. Show your work. You will not receive full credit for directly copying a formula from your cheat sheet.

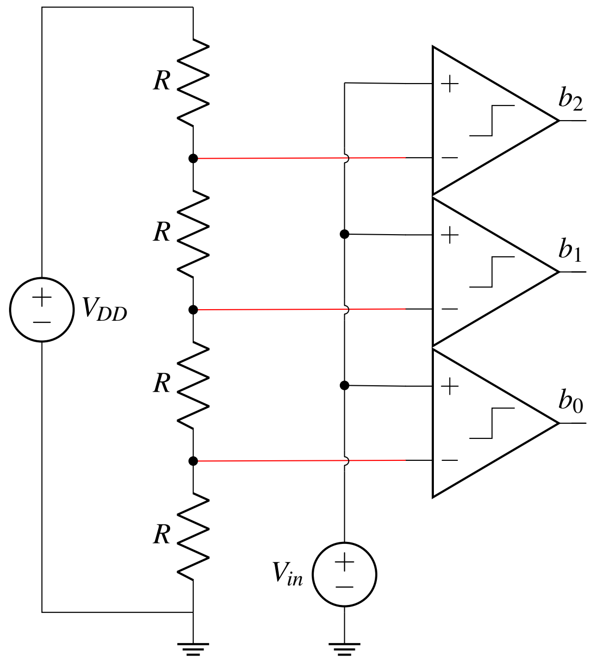
(c) (4 points) We need some circuits to convert between our analog voltage values and some digital representation stored in 1s and 0s. We mentioned digital-to-analog converter circuits, or DACs, in lecture. Let's inspect one here:



Note that there are N resistors and N switches in the circuit. Depending on some input digital code, one of the switches is closed, connecting the output to some node in the resistor ladder.

If only the ith switch is closed ($0 \leq i \leq N-1$), **what is the output voltage V_{out} in terms of V_{DD}, i, N, and R?**

(d) (4 points) The dual to DAC circuits are analog-to-digital converters, or ADC circuits. Here is an example of one, using resistors and comparators:

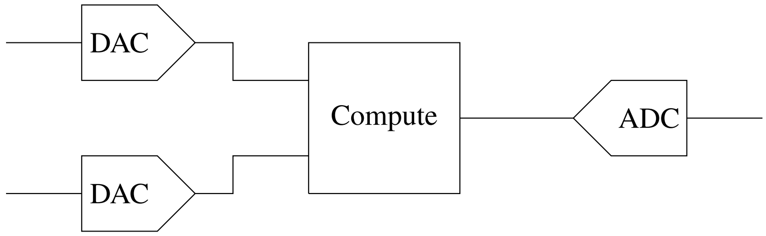


Note: The red wires in the diagram are regular wires, but have been colored to show that they do not touch the crossing black wires.

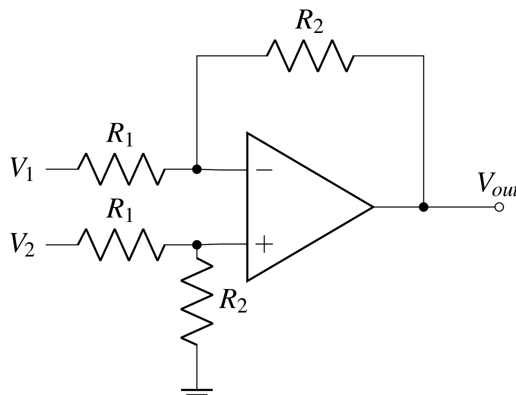
The resistor ladder gives us a set of reference voltages to compare against. We use a set of comparators to compare the input voltage V_{in} against these reference levels, and we get out a corresponding digital code b_0, b_1, and b_2.

Assume that $V_{DD} = 1\,\text{V}$, and that the comparators are connected to rails $V_{DD} = 1\,\text{V}$ and $V_{SS} = 0\,\text{V}$. If V_{in} is $0.3\,\text{V}$, **what are the outputs** b_0**,** b_1**, and** b_2?

(e) (6 points) These DAC and ADC circuits help us represent numbers using voltage values. We would like to build some blocks that let us compute, e.g. add and multiply, with these numbers. We call this "analog computing," and we saw an example of this previously in the "artificial neuron" circuit. These analog compute circuits have potentially massive speed benefits over comparable digital compute circuits. (Take EECS151 and EE140 for more details).

