Announcements

• Last time:
  - Vector spaces
  - Subspaces
  - Null spaces

• Today:
  - Computing the determinant
  - Eigen Values and Eigen Vectors of a Matrix
    • Example via page-rank
Jargon from Last time

• **Rank** a matrix $A$ is the number of linearly independent columns.

• **Nullspace** of a matrix $A$ is the set of solutions to $A\vec{x} = 0$.

• A **vector space** is a set of vectors connected by two operators (+,x).

• A vector **subspace** is a subset of vectors that have “nice properties”.

• A **basis** for a vector space is a minimum set of vectors needed to represent all vectors in the space.

• **Dimension** of a vector space is the number of basis vectors.

• **Column space** is the span (range) of the columns of a matrix.

• **Row space** is the span of the rows of a matrix.
Null Space

- Definition: The null-space of $A \in \mathbb{R}^{N \times M}$ is the set of all vectors $\vec{x} \in \mathbb{R}^{M}$ such that: $A \vec{x} = 0$
Examples

\[
\begin{bmatrix}
1 & 0 \\
0 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Linearly independent!

\[\vec{x} = \begin{bmatrix}
0 \\
0
\end{bmatrix}\]

\(\vec{0}\) is always in the null space — trivial Null space
Examples

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Linearly dependent!

Gaussian elimination:

$$\begin{bmatrix} 1 & -2 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \Rightarrow x_1 = 2x_2$$

$$\Rightarrow \vec{x} = \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

A has a non-trivial null-space, span \( \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \)
We know that $\vec{v}_0 \in \text{Null}(A)$

$\rightarrow A\vec{v}_0 = \vec{0}$

We know 1 solution: $\vec{x}_0$

$\rightarrow A\vec{x}_0 = b$
Example

\[ A \vec{x} = \vec{b} \]

We know that \( \vec{v}_0 \in \text{Null}(A) \)

\[ \rightarrow A \vec{v}_0 = \vec{0} \]

We know 1 solution: \( \vec{x}_0 \)

\[ \rightarrow A \vec{x}_0 = \vec{b} \]

Then: \( \vec{x}_0 + \alpha \vec{v}_0 \) is also a solution

\[ \rightarrow A(\vec{x}_0 + \alpha \vec{v}_0) = A \vec{x}_0 + A(\alpha \vec{v}_0) \]

\[ = \vec{b} + \alpha A \vec{v}_0 \]

\[ = \vec{b} \]
Back to Tomography

\[
\begin{align*}
1 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 0 \cdot x_4 &= 4 \\
0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 &= 3 \\
1 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 0 \cdot x_4 &= 2 \\
0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + 1 \cdot x_4 &= 5
\end{align*}
\]

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & | & 4 \\
0 & 0 & 1 & 1 & | & 3 \\
1 & 0 & 1 & 0 & | & 2 \\
0 & 1 & 0 & 1 & | & 5
\end{bmatrix}
\]

\[A\]
Null Space of the Tomography System (4 measur.)

<table>
<thead>
<tr>
<th>Step I</th>
<th>Step II</th>
<th>Step IV</th>
<th>Step V</th>
<th>Step VI</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & -1 & 1 & 0 \\
0 & 1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
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\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 \\
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\end{bmatrix}
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\] | \[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
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0 & 0 & 1 & 1 \\
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\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\] |
Null Space of the Tomography System (4 measur.)

\[
\begin{bmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\(x_4\) is the free variable:

\[\Rightarrow \overline{x} = \alpha \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\]

Possible reconstruction

\[
\begin{bmatrix}
1 & 