
EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 5A

1. Identifying a Subspace: Proof

Is the set

$$V = \left\{ \vec{v} \mid \vec{v} = c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } c, d \in \mathbb{R} \right\}$$

a subspace of \mathbb{R}^3 ? Why or why not?

2. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that when multiplied with the matrix result in the zero vector.

For the following matrices, answer the following questions:

- What is the column space of \mathbf{A} ? What is its dimension?
- What is the null space of \mathbf{A} ? What is its dimension?
- Are the column spaces of the row reduced matrix \mathbf{A} and the original matrix \mathbf{A} the same?
- Do the columns of \mathbf{A} span \mathbb{R}^2 ? Do they form a basis for \mathbb{R}^2 ? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$(b) \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$(c) \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

$$(d) \begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$$

$$(e) \begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$$

- (f) What do you notice about the dimension of the column space, the dimension of the nullspace, and their sum in all of these matrices?