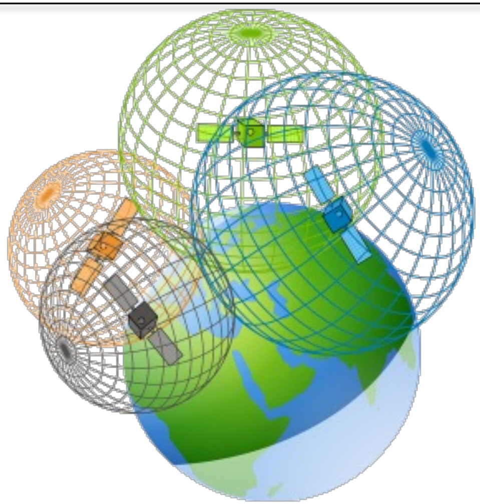



# Welcome to EECS 16A!

## Designing Information Devices and Systems I

**Ana Claudia Arias and Miki Lustig**

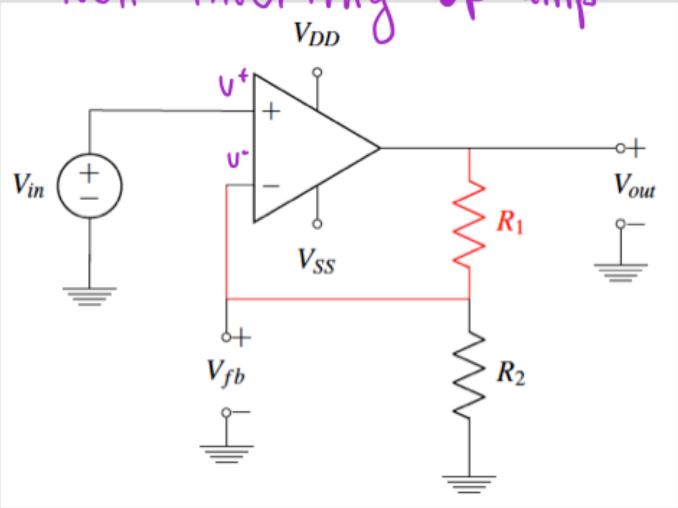
**Fall 2022**

Module 2
Lecture 11
Op-amp circuit analysis (Note 19)

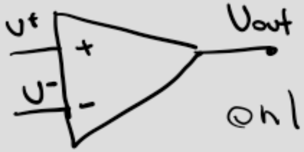
# Last Lecture…

## Op-Amp in negative feedback

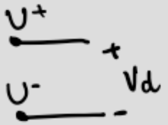
Non-inverting op-amp

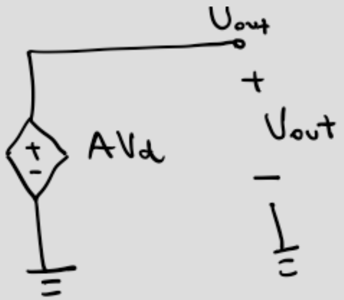



Model :




only for $V_{SS} < V_{out} < V_{DD}$




Simpler model as the second source is not "needed".

(1) $V_d = V^+ - V^- = V_{in} - V_{fb}$

(2) $V_{out} = A V_d$

(3) $V_{fb} = \underbrace{\frac{R_2}{R_1+R_2}}_{f} \cdot V_{out}$

"BUFFER circuit"

$V_{out} = A\,(V_{in} - f \cdot V_{out})$

$V_{out}\,(1 + Af) = A V_{in}$

$A_v = \text{Gain} = \frac{V_{out}}{V_{in}} = \frac{A}{1+Af}$

$A_{v\,A\to\infty} = \frac{1}{f} \to \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}$

