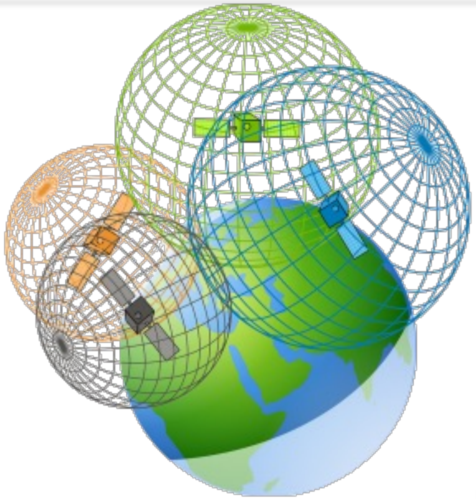


Welcome to EECS 16A!

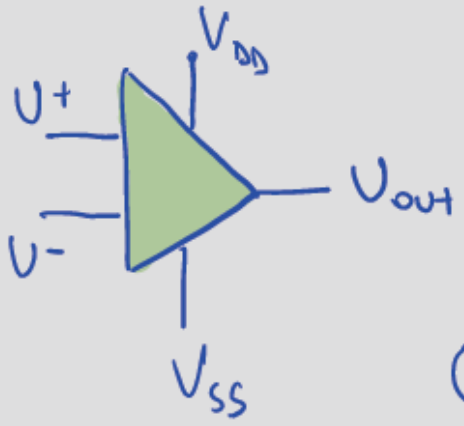
Designing Information Devices and Systems I

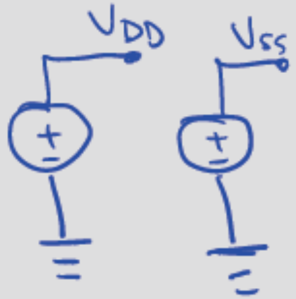
Ana Claudia Arias and Miki Lustig

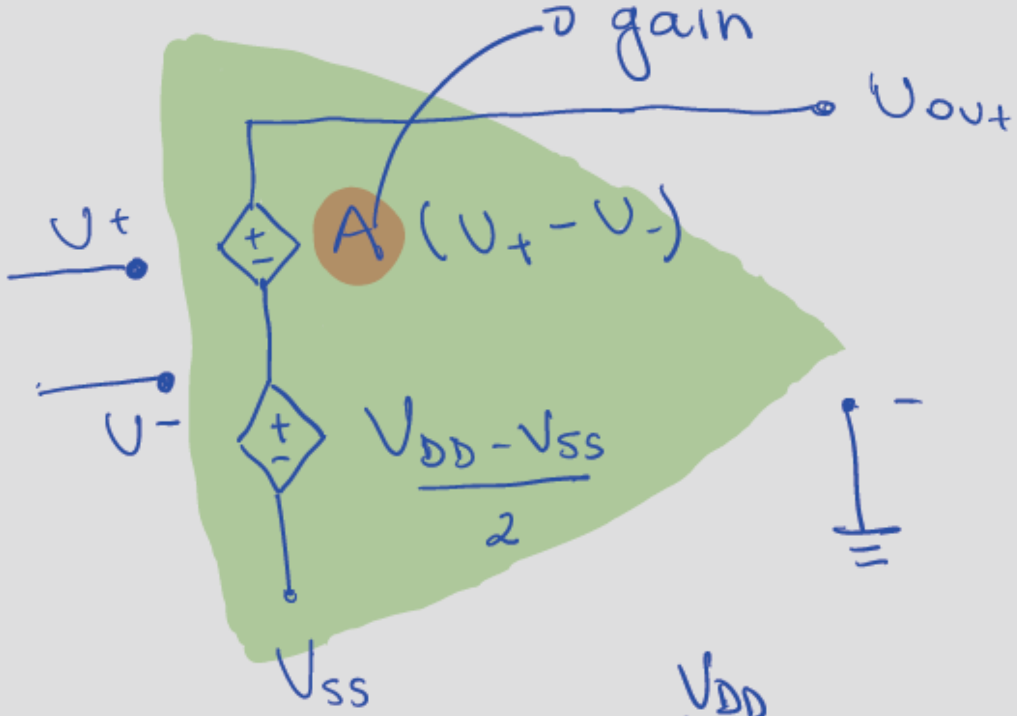
Fall 2022

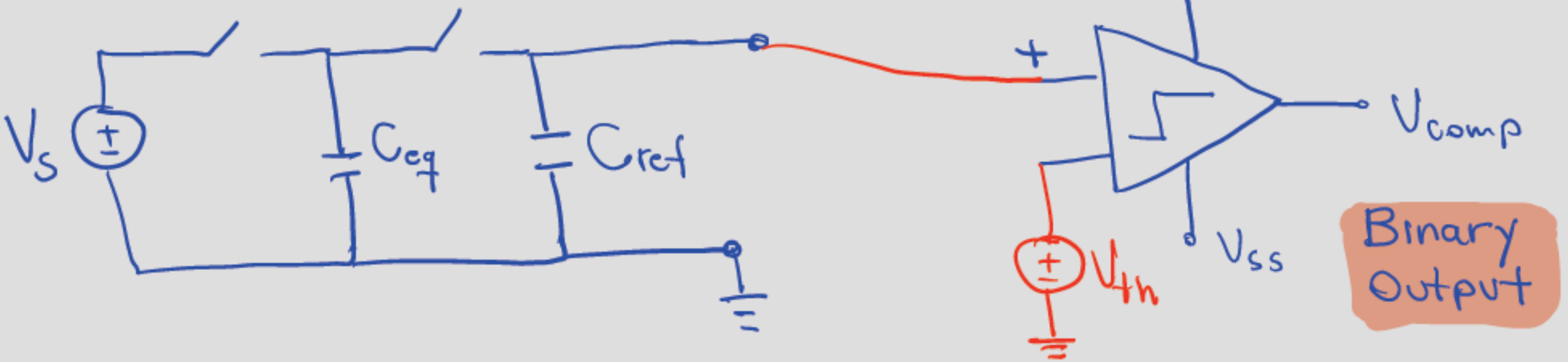
Module 2
Lecture 10
Negative Feedback
(Note 17 and 17B)

Last Lecture…









Last Lecture…

Problem

- We want to play music loud!
- Music is stored as digital signal
- Speakers are analog

Tools

- Resistors
- Capacitors
- Open-circuits
- Voltage Dividers
- Op-Amps
- Thevenin Equivalence
- Norton Equivalene
- KCL / KVL
- Element Def.

Specs

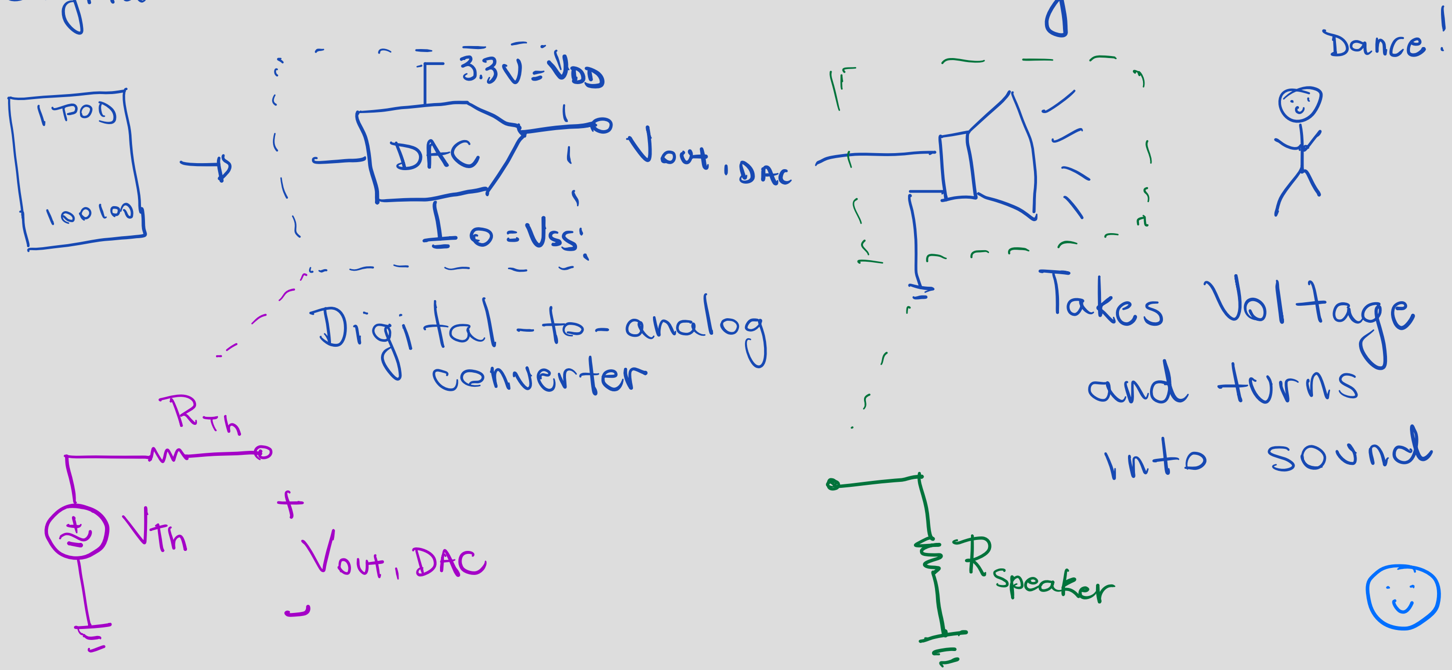
- Speaker takes 0-10V ✓
- Need to go from digital to analog. ?

