1. Identifying a Subspace: Proof

Is the set

\[ V = \left\{ \vec{v} \mid \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\} \]

a subspace of \( \mathbb{R}^3 \)? Why or why not?

2. Exploring Column Spaces and Null Spaces

- The **column space** is the span of the column vectors of the matrix.
- The **null space** is the set of input vectors that when multiplied with the matrix result in the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of \( A \)? What is its dimension?
ii. What is the null space of \( A \)? What is its dimension?
iii. Are the column spaces of the row reduced matrix \( A \) and the original matrix \( A \) the same?
iv. Do the columns of \( A \) span \( \mathbb{R}^2 \)? Do they form a basis for \( \mathbb{R}^2 \)? Why or why not?

(a) \[
\begin{bmatrix}
1 & 0 \\
0 & 0 \\
\end{bmatrix}
\]
(b) \[
\begin{bmatrix}
0 & 1 \\
0 & 1
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 2 \\
-1 & 1
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
-2 & 4 \\
3 & -6
\end{bmatrix}
\]

(e) \[
\begin{bmatrix}
1 & -1 & -2 & -4 \\
1 & 1 & 3 & -3
\end{bmatrix}
\]
(f) What do you notice about the dimension of the columnspace, the dimension of the nullspace, and their sum in all of these matrices?