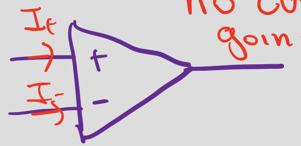
# Golden Rules of Op-Amps

For our design we want $A = 3$

$$V_d = \frac{V_{out}}{A} \quad \text{if } A \to \infty$$

$$V_d = \frac{1}{A} \cdot \frac{A}{1+Af} V_{in} = \frac{V_{in}}{1+Af} = 0$$

In NFB: $V^+ = V^-$ and $A \to \infty$




no current going in

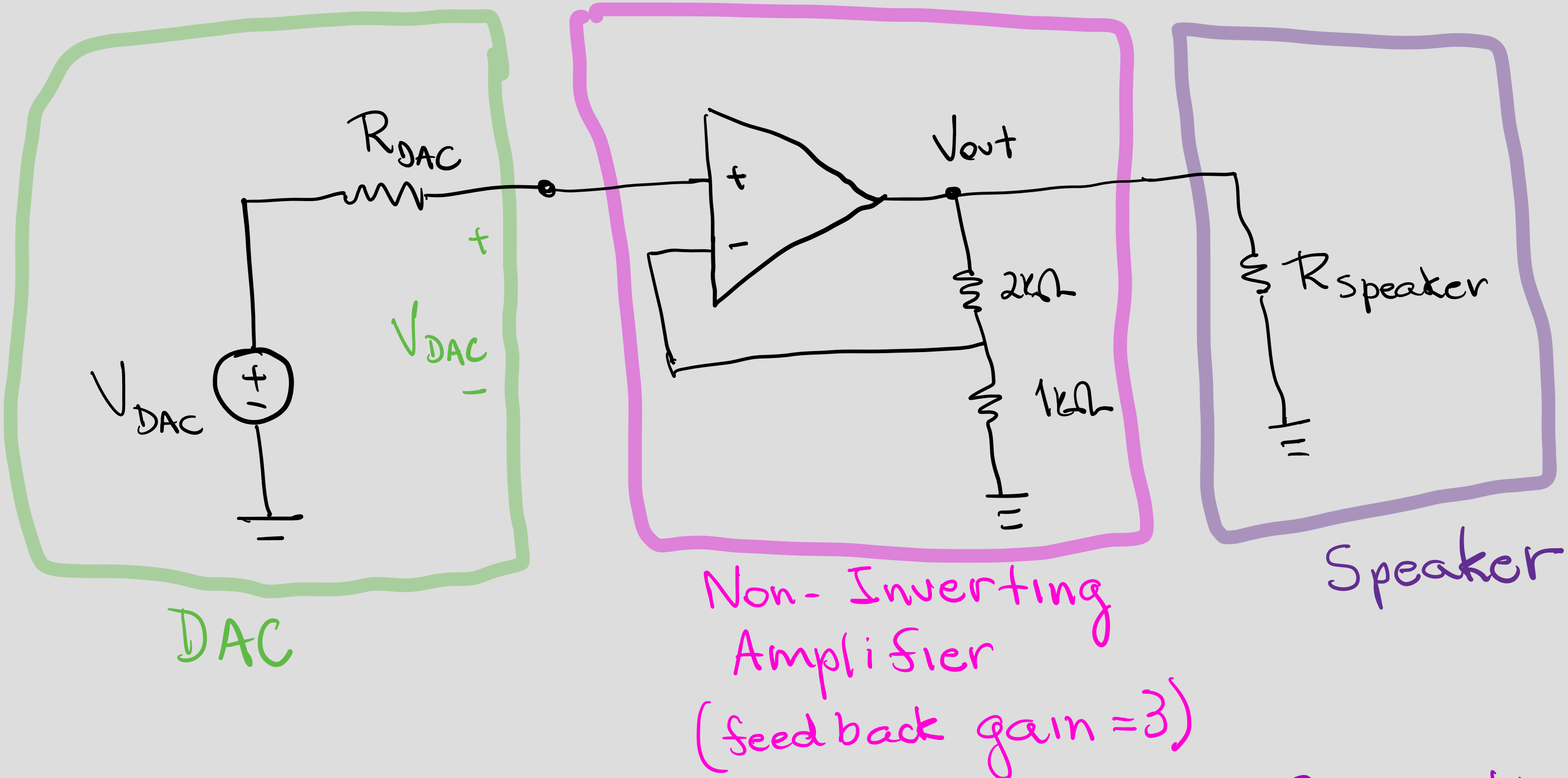
$I^+ = I^- = 0$

Rules: (Golden Rules)

(1) $I^+ = I^- = 0$ (always true)

(2) $V^+ = V^-$ (only in NFB & $A \to \infty$)

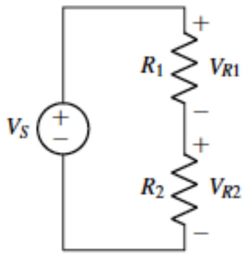
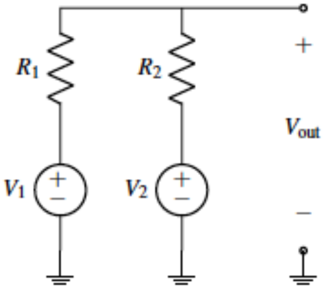
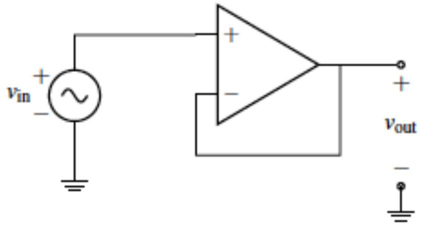
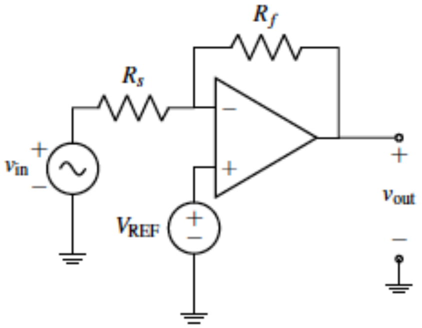
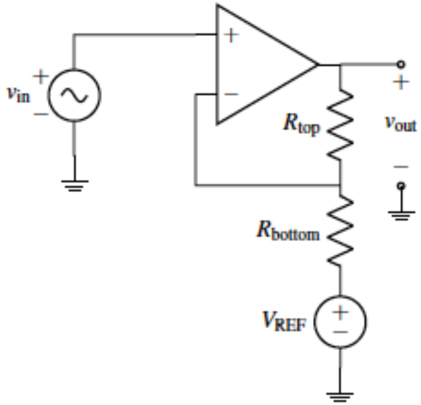
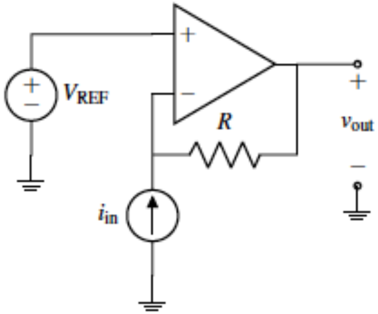
# Let's go back to playing music




Party time!

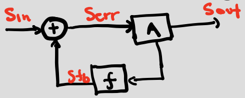
Yay!