New Design – Let's play music

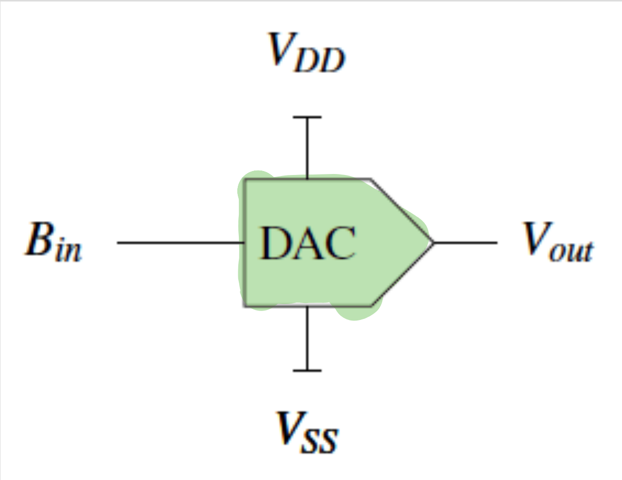
* Want to play music LOUD

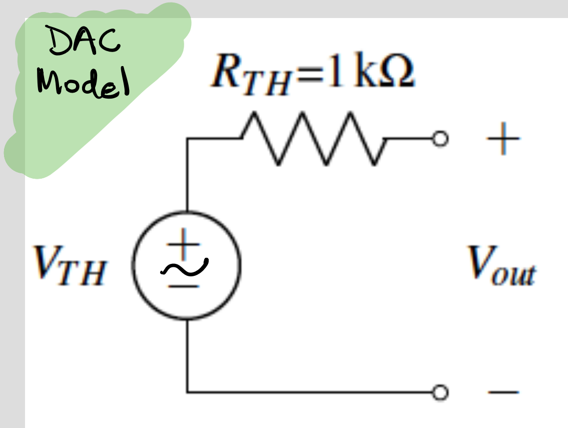
↳ Music is stored as digital signal

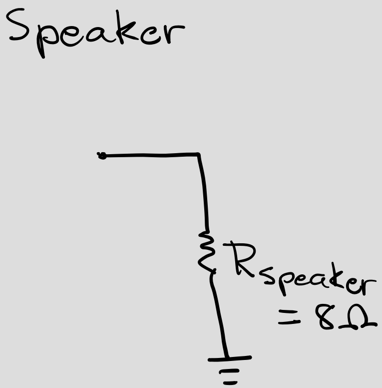
Digital ⟶ Analog

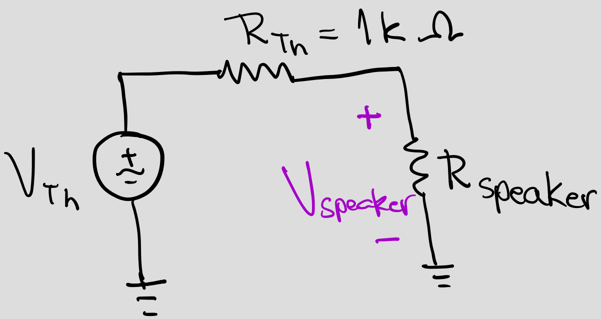


Digital to Analog Converter - DAC









Voltage Divider

$$V_{speaker} = \frac{\overset{8\,\Omega}{\cancel{R_{speaker}}}}{\overset{1000\,\Omega}{\cancel{R_{Th}}} + \overset{8\,\Omega}{\cancel{R_{speaker}}}} \cdot V_{Th}$$

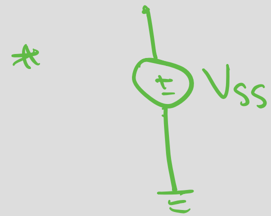
$$V_{speaker} = \frac{V_{th}}{126}$$

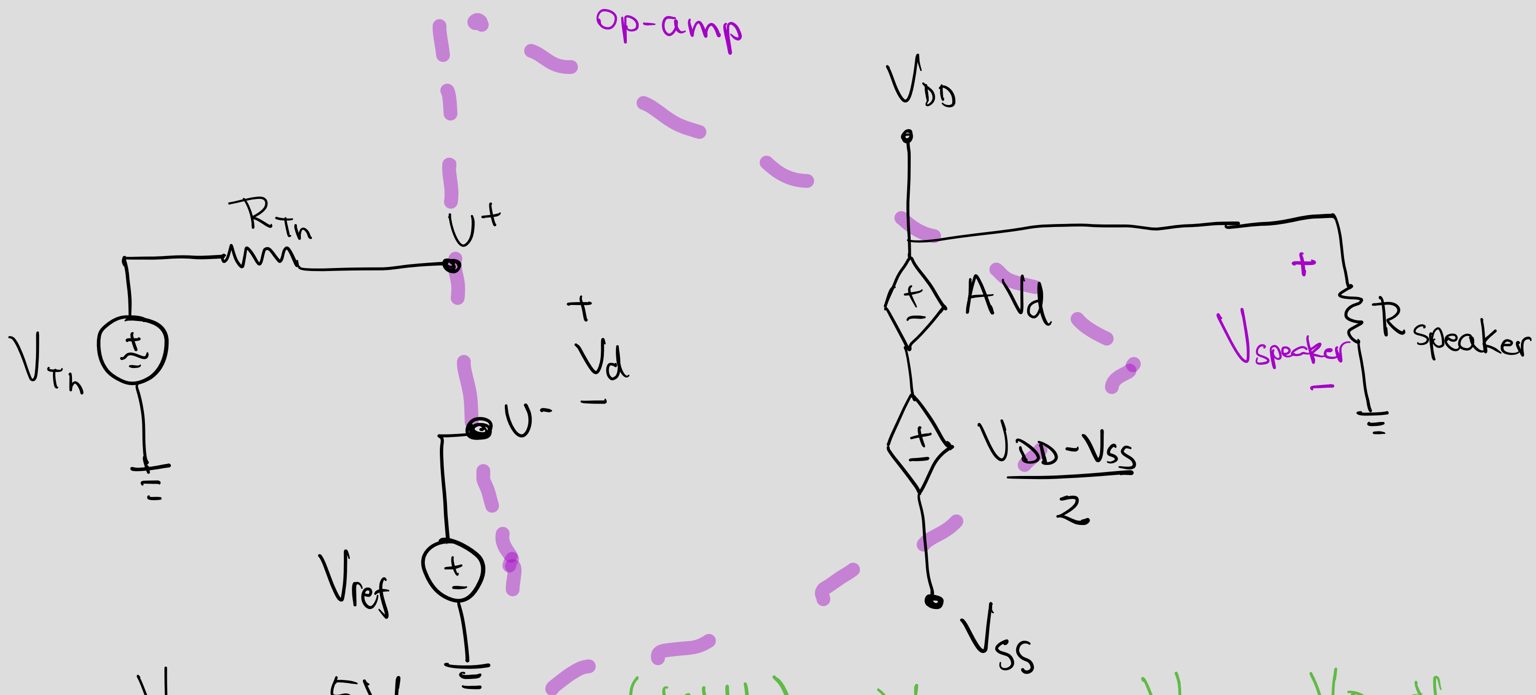
Not loud!