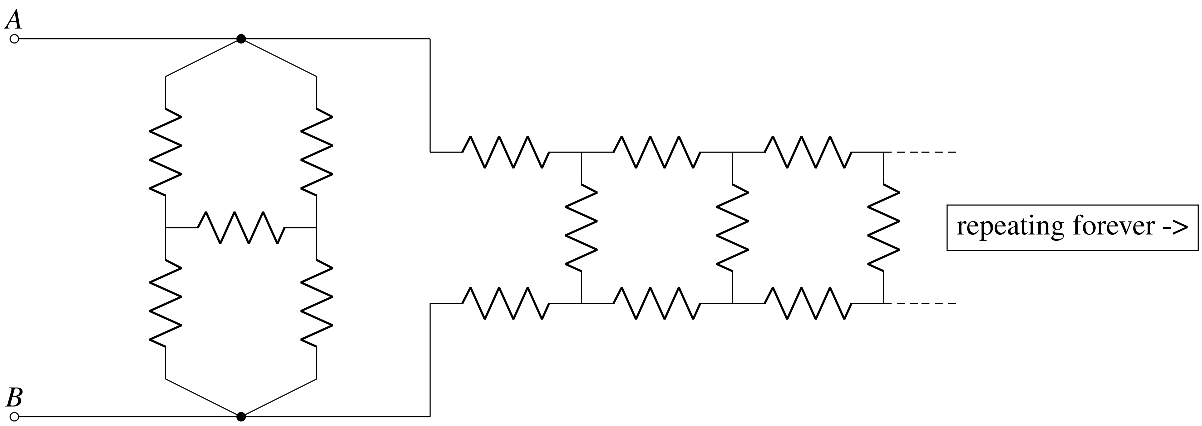


Consider one such compute circuit below, the differential amplifier. This is a common circuit in audio amplifiers but is also a useful tool for mathematical computing. **Find the output V_{out} in terms of V_1, V_2, R_1, and R_2.**



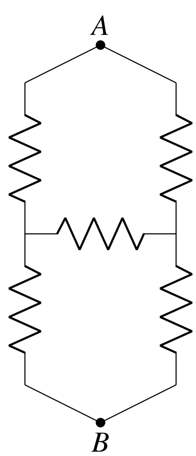
7. Aid to the Resistance (13 points)

The main ship of the Resistance Fleet is in trouble! They have recruited you to help fix the issue. The on-board technicians have determined that the resistor grid in the main console is faulty (one of the resistors must be fried). It is your job to replace the grid with something of equivalent resistance. However, because of severe budget cuts in the Resistance's EE department, **you can only use a single resistor connected between nodes A and B to replace the resistor grid**. The technicians hand you the diagram below of what the resistor grid looked like. **All resistors in the diagram have resistance value R.**



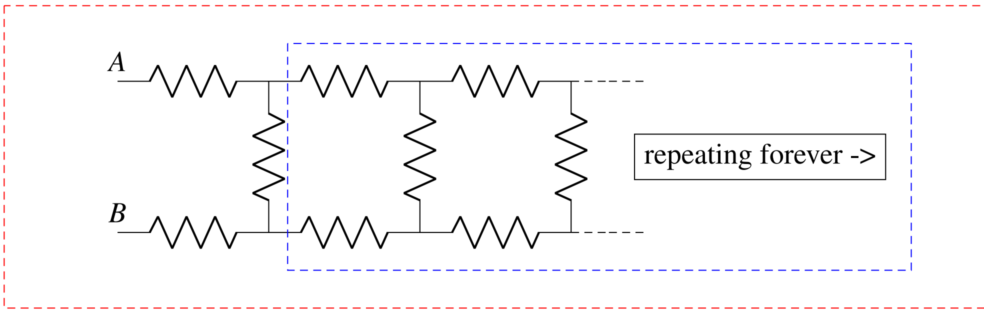
(a) (5 points) **Find the equivalent resistance** of the following piece of the resistor grid between nodes A and B in terms of R.

Hint: If a resistor has no current flowing through it, what is it equivalent to?



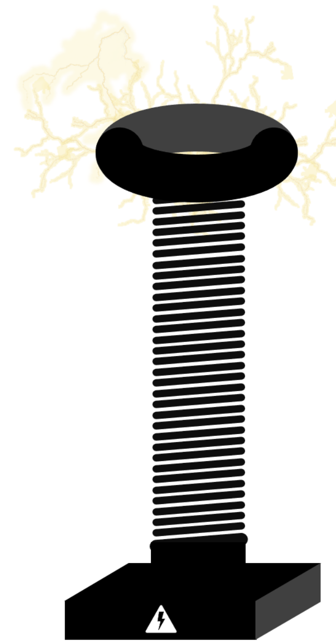
(b) (5 points) **Find the equivalent resistance** of the following piece of the resistor grid between nodes A and B in terms of R.