# Today

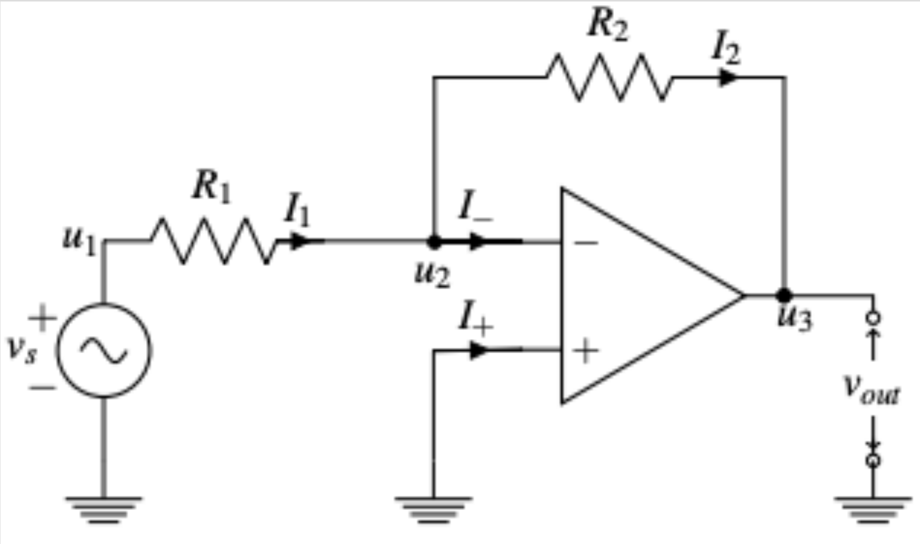
| Voltage Divider | Voltage Summer | Unity Gain Buffer |
|---|---|---|
|  |  |  |
| $V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$ | $V_{\text{out}} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$ | $\frac{v_{\text{out}}}{v_{\text{in}}} = 1$ |
| **Inverting Amplifier** | **Non-inverting Amplifier** | **Transresistance Amplifier** |
|  |  |  |
| $v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right) + V_{\text{REF}}\left(\frac{R_f}{R_s}+1\right)$ | $v_{\text{out}} = v_{\text{in}}\left(1+\frac{R_{\text{top}}}{R_{\text{bottom}}}\right) - V_{\text{REF}}\left(\frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$ | $v_{\text{out}} = i_{\text{in}}(-R) + V_{\text{REF}}$ |

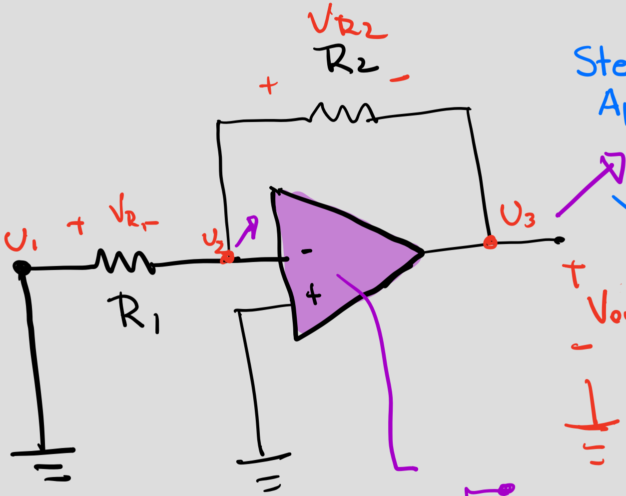
# Checking for Negative Feedback (Determing the polarity of NFB)

Step 1 – Zero out all independent sources : replacing voltage sources with wires and current sources with open circuits as in superposition




Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.
- if the error signal decreases, the output must also decrease. The circuit is in negative feedback
- if the error signal increases, the output must also increase. The circuit is in positive feedback



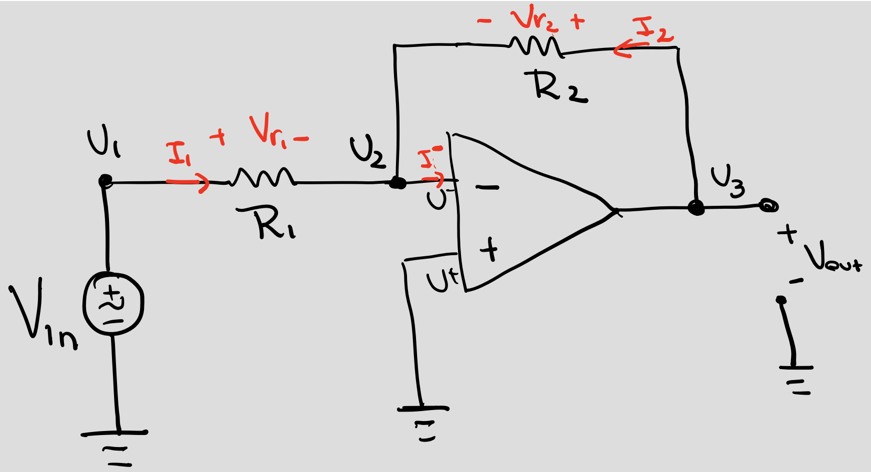




Step 2 :
Apply disturbance
$U_3$ increases
$U_2$ increases ($V^-$)
$U_3$ decreases
(NFB) ✓

$A\,(V^+ - V^-)$

(when $V^-$ increase
$A(V^+ - V^-)$ decreases)

Now lets solve it...