Too quiet!

Need to isolate DAC.

Digital to Analog Converter - DAC





$V_{DD} = -V_{SS} = 5V$

10V output

(Input)

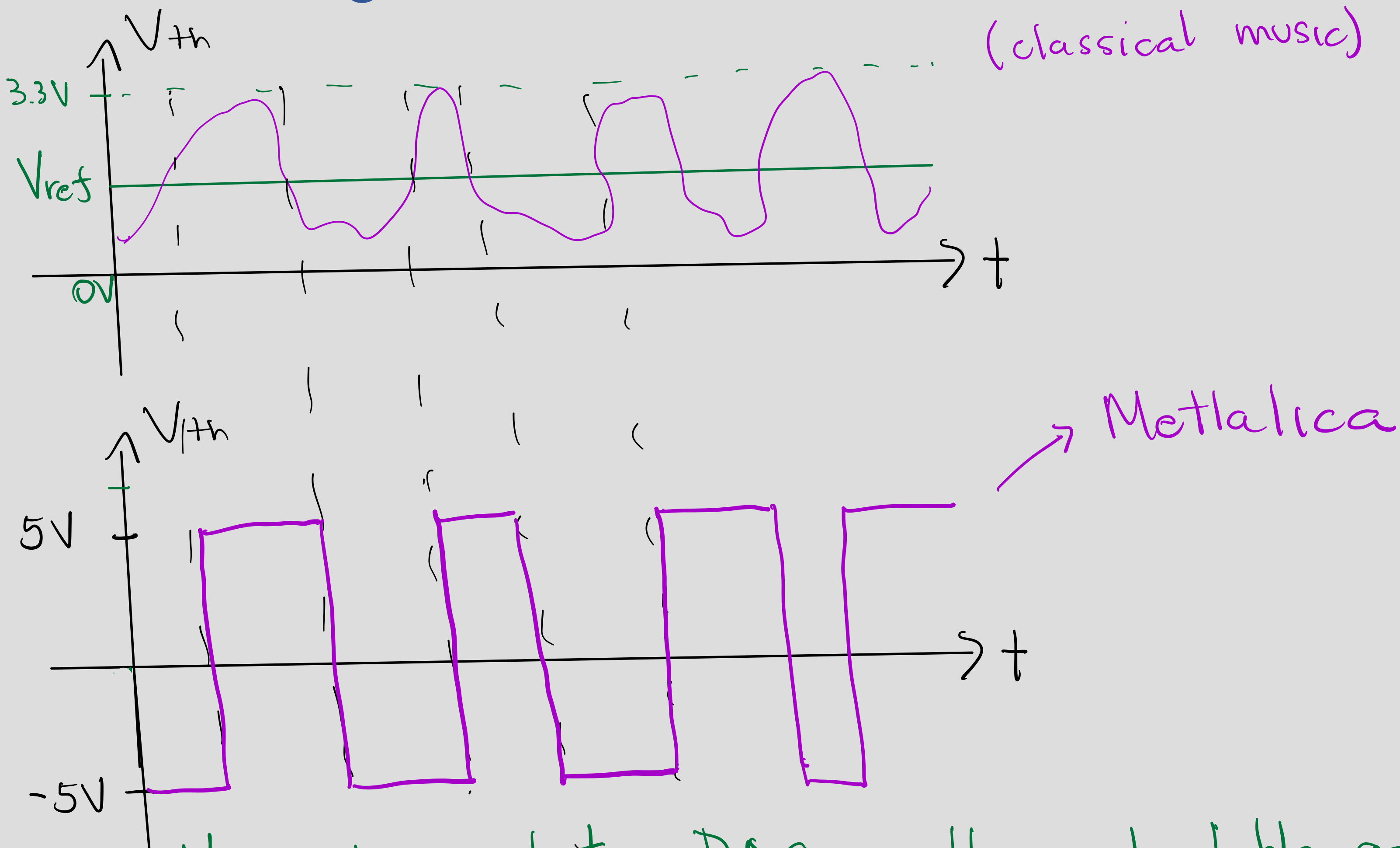
$V_d = U^+ - U^- = V_{th} - V_{ref}$

(KVL) $V_{speaker} = \underbrace{V_{SS} + \frac{V_{DD}-V_{SS}}{2}}_{\frac{V_{DD}+V_{SS}}{2} \to 0} + AV_d = AV_d$

when:

$V_{SS} < AV_d < V_{DD}$

Digital to Analog Converter - DAC



Need to isolate DAC with controlable gain!

e. 3x

Negative Feedback

$$S_{err} = S_{in} - S_{fb}$$

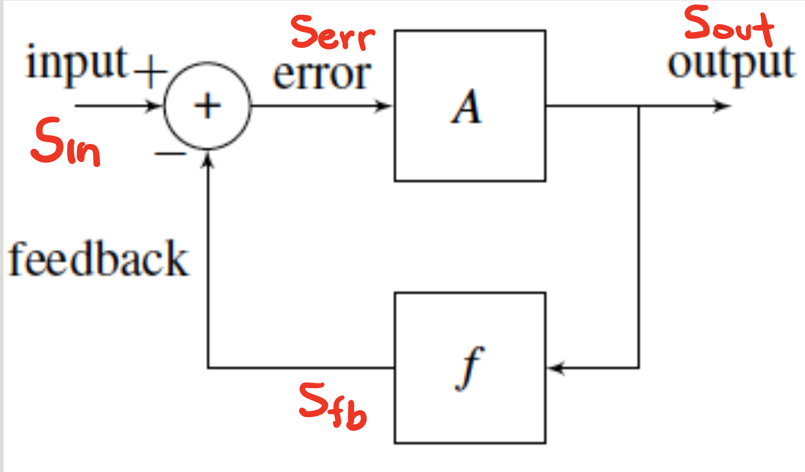
$$S_{out} = A \cdot S_{err}$$

$$S_{fb} = f \cdot S_{out}$$

$$\frac{S_{out}}{A} = S_{in} - S_{fb}$$

$$S_{out}\left(\frac{1}{A} + f\right) = S_{in}$$

$$\frac{S_{out}}{S_{in}} = \frac{1}{\frac{1}{A} + f} = \frac{A}{1 + Af}$$



- Making small adjustments to correct output on the fly
- Basis of control theory
- Many examples in daily life:
  - Biology
  - Self-driving car
  - Human driving car
  - Hand-eye coordination
  - ....

Negative Feedback

$$\frac{S_{out}}{S_{in}} = \frac{A}{1+Af}$$

- Describes the behaviour of the system - transfer function.
- How Sout depends on Sin

$$\frac{S_{out}}{S_{in}} \underset{A \to \infty}{=} \frac{1}{f}$$

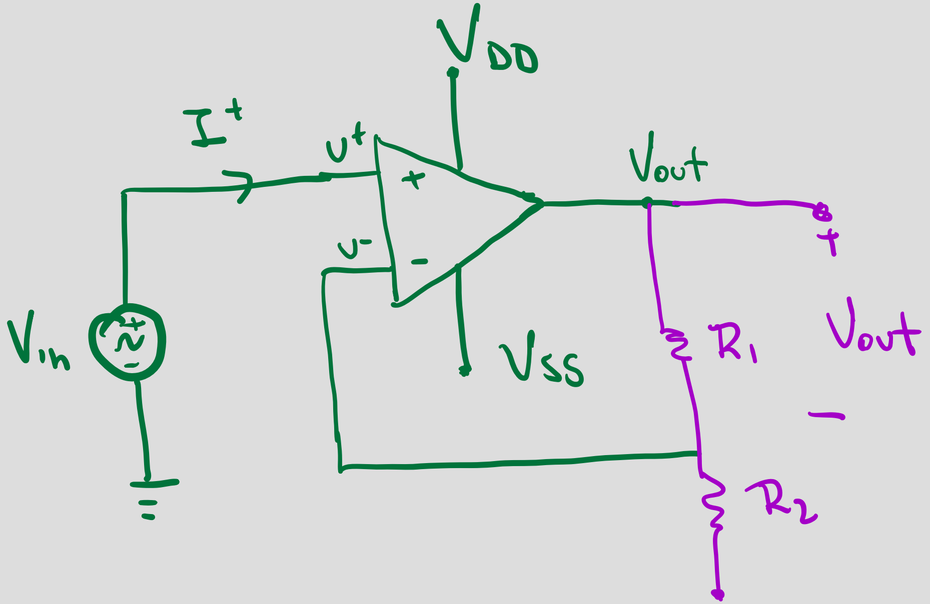
↳ We control the output via block $\boxed{f}$!