Hint: (i) Because the pattern is infinite, the equivalent resistance of the circuit in the red box and the equivalent resistance of the circuit in the blue box are equal.
(ii) The solutions to the quadratic equation $ax^2 + bx + c = 0$ *are* $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.



(c) (3 points) Suppose the equivalent resistance for the piece of resistor grid in part (a) is αR, and the equivalent resistance for the piece of resistor grid in part (b) is βR, where α and β are known real numbers for this part. **What should be the value of the resistor you use to replace the entire grid with, in terms of R, α, and β?**

8. Explosive Tesla Coil!! (19 points)



Renowned as the father of alternating current technology, the brilliant Nikola Tesla created countless inventions which were truly beyond his time. One of such inventions was the Tesla Coil, a device that could continuously generate safe lightning! While we do not yet have all the background knowledge needed for the full circuitry, we can still make an effective model of this device with a capacitor network.

Loosely stated, the Tesla coil circuit charges up a specially-designed, massive capacitor (labeled C_T here) until it reaches a certain threshold of charge. The effective capacitor model is shown in Figure 8.1, where V_S is the charging voltage source, each C are identical charge-loading capacitors, and C_T is the capacitor that models our Tesla Coil.

The capacitor C_T charges over a repeated series of cycles.
Each cycle involves two stages:

- Stage A – The ϕ_B switches in Figure 8.1 open and then the ϕ_A switches close.
  In this stage the loading capacitors C are charged by V_s.
- Stage B – The ϕ_A switches in Figure 8.1 open and then the ϕ_B switches close.
  In this stage the loading capacitor C charges are shared with the Tesla coil capacitor C_T.

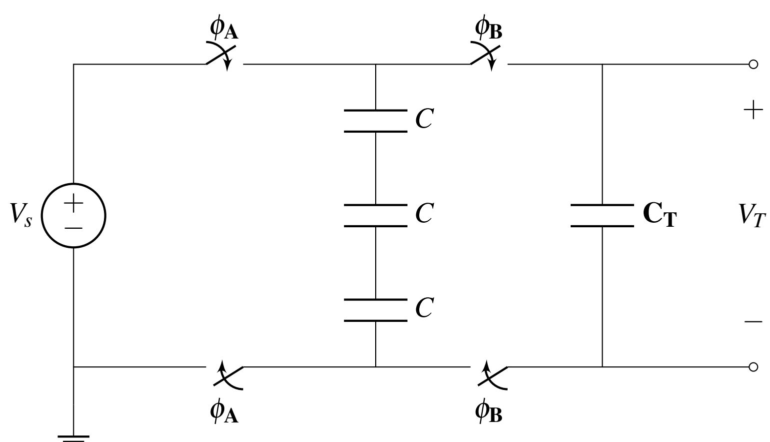


Figure 8.1: Tesla coil effective circuit model.

(a) (4 points) **Draw the equivalent circuit diagram for each stage (A and B).** To receive full credit, make sure each diagram you draw has only the relevant circuit elements, i.e. the diagram should not include any elements that are not connected in a closed circuit.

(b) (3 points) If we would like to model the three identical loading capacitors as a single equivalent capacitor labeled C_L, **what value should we choose for C_L to ensure that the circuit has the same behavior**? Your answer should be in terms of C.

(c) (6 points) Suppose that all capacitors in the device start off without any charge. **Compute the charge Q_T and the voltage V_T** across the Tesla coil capacitor C_T after the circuit completes a cycle (going from stage A to stage B).

All final solutions should be in terms of the known circuit constants (V_s, C, and C_T).

Hint: You may use the simplified equivalent loading capacitor model C_L through your work, then plug C back in at the end.

(d) (6 points) Now suppose that the Tesla coil capacitor C_T starts with an initial charge Q_0, which was collected as a result of previous cycles.

i. **Compute the charge Q_T and the voltage** V_T across the Tesla coil capacitor C_T after the circuit completes a cycle (going from stage A to stage B). For simplicity, you can assume the loading capacitors still start off without any charge.

ii. **Compute the ratio σ of energy stored on C_T before and after this cycle** (so $\sigma = E_{\text{after}}/E_{\text{before}}$). You can get partial credit for writing the expressions for E_{before} and E_{after}.
At which value of initial charge Q_0 do we no longer add energy to the coil after a cycle?

All final solutions should be in terms of the known circuit constants (V_s, C, C_T, and Q_0).

Hint: It may be helpful to simplify the final answer of σ in the form:

$$\sigma = \left(\frac{3\left(\frac{C_T}{C}\right) + \_\_\_}{3\left(\frac{C_T}{C}\right) + \_\_\_} \right)^2$$

where the ___ spaces are yet to be discovered by you!