NFB $\Rightarrow$ GR#2 applies

$U^+ = U^-$

① $U_1 = V_{in}$

$U_3 = V_{out}$

$U_2 = 0$ (circuit in NFB $\Rightarrow$ GR#2 applies $U^+ = U^-$

$\hookrightarrow U_2 = U^-$ we know $U^+ = 0 \Rightarrow U^- = 0$

$U^- = U_2 \Rightarrow U_2 = 0$ )

② Element Definitions:

$V_{R_1} = I_1 R_1$

$V_{R_2} = I_2 R_2$

Voltage Def.

$V_{R_1} = U_1 - \cancel{U_2}^{\to 0} = U_1 = V_{in}$

$V_{R_2} = U_3 - \cancel{U_2}^{\to 0} = U_3 = V_{out}$

③ (KCL)

$I_1 + I_2 = \cancel{I^-}^{\to 0}$ (GR#1)

$V_{in} = U_1 = I_1 R_1$

$V_{out} = U_3 = I_2 R_2$

$I_1 + I_2 = 0$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

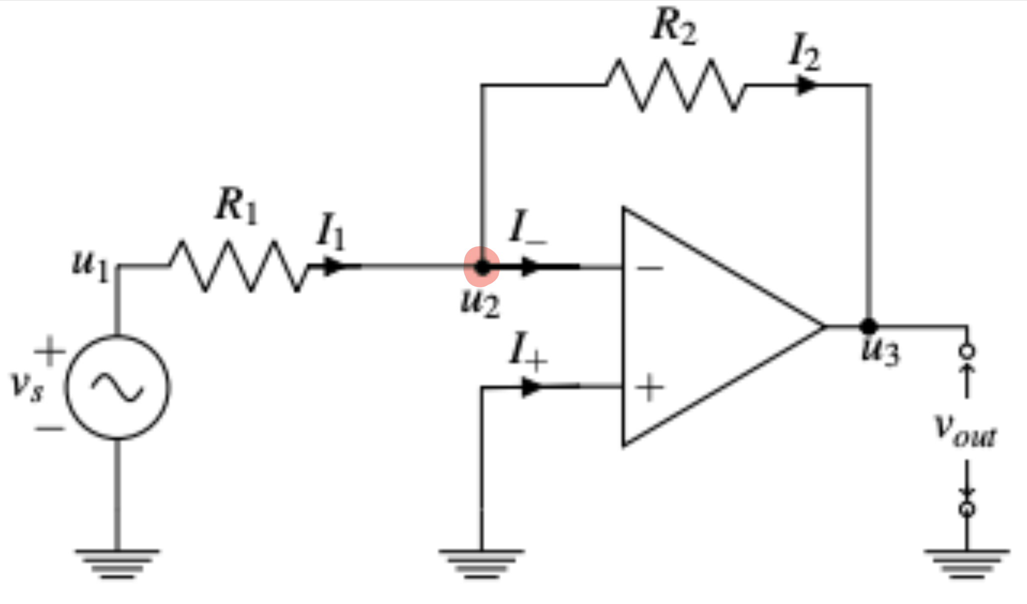
$$V_{out} = R_2 \cdot \left(-\frac{V_{in}}{R_1}\right)$$

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

Inverting Amplifier

$$\boxed{A_v = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

# A faster way…




$\boxed{GR2}$: $U^+ = U^-$

$U_2 = U^-$

$U^+ = 0 \Rightarrow \boxed{U_2 = 0}$

GR1 + KCL $\quad (I_1 = I_2 + I^-)$

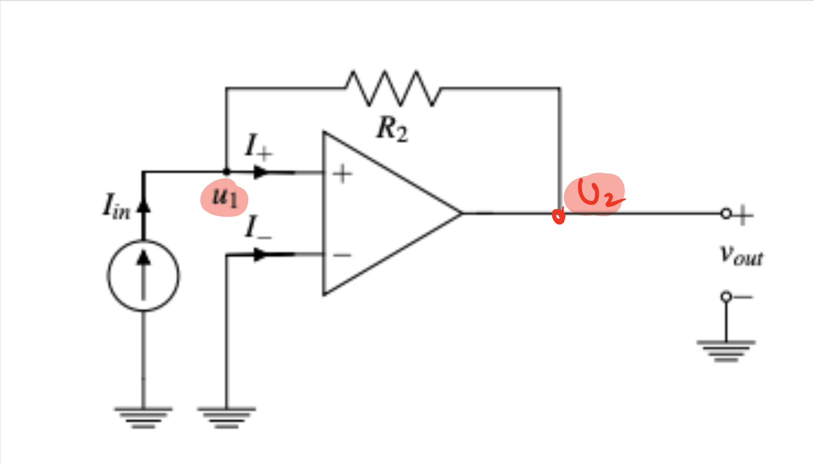
$$\frac{\cancel{U_2}^{\to 0} - U_1}{R_1} = \frac{U_3 - \cancel{U_2}^{\to 0}}{R_2} + \cancel{I^-}^{\to 0}$$

$$-\frac{\cancel{U_1}^{V_{in}}}{R_1} = \frac{\cancel{U_3}^{V_{out}}}{R_2}$$

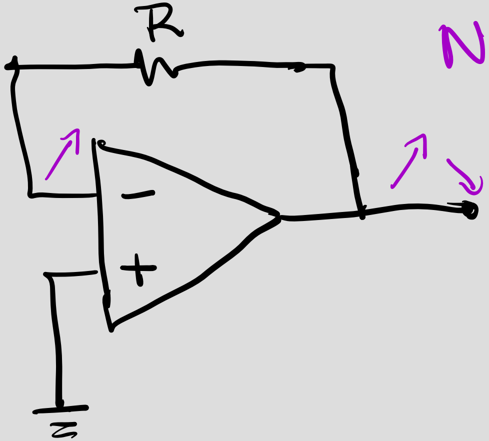
$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