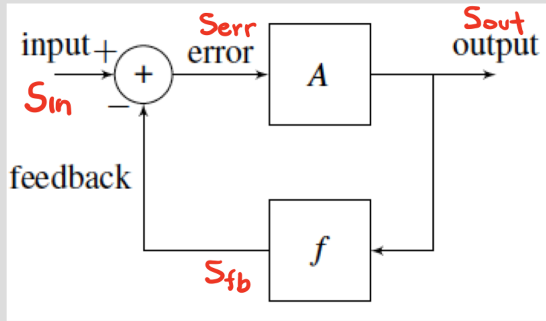
So $V_{out} = \frac{1}{f} V_{in}$ for very large gain.

↳ we can set f to get any output.

(Beautiful result). ☺

Need to isolate the DAC from speaker – OP-Amp with NFB



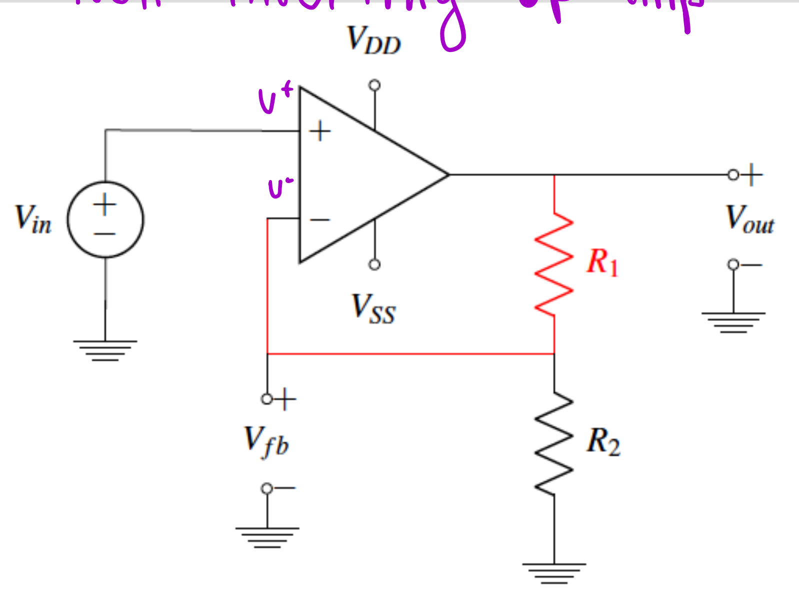


- We want to measure Vout, take a portion of the signal and feedback as V^-

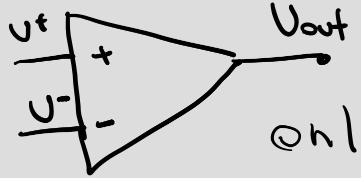
$$V^+ = S_{in}$$
$$V_{out} = S_{out}$$
$$V^- = S_{fb}$$
$$V^+ - V^- = S_{err}$$

Op-Amp in negative feedback

Non-inverting op-amp

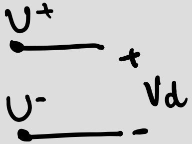


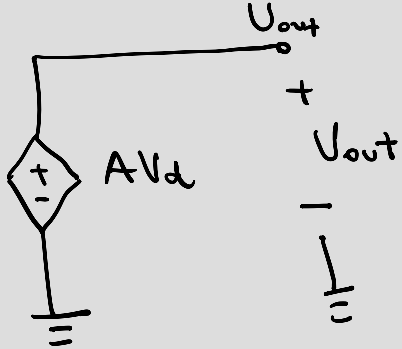
Model:



only for

$$V_{SS} < V_{out} < V_{DD}$$





Simpler model as the second source is not "needed".

(1) $V_d = U^+ - U^- = V_{in} - V_{fb}$

(2) $V_{out} = A V_d$

(3) $V_{fb} = \underbrace{\frac{R_2}{R_1+R_2}}_{f} \cdot V_{out}$

"BUFFER CIRCUIT"

$$V_{out} = A\,(V_{in} - f \cdot V_{out})$$

$$V_{out}\,(1 + Af) = A\,V_{in}$$

$$A_v = \text{Gain} = \frac{V_{out}}{V_{in}} = \frac{A}{1+Af}$$

$$A_v \underset{A\to\infty}{=} \frac{1}{f} \rightarrow \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

Golden Rules of Op-Amps

For our design we want $A = 3$

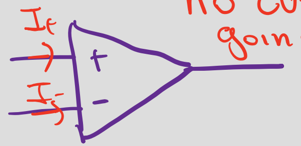
$$V_d = \frac{V_{out}}{A} \quad \text{if} \quad A \to \infty$$

$$V_d = \frac{1}{A} \cdot \frac{A}{1+Af} V_{in} = \frac{V_{in}}{1+Af} = 0$$

In NFB: $U^+ = U^-$ and $A \to \infty$

Rules: (Golden Rules)

(1) $I^+ = I^- = 0$ (always true)

