EECS 16A Designing Information Devices and Systems I Fall 2022 Discussion 5B

1. Mechanical Determinants

(a) Compute the determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(b) Compute the determinant of
$$\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}$$
.

1

2. Mechanical Eigenvalues and Eigenvectors

In each part, find the eigenvalues of the matrix \mathbf{M} and their associated eigenvectors.

(a)
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$$

Do you observe anything about the eigenvalues and eigenvectors?

(b)
$$\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

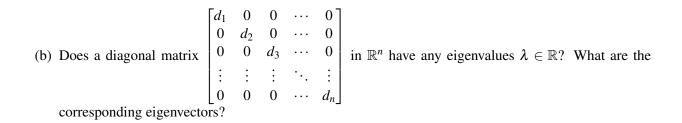
3. Eigenvalues and Special Matrices – Visualization

An eigenvector \vec{v} belonging to a square matrix **A** is a nonzero vector that satisfies

 $\mathbf{A}\vec{v} = \lambda\vec{v}$

where λ is a scalar known as the **eigenvalue** corresponding to eigenvector \vec{v} . Rather than mechanically compute the eigenvalues and eigenvectors, answer each part here by reasoning about the matrix at hand.

(a) Does the identity matrix in \mathbb{R}^n have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?



(c) Conceptually, does a rotation matrix in \mathbb{R}^2 by angle θ have any eigenvalues $\lambda \in \mathbb{R}$? For which angles is this the case?

(d) (**PRACTICE**) Now let us mechanically compute the eigenvalues of the rotation matrix in \mathbb{R}^2 . Does it agree with our findings above? As a refresher, the rotation matrix **R** has the following form:

 $\mathbf{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

(e) Does the reflection matrix **T** across the x-axis in $\mathbb{R}^{2\times 2}$ have any eigenvalues $\lambda \in \mathbb{R}$?

$$\mathbf{T} = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right]$$

(f) If a matrix **M** has an eigenvalue $\lambda = 0$, what does this say about its null space? What does this say about the solutions of the system of linear equations $\mathbf{M}\vec{x} = \vec{b}$?

(g) (**Practice**) Does the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ have any eigenvalues $\lambda \in \mathbb{R}$? What are the corresponding eigenvectors?