# Example circuit 2 (trans-resistance amplifier)

$I^+ = 0 \Rightarrow U_1 = U_2$

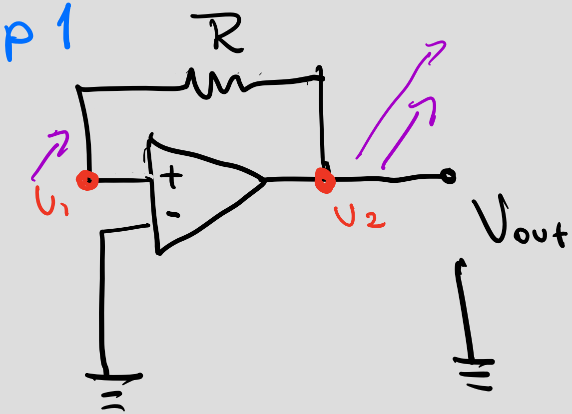



Invert polarity
$\Rightarrow$




NFB

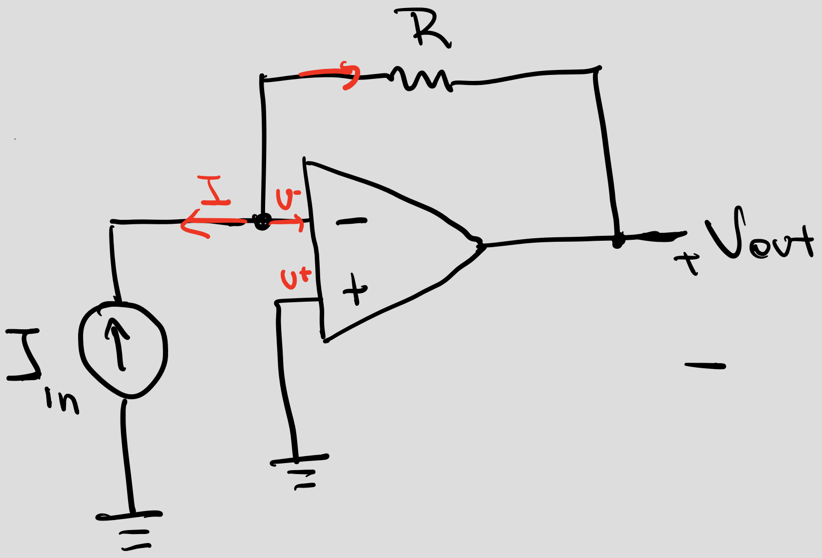
Step 1




Step 2: Check for NFB

Increase output →
+ moves up
output increases
by a lot

X Not in NFB




NFB : $U^+ = U^-$

$U^+ = 0 \rightarrow U^- = 0$

GR # 2

$$\frac{\cancel{U^-}^{\,0} - V_{out}}{R} + (-I_{in}) + \cancel{I^-}^{\,0} = 0$$

GR2 GR1

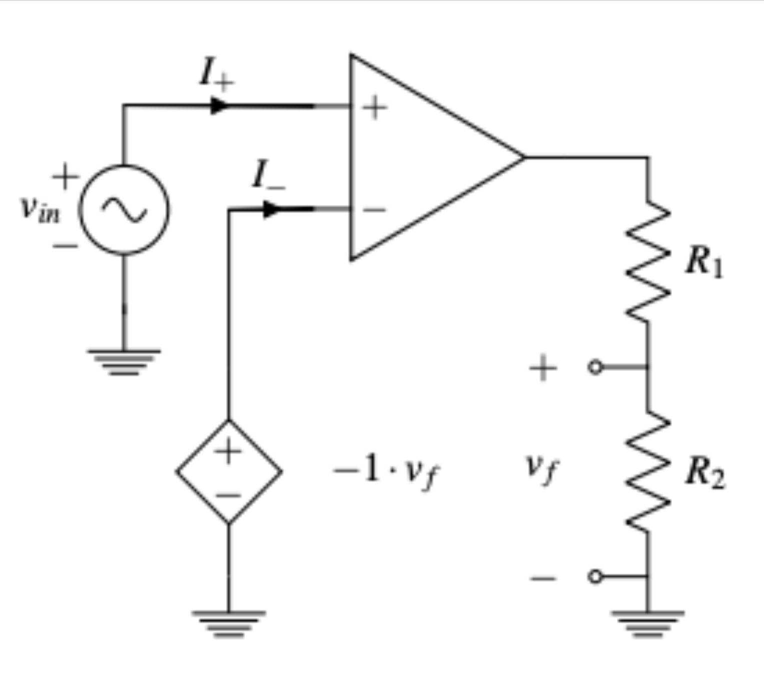
$$-\frac{V_{out}}{R} = I_{in}$$

$$V_{out} = -I_{in} R$$

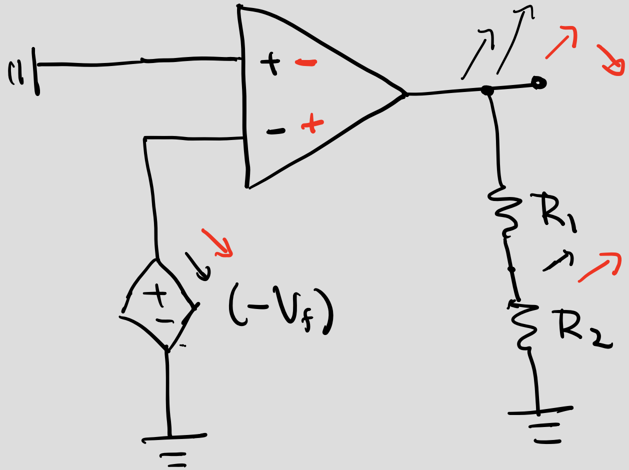
$$\boxed{\frac{V_{out}}{I_{in}} = -R}$$