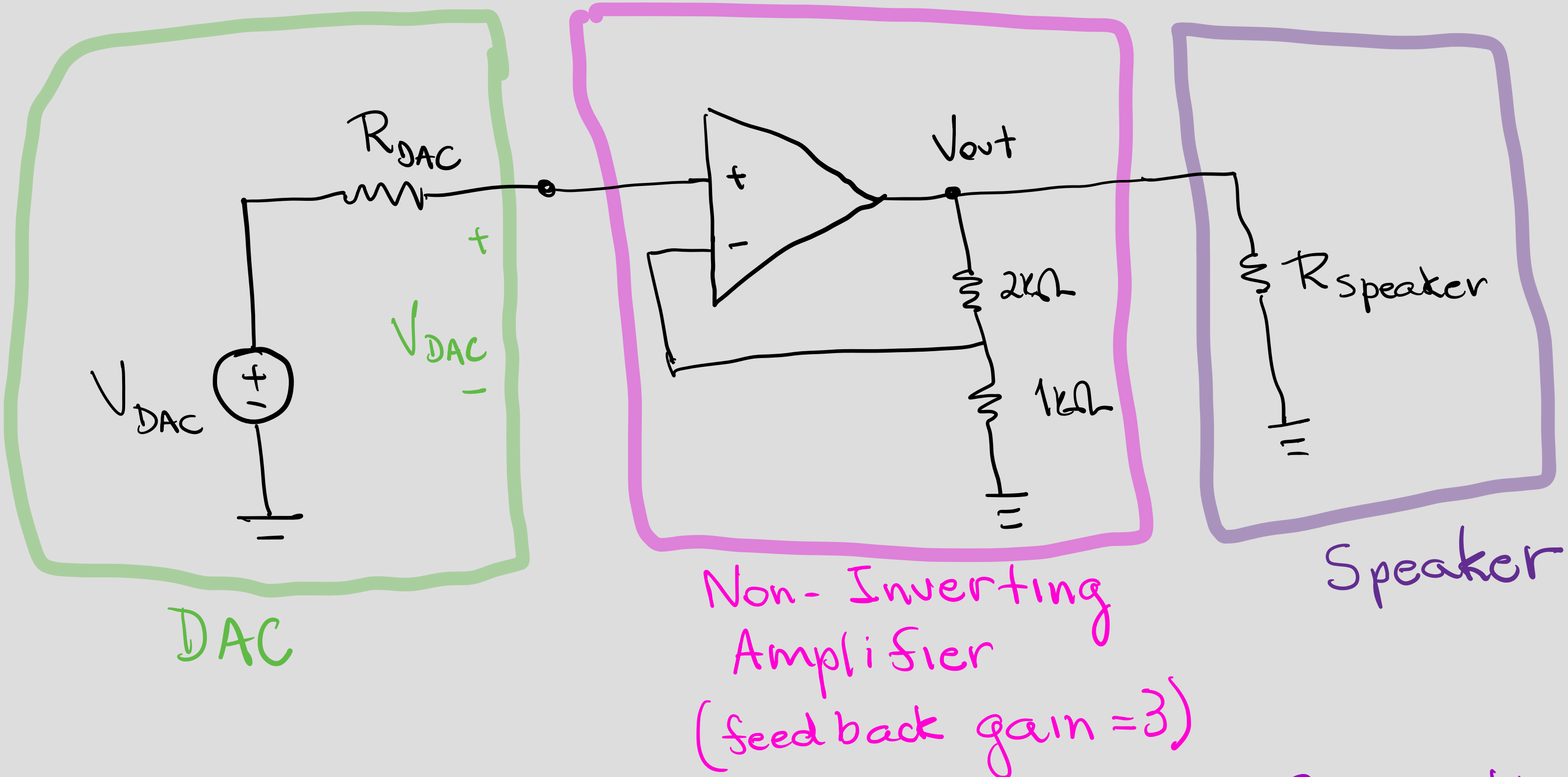


no current going in

$I^+ = I^- = 0$

(2) $U^+ = U^-$ (only in NFB & $A \to \infty$)

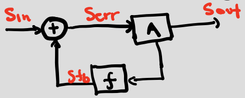
Let's go back to playing music



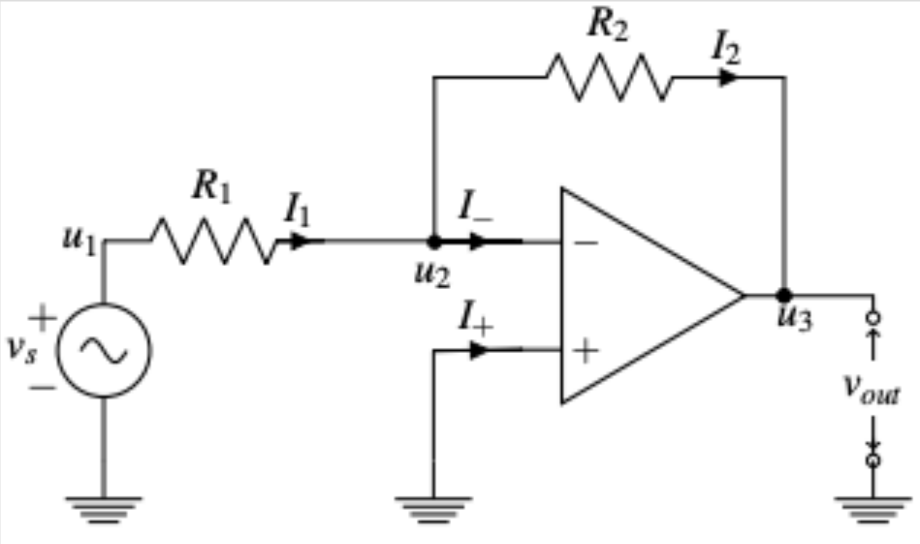
Party time! Yay!

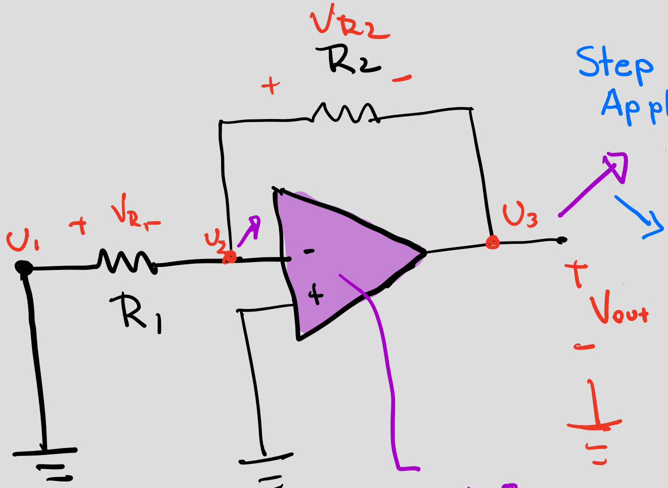
Checking for Negative Feedback (Determing the polarity of NFB)

Step 1 – Zero out all independent sources : replacing voltage sources with wires and current sources with open circuits as in superposition



Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.
- if the error signal decreases, the output must also decrease. The circuit is in negative feedback
- if the error signal increases, the output must also increase. The circuit is in positive feedback



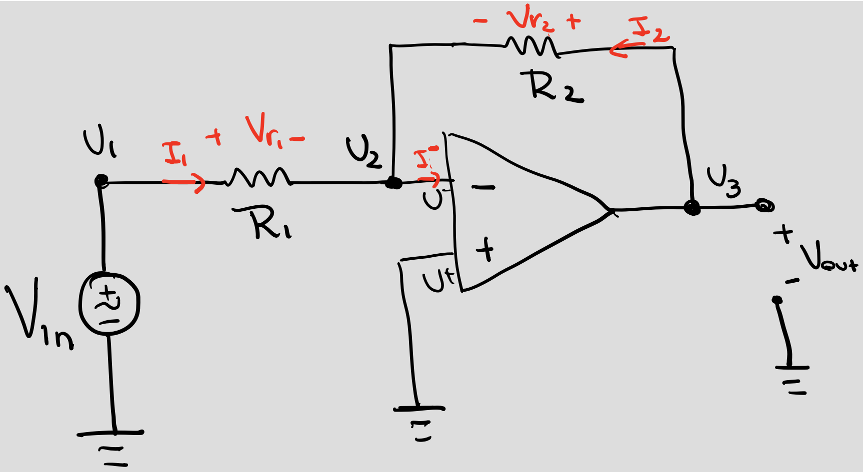


Step 2:
Apply disturbance
U_3 increases
U_2 increases (V^-)
U_3 decreases
(NFB) ✓

$A(U^+ - U^-)$

(when U^- increase $A(U^+ - U^-)$ decreases)

Now lets solve it...



NFB ⇒ GR#2 applies
$U^+ = U^-$

① $U_1 = V_{in}$
$U_3 = V_{out}$
$U_2 = 0$ (circuit in NFB ⇒ GR#2 applies $U^+ = U^-$
↳ $U_2 = U^-$ we know $U^+ = 0$ ⇒ $U^- = 0$
$U^- = U_2$ ⇒ $U_2 = 0$)

② Element Definitions:
$V_{R_1} = I_1 R_1$
$V_{R_2} = I_2 R_2$

Voltage Def.
$V_{R1} = U_1 - \cancel{U_2}^{\nearrow 0} = U_1 = V_{in}$
$V_{R2} = U_3 - \cancel{U_2}^{\nearrow 0} = U_3 = V_{out}$

③ (KCL)
$I_1 + I_2 = \cancel{I^-}^{\nearrow 0}$ (GR#1)

