Recipe: Nodal Analysis!

March 16, 2017

1 What Is Nodal Analysis?

Nodal analysis is a systematic way to solve for the voltages and currents in a circuit. It combines KVL, KCL, and Ohm's law.

2 Total Prep Time

We'll usually give you circuits that'll take fifteen minutes, or less, to solve by hand. If you really hate somebody and want to make him/her suffer, you can always come up with crazy circuits that will make them tear their hair out! But if you give *any* of these circuits to a computer, it'll take a few seconds. [1] In fact, the simulation program used to solve circuits, SPICE (Simulation Program with Integrated Circuits Emphasis) was developed here at Berkeley in the 1970s. Check out the plaque at the front door of Cory Hall!

3 Ingredients

- **Your circuit, which, for now, can have independent/dependent voltage sources, independent/dependent current sources, and resistors. Later on you'll find that nodal analysis works just fine with more advanced elements**
- Identification of nodes in a circuit (and setting of the "reference" node)
- Branch currents and KCL
- Sign conventions, and Ohm's Law
- Solving systems of linear equations

[1] See? Computers are wizards. You might want to be nice to them to get them on your side. We'll need them in our arsenal from time to time on our journey through the mysterious and wonderful land of circuits.

4 Directions

1. **Identify all the nodes in the circuit, and label them.** An intersection of *two or more circuit elements* constitutes a node. These elements can be voltage sources, current sources, or resistors. In nodal analysis, we will be solving for the voltages at all the nodes, or, in other words, the *node voltages*. We denote the node voltage for node j by U_j. But, wait! Remember how we said that voltages only mean something with respect to a reference?

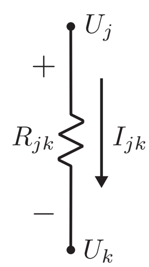
2. So, we next **select a reference node, or ground node.** We set the voltage at the selected reference node to be 0. [2]

3. Select **current flow directions** and label the currents in each branch of the circuit. Match these branch currents with **voltage differences** across each of the resistors in the circuit. Mark the polarity of the voltage difference according to the passive sign convention!

4. For every node that is *not connected to a voltage source (independent, dependent or otherwise)*, **write KCL in terms of the branch currents.** As a refresher, your KCL equation will look like this:

$$\sum(\text{currents entering node}) = \sum(\text{currents leaving node})$$

5. **Translate each KCL equation using Ohm's Law** as follows:



For resistor R_{jk} connecting nodes j (marked with + voltage polarity) and k (marked with − voltage polarity),

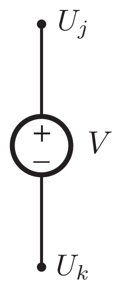
$$I_{jk} = \frac{U_j - U_k}{R_{jk}}$$

[2] You can set the reference node to be any node you want. Your *branch voltages* and *branch currents* will not change. Here are some good choices for the reference node (in practice, they make your equations much simpler):

- negative terminal of a voltage source
- a node with lots of elements connected to it
- somewhere near the "bottom" of the circuit

Note that we do not need to do this step for branches that contain a current source. For these branches, we already know the exact value of the current, and we retain this exact value in the equation(s).

6. Now, what do we do for the nodes that *are* connected to voltage sources? We **write a voltage difference equation**.



For instance, if node j is at the positive terminal of a voltage source V, and node k is at the negative terminal of the voltage source V, we will have

$$U_j - U_k = V$$

.

7. *Stop to taste:* Sanity check! Make sure that you have as many equations as unknowns (number of node voltages). You should also check to see that the units in your equations match on both sides!

8. **Set up the system of equations and solve them! You may use a matrix for this step.** You have now solved uniquely for *every* branch voltage, and every branch current.

We're done! Now, before you serve up your solution for us to taste, you might want to check if everything's correct. How might you do that? You have the following options:

- Solve for the branch currents through each resistor. Make sure that KCL is satisfied at each node that is not connected to a voltage source.
- Solve the circuit using superposition, and see if you get the same answer.
- Solve the circuit using observations of resistors being in series/parallel (this is not always possible, though)
- Ask a friend to taste your solution and see if s/he likes it.[3]

[3] You are not allowed to use this method on the exam ;)