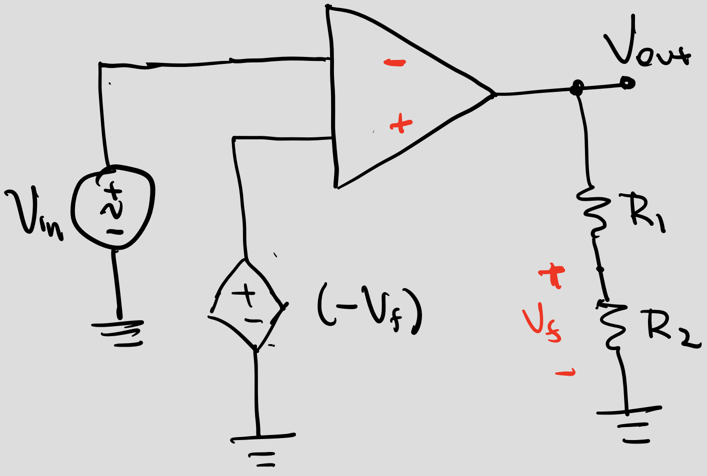
The input is current ; output is Voltage : we use this model in the lab for photo sensors !

# Example circuit 3 -




Check NFB:






## Voltage Divider

$$V_f = \frac{R_2}{R_1+R_2} \cdot V_{out}$$

NFB (GR#2) $V^- = V^+$

$$\underbrace{V_{in}}_{V^-} = \underbrace{-V_f}_{V^+}$$

$$V_{in} = -\frac{R_2}{R_1+R_2} V_{out} \Rightarrow \frac{V_{in}}{V_{out}} = -\frac{R_2}{R_1+R_2}$$

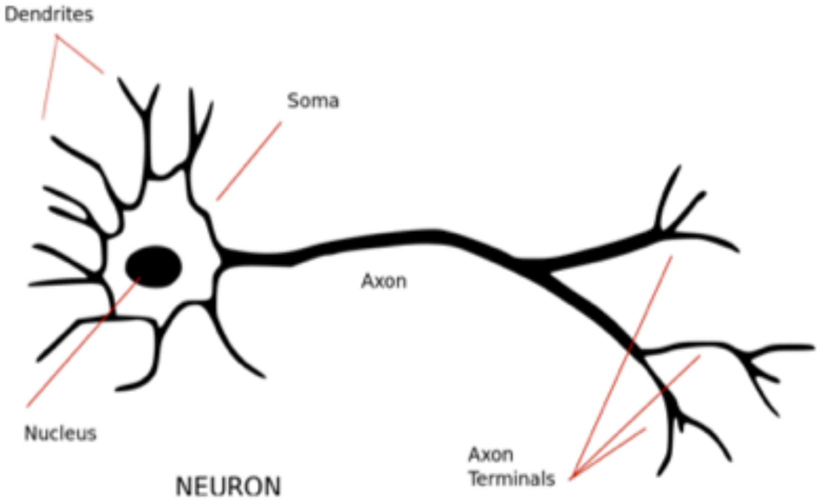
$$\boxed{A_V = \frac{V_{out}}{V_{in}}} = -\frac{R_1+R_2}{R_2} = \boxed{-\left(1+\frac{R_1}{R_2}\right)}$$

# Artificial Neuron

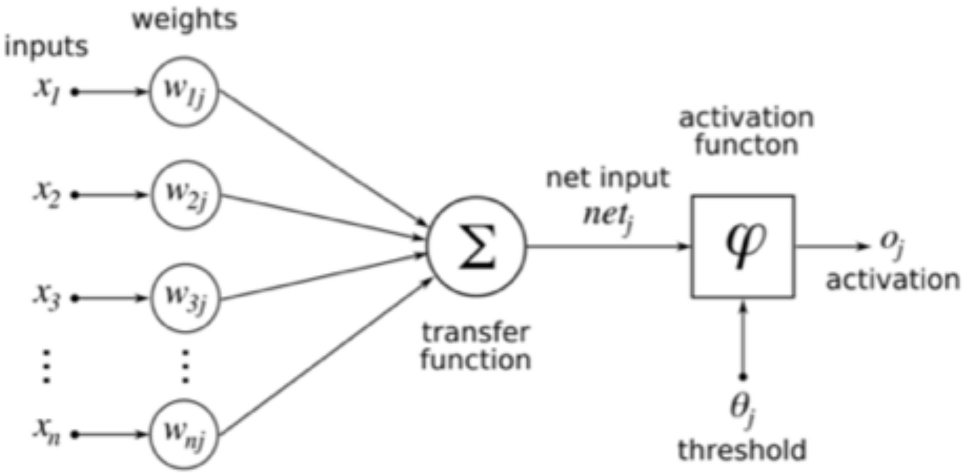
(Energy Efficient Neural Networks) — Yes we can!

