EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 3A

1. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, *if the product exists*, find the product by hand. Otherwise, explain why the product does not exist.

(a) **A B**

(b) **C D**

(c) $\mathbf{D}\,\mathbf{C}$

(d) $\mathbf{C}\,\mathbf{E}$

(e) $\mathbf{F}\,\mathbf{E}$ (only note whether or not the product exists and optionally compute the product if it does)

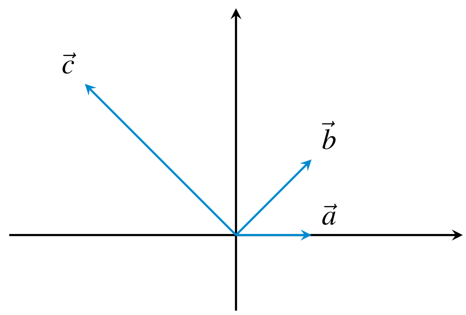
(f) $\mathbf{E}\,\mathbf{F}$ (only note whether or not the product exists and optionally compute the product if it does)

(g) $\mathbf{G}\,\mathbf{H}$ (Practice on your own)

(h) $\mathbf{H}\,\mathbf{G}$ (Practice on your own)

2. Visualizing Span

We are given a point $\vec{c}$ that we want to get to, but we can only move in two directions: $\vec{a}$ and $\vec{b}$. We know that to get to $\vec{c}$, we can travel along $\vec{a}$ for some amount α, then change direction and travel along $\vec{b}$ for some amount β. We want to find these two scalars α and β, such that we reach point $\vec{c}$. That is, $\alpha\vec{a}+\beta\vec{b}=\vec{c}$.



(a) First, consider the case where $\vec{x}=\begin{bmatrix}1\\1\end{bmatrix}$, $\vec{y}=\begin{bmatrix}2\\1\end{bmatrix}$, and $\vec{z}=\begin{bmatrix}-2\\2\end{bmatrix}$. Draw these vectors on a sheet of paper.

(b) We want to find the two scalars α and β, such that by moving α along $\vec{x}$ and β along $\vec{y}$, we can reach $\vec{z}$. Write a system of equations to find α and β in matrix form.

(c) Solve for α, β.

3. Span Basics

(a) What is span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(b) Is $\begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(c) What is a possible choice for $\vec{v}$ that would make $\text{span}\left\{ \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$?

(d) For what values of b_1, b_2, b_3 is the following system of linear equations consistent? ("Consistent" means there is at least one solution.)

$$\begin{bmatrix}1 & 2\\2 & 1\\0 & 0\end{bmatrix}\vec{x} = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$$