$V_{in} = U_1 = I_1 R_1$
$V_{out} = U_3 = I_2 R_2$

$I_1 + I_2 = 0$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

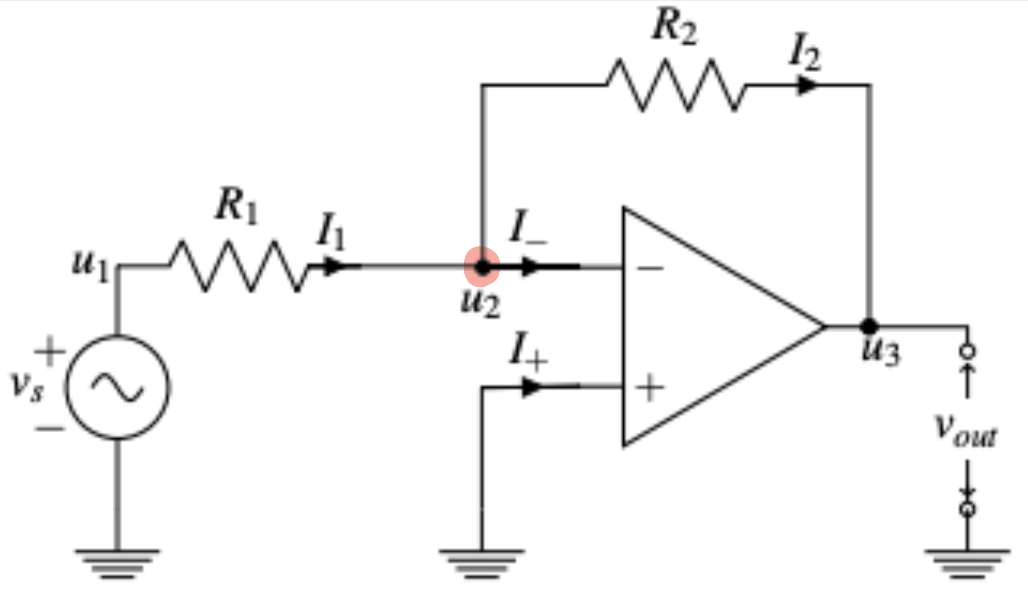
$$V_{out} = R_2 \cdot \left(-\frac{V_{in}}{R_1}\right)$$

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

Inverting Amplifier

$$\boxed{A_v = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

A faster way…



$\boxed{GR2}$: $U^+ = U^-$

$U_2 = U^-$
$U^+ = 0$ $\Rightarrow$ $\boxed{U_2 = 0}$

GR1 + KCL $\quad (I_1 = I_2 + I^-)$

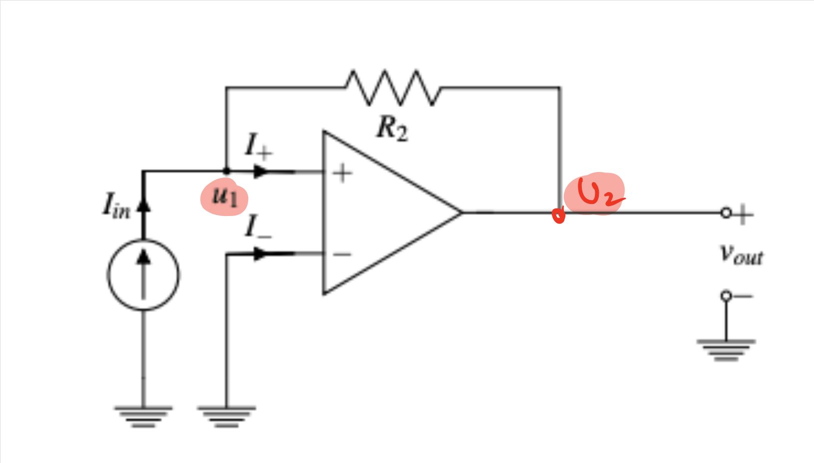
$$\frac{\cancel{U_2}^{\to 0} - U_1}{R_1} = \frac{U_3 - \cancel{U_2}^{\to 0}}{R_2} + \cancel{I^-}^{\to 0}$$

$$-\frac{\cancel{U_1}^{V_{in}}}{R_1} = \frac{\cancel{U_3}^{V_{out}}}{R_2}$$

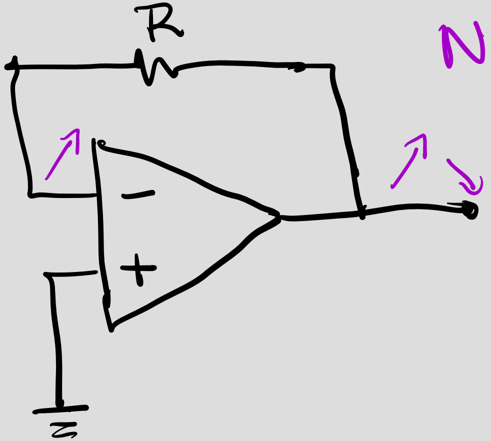
$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

Example circuit 2 (trans-resistance amplifier)

$I^+ = 0 \Rightarrow U_1 = U_2$

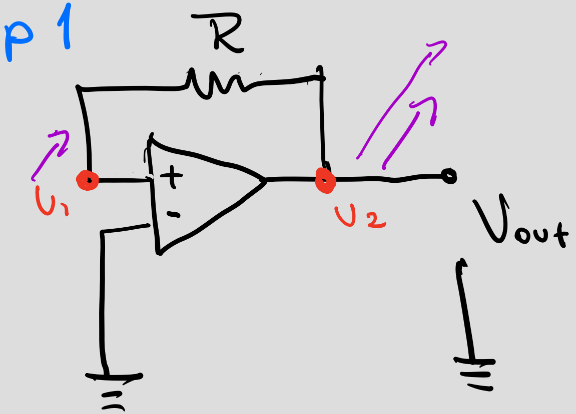


Invert polarity $\Rightarrow$



NFB

Step 1

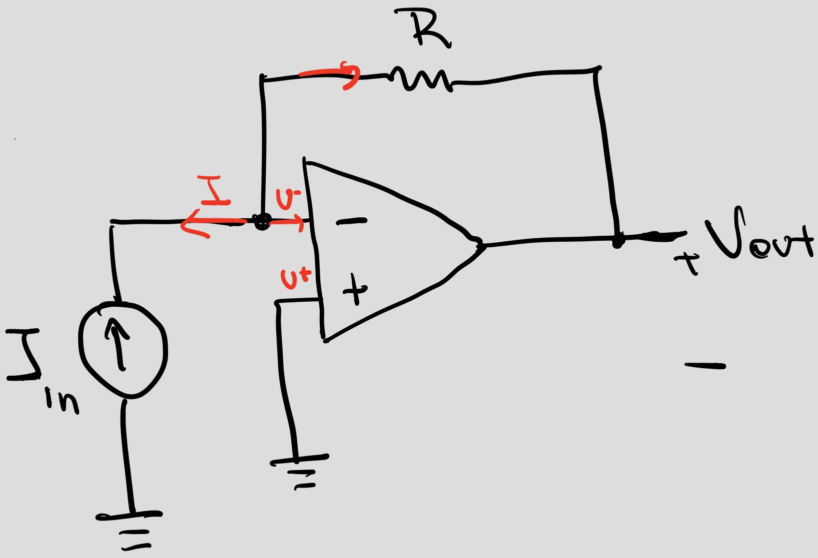


Step 2: Check for NFB

Increase output → + moves up

output increases by a lot

X Not in NFB



NFB : $U^+ = U^-$

$U^+ = 0 \rightarrow U^- = 0$

GR # 2

$$\frac{\overbrace{\cancel{U^-}}^{0 \text{ (GR2)}} - V_{out}}{R} + (-I_{in}) + \overbrace{\cancel{I^-}}^{0 \text{ (GR1)}} = 0$$