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ V_2 \end{bmatrix} = a_1 V_1 + a_2 \cdot V_2$$




A biological Neuron




An Artificial Neuron

# Artificial Neuron

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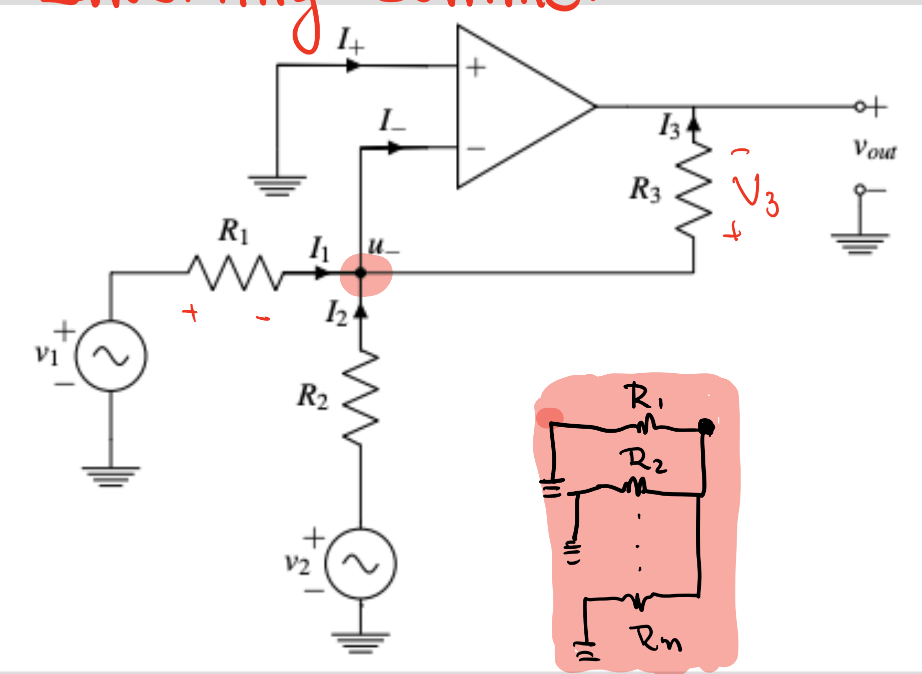
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$V_3 = V_{out} - V^-$

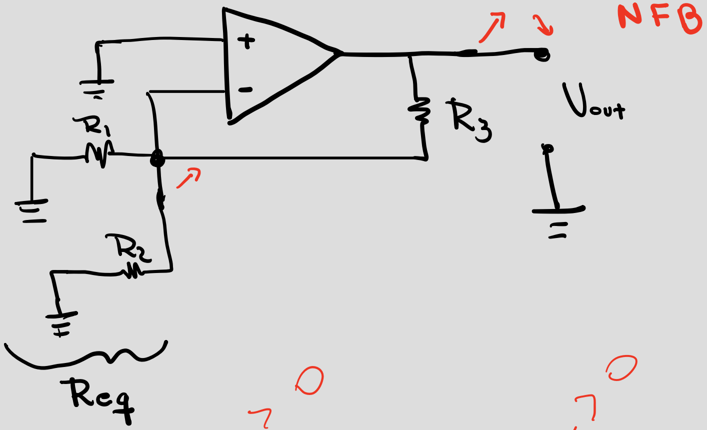
Inverting summer




Check for NFB:

$V^+ = V^-$ : GR2

$V^+ = 0 \Rightarrow V^- = 0$




KCL : $\frac{\cancel{V^-}^{0} - V_1}{R_1} + \frac{\cancel{V^-}^{0} - V_2}{R_2} = \cancel{I^-}^{0} + \frac{V_{out} - \cancel{V^-}^{0}}{R_3}$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3}$$

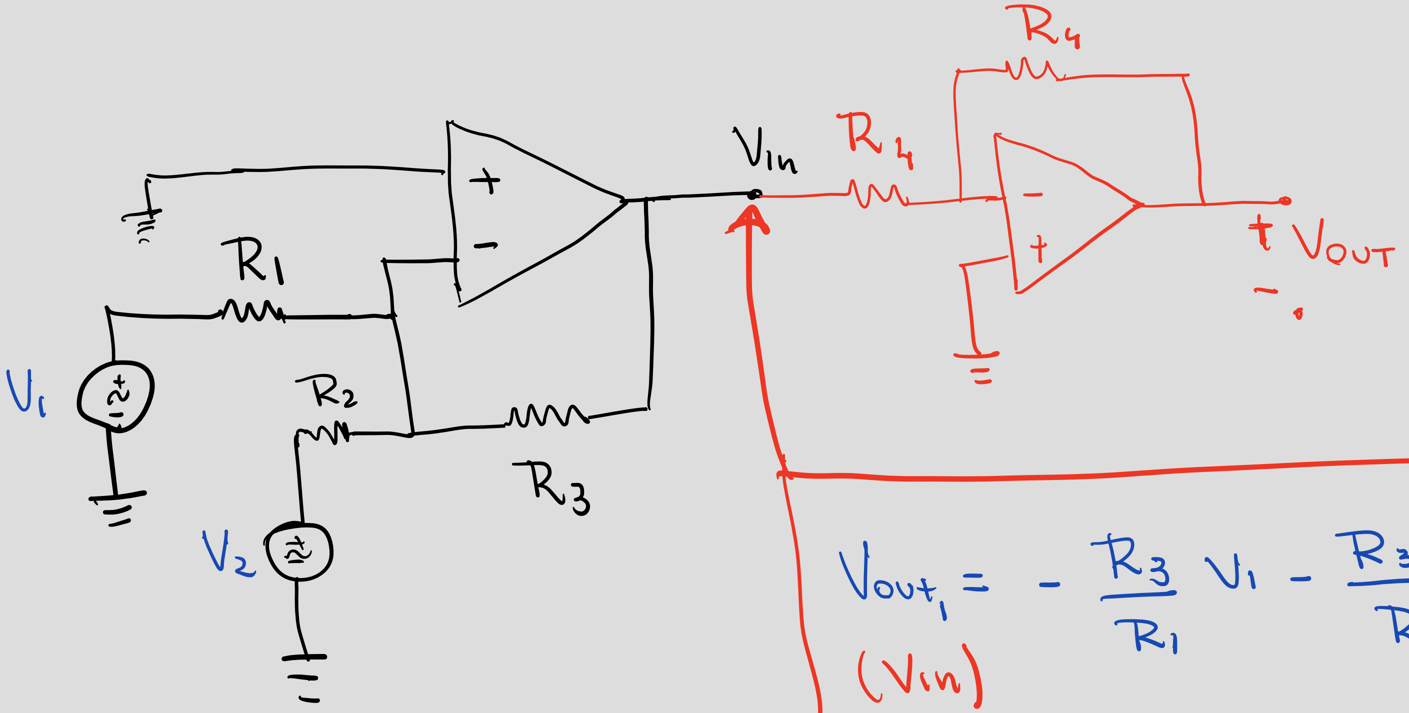
$$\boxed{V_{out}} = \underbrace{-\frac{R_3}{R_1}}_{\text{weights}} \cdot V_1 + \left(\underbrace{-\frac{R_3}{R_2}}\, V_2\right) + \cdots + \left(\underbrace{-\frac{R_3}{R_N}}\, V_N\right)$$

