Recipe: Thevenin and Norton Equivalents!

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1 Who were Thevenin and Norton?

Wikipedia tells me they were famous circuit theorists. Wikipedia also tells me the following: "Thvenin is remembered as a model engineer and employee, hard-working, of scrupulous morality, strict in his principles but kind at heart."

Yeah, sure, that's circuits people for you.

2 What are Thevenin and Norton equivalents?

Basically, Thevenin and Norton discovered something that is potentially very useful for us 16A Intrepid Circuit Solvers. They discovered that any resistive circuit with current and voltage sources that has two output terminals, A and B, can be equivalently represented as a voltage source with a resistance in series (Thevenin equivalent) or a current source with a resistance in parallel (Norton equivalent).

Why is this useful? Think of the circuit as a *black box*, something that we cannot look inside and see. If we applied some sort of load across the terminals A and B of the black box (for example, a resistor), we might be interested in the voltage difference that develops across the load, or the current that flows through the load. How would we answer this question easily and quickly for a generic load resistance? It turns out that if we invoke the Thevenin and Norton equivalent of the circuit, this becomes pretty easy to do, as you will see!

3 Total Prep Time

Usually about five to ten minutes to find each equivalent. If you have a physical circuit and an ammeter and voltmeter, a lot less!

4 Ingredients

- **Your circuit, which basically contains resistors, voltage sources and current sources. The sources can be dependent or independent**

- Identification of nodes in a circuit (optional: setting the "reference" node)
- Solving for node voltages in a circuit using one of **nodal analysis, superposition, or inspection (identifying voltage/current dividers, etc)**
- Knowledge of short circuit and how short circuiting two nodes modifies a circuit
- Solving for currents in a circuit using one of **nodal analysis, superposition, or inspection (identifying voltage/current dividers, etc)**
- Finding equivalent resistance between two terminals in a circuit

5 Directions

5.1 Part 1: Finding the Thevenin voltage

Remember that the Thevenin voltage is the *open circuit voltage* between terminals A and B. This amounts to simply taking a voltmeter, placing its probes at terminals A and B, and measuring the voltage.

So, in theory, what do you do? Just take your original circuit, and find the voltage difference between A and B!

1. **Identify all the nodes in the circuit.** Remember that terminals A and B are going to correspond to two different nodes in the circuit. What you are trying to solve for is $V_A - V_B$!!
2. Solve for $V_{th} = V_A - V_B$ using one of the following methods:
   - Use nodal analysis to solve for V_A and V_B. (Don't forget to set a reference node!)
   - Use superposition to solve for $V_A - V_B$. In a nutshell: love the circuit with only one of the independent sources on in each step, and add all your resulting solutions.
   - Any other method! The world of circuits is your oyster.

5.2 Part 2: Finding the Norton current

Recall that the Norton current is the *short circuit current* between terminals A and B. This amounts to simply taking an ammeter, placing it between terminals A and B, and measuring the current through it!

So, in theory, what do you do? Just take your original circuit, short terminals A and B with a wire, and find the current through the wire.

1. If you've not done it already, **identify all the nodes in the circuit**.

2. Connect a wire between A and B. This is called a **short circuit**, and you want to find the current through this wire!

3. (Remember that you have actually changed the circuit by adding this wire!) Use the newly added wire to **simplify your circuit.** Here's how you do it: any resistors that are connected in parallel with the wire between terminals A and B are just not going to matter! Why is this? Remember that a short-circuit wire effectly produces a path of zero resistance between its end points. Therefore, the charges flowing between A and B are always going to choose the path of least resistance, which is the short-circuit wire. Ultimately, for the purpose of this part of the problem, you can remove any such resistors. :)

4. Solve for $I_N = I_{AB}$ using nodal analysis, superposition, or any other method you wish to use!

5.3 Part 3: Finding the Thevenin/Norton equivalent resistance, R_{eq}

There are two ways to do this. One is very simple. Thevenin's and Norton's theorem tells us that $R_{eq} = \frac{V_{th}}{I_N}$. Therefore, apart from dividing two numbers, you have to do no extra work! No solving circuits! Yay.

But what if you wanted to calculate only the Thevenin voltage, and not the Norton current (or vice versa), and you still wanted to know R_{eq}? Here's what you would do to find the *equivalent resistance* between terminals A and B:

1. Turn off all **independent sources** in the circuit. Your modified circuit is now going to be just a network of resistors (and maybe dependent sources, but let's cross that bridge when we come to it :)) [1]

2. Apply a **test voltage source** V_{test} to the circuit. Calculate the **test current** I_{test} that flows through the test voltage source. Then, $R_{eq} = \frac{V_{test}}{I_{test}}$. As before, you can use any method you want to solve for I_{test}!

3. Note that if your resistive network is relatively simple, you can just combine resistors in series and parallel to find the equivalent resistance *between A and B*. For instance, if your resistive network is just two resistors connected in parallel between A and B, R_{eq} will be the parallel combination of these resistors. Similarly for series!

That's it! You're done. And now, you can draw your beautifully simple Thevenin and Norton equivalents as follows:

INSERT FIGURE

And now, how would you calculate V_L and I_L?

[1] Remember that turning off an independent voltage source is equivalent to shorting its terminals, and turning off an independent current source is equivalent to opening its terminals. Why? Think about it!

From the Thevenin equivalent, you'll get $V_L = \frac{V_{th} R_L}{R_{eq} + R_L}$. (using the formula for a voltage divider!)

And from the Norton equivalent, you'll get $I_L = \frac{I_N R_{eq}}{R_{eq} + R_L}$ (using the formula for a current divider!)

Told you it would become simple. Yes, Thevenin and Norton were geniuses. No, we do not expect you to become like them.