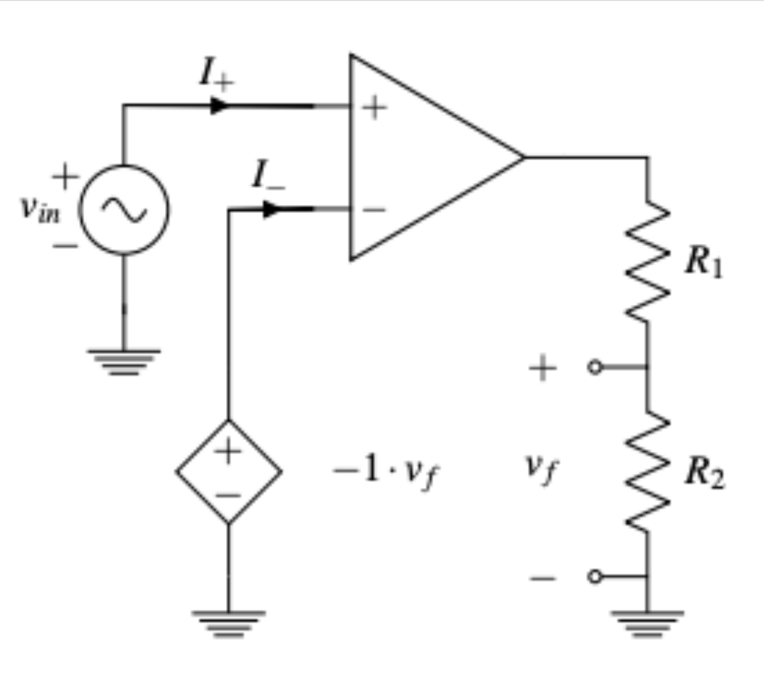
$$-\frac{V_{out}}{R} = I_{in}$$

$$V_{out} = -I_{in} R$$

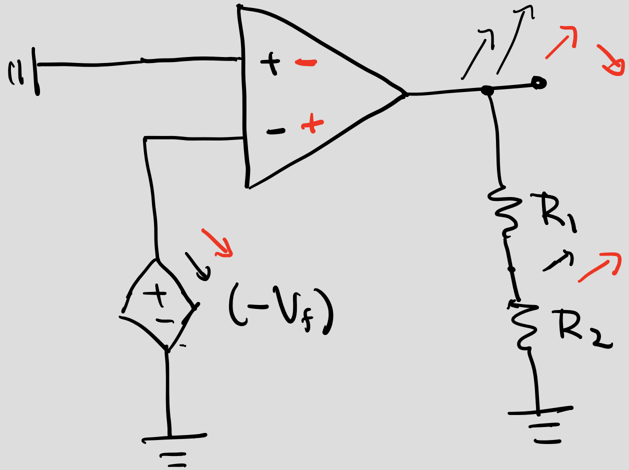
$$\boxed{\frac{V_{out}}{I_{in}} = -R}$$

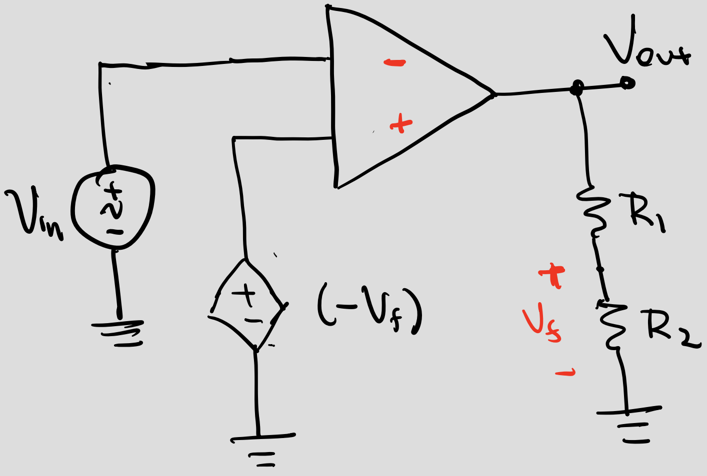
The input is current ; output is Voltage : we use this model in the lab for photo sensors !

Example circuit 3 -



Check NFB:





Voltage Divider

$$V_f = \frac{R_2}{R_1+R_2} \cdot V_{out}$$

NFB (GR#2) $V^- = V^+$

$$\underbrace{V_{in}}_{V^-} = -\underbrace{V_f}_{V^+}$$

$$V_{in} = -\frac{R_2}{R_1+R_2} V_{out} \Rightarrow \frac{V_{in}}{V_{out}} = -\frac{R_2}{R_1+R_2}$$

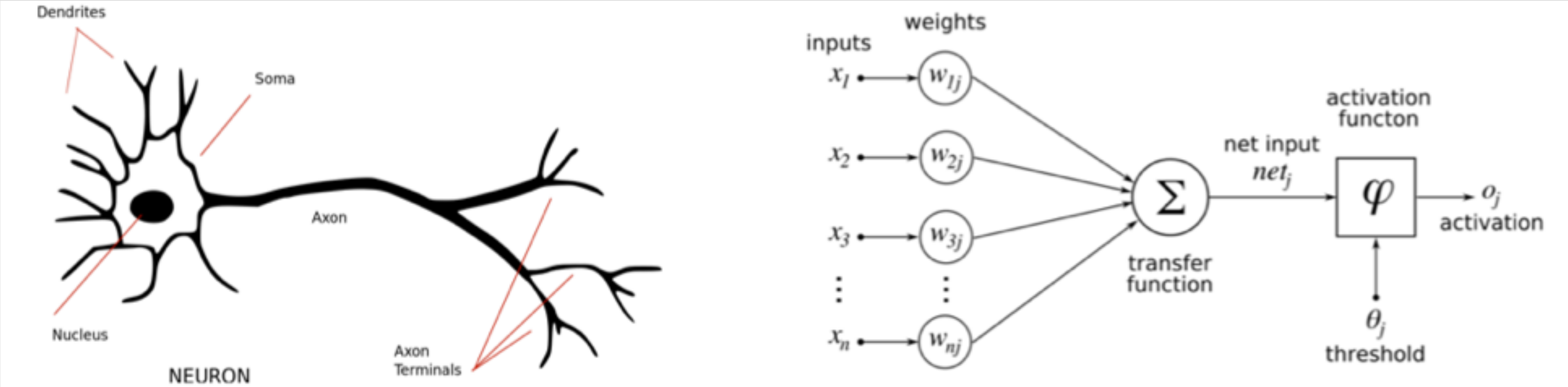
$$\boxed{A_v = \frac{V_{out}}{V_{in}}} = -\frac{R_1+R_2}{R_2} = \boxed{-\left(1+\frac{R_1}{R_2}\right)}$$

Artificial Neuron

(Energy Efficient Neural Networks) — Yes we can!

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = a_1 v_1 + a_2 \cdot v_2$$



A biological Neuron

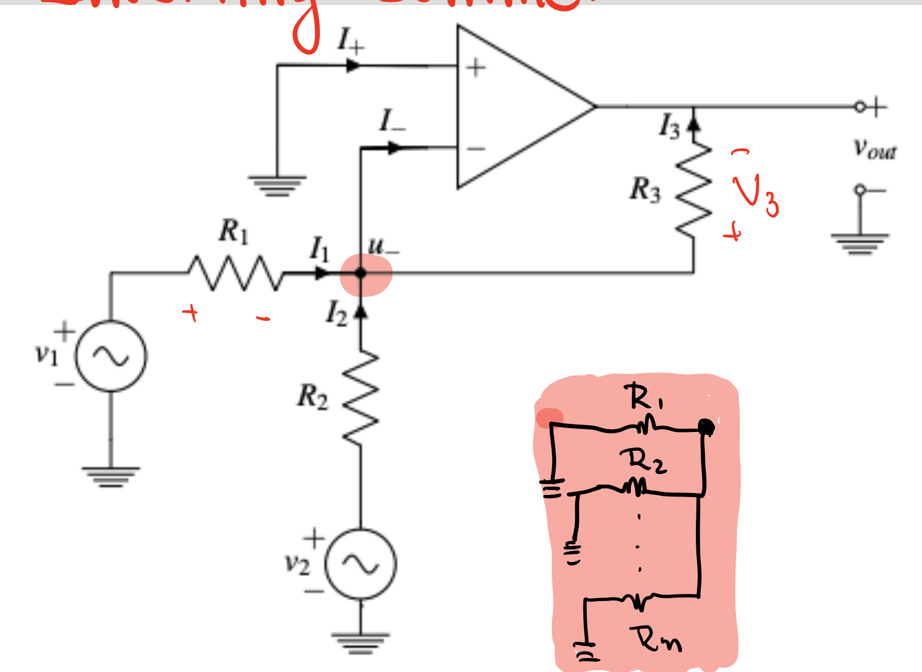
An Artificial Neuron

Artificial Neuron

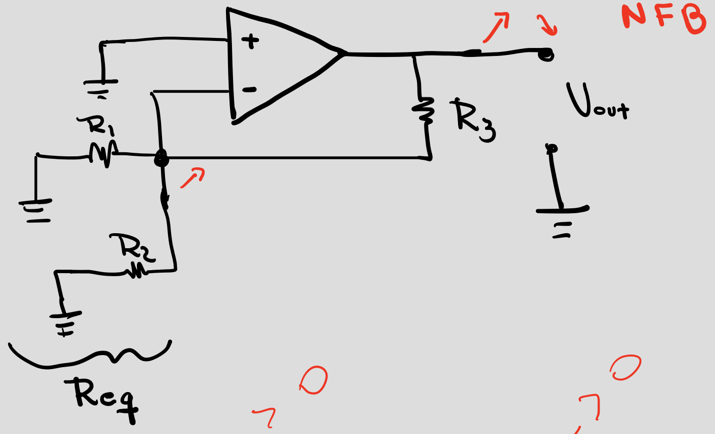
- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

Inverting summer

$V_3 = V_{out} - V^-$



Check for NFB:



$U^+ = U^-$: GR2

$U^+ = 0 \Rightarrow U^- = 0$

$$\text{KCL}: \frac{\cancel{V^-}^{\,0} - V_1}{R_1} + \frac{\cancel{V^-}^{\,0} - V_2}{R_2} = \cancel{I^-}^{\,0} + \frac{V_{out} - \cancel{V^-}^{\,0}}{R_3}$$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3}$$

$$\boxed{V_{out}} = \underbrace{-\frac{R_3}{R_1}}_{} \cdot V_1 + \left(\underbrace{-\frac{R_3}{R_2}}_{} V_2\right) + \cdots + \left(\underbrace{-\frac{R_3}{R_N}}_{} V_N\right)$$

only negative coef.

weights

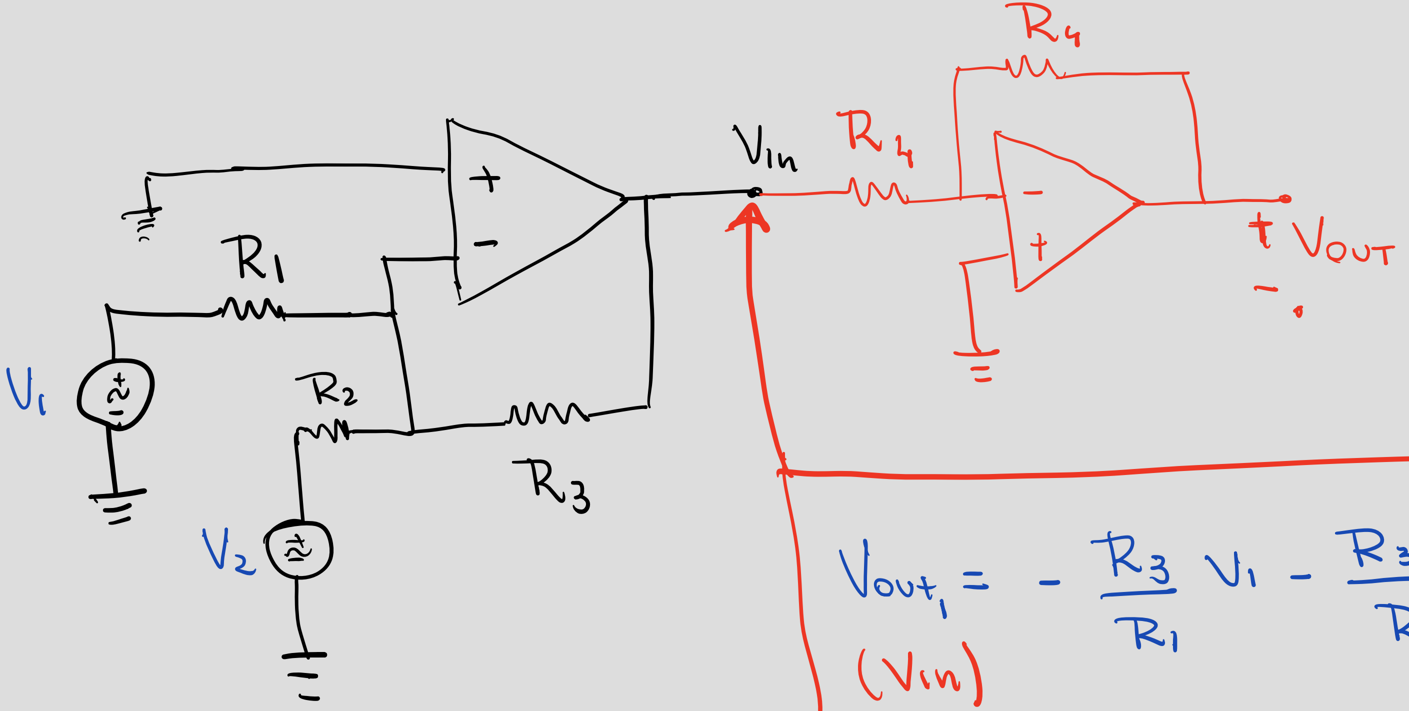
a_{11} v_1

a_{12} v_2

a_{1N} v_N

All weights are negative: How can we make a_1 and a_2 positive?

Add another inverting amplifier circuit.



$$V_{out_1} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} \cdot V_2$$
(V_{in})

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

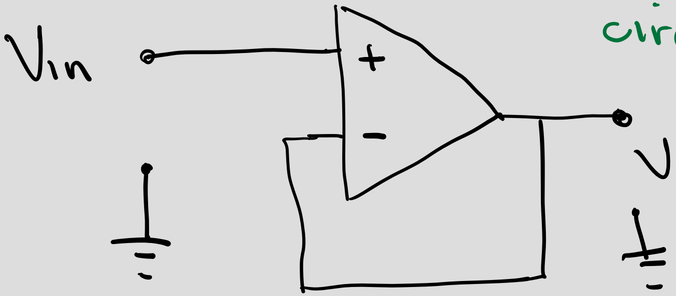
↳ result from Inverting amplifier

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

$V_{out} = -V_{in}$ (when R_1 and R_2 are the same)

Unity Gain Buffer

↳ Allows us to isolate circuits



$V^+ = V_{in}$

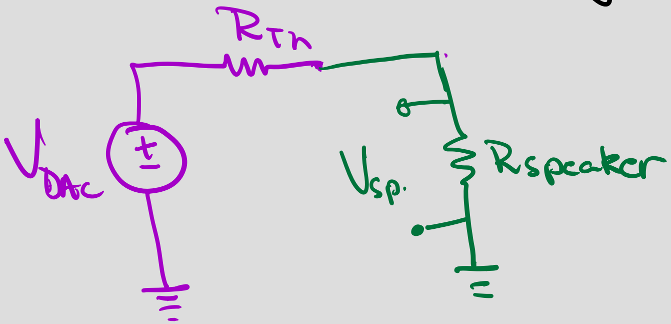
$V^- = V_{out}$

GR2

$V^+ = V^-$

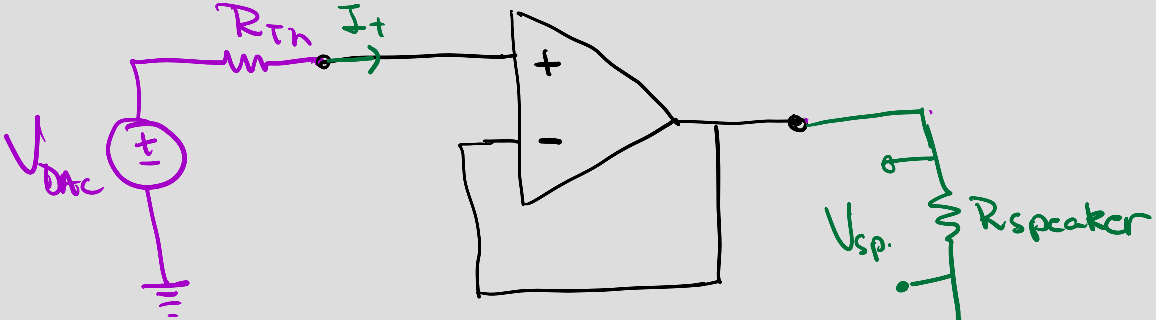
$V_{in} = V_{out}$

Speaker Design



$V_{speaker} = \frac{V_{DAC}}{126}$

loading



$I^+ = 0 \Rightarrow V^+ = V_{DAC}$

$V_{out} = V_{speaker} = V^- \Rightarrow V^+ = V^-$

$V_{DAC} = V_{speaker}$