only negative coef.

$a_{11}$ $v_1$ $a_{12}$ $v_2$ $a_{1N}$ $v_N$

All weights are negative : How can we make $a_1$ and $a_2$ positive?

Add another inverting amplifier circuit.




$$V_{out_1} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2$$
$(V_{in})$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

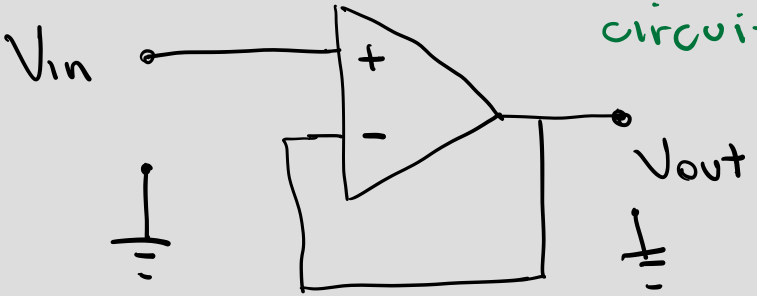
↳ result from Inverting amplifier

$$V_{out} = -\frac{R_2}{R_1}V_{in}$$

$V_{out} = -V_{in}$ (when $R_1$ and $R_2$ are the same)

# Unity Gain Buffer

↳ Allows us to isolate circuits




$V^+ = V_{in}$

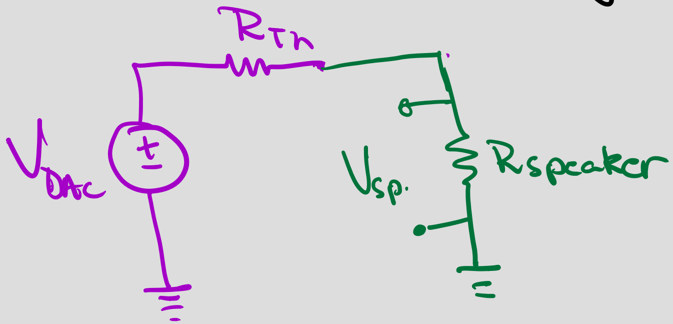
$V^- = V_{out}$

GR2

$V^+ = V^-$

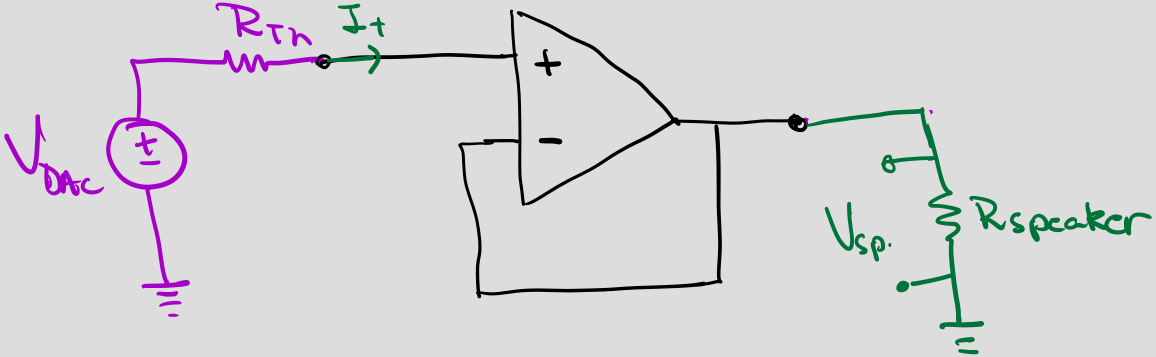
$V_{in} = V_{out}$

## Speaker Design




$V_{speaker} = \frac{V_{DAC}}{126}$

loading




$I^+ = 0 \Rightarrow V^+ = V_{DAC}$

$V_{out} = V_{speaker} = V^- \Rightarrow V^+ = V^-$

$V_{DAC} = V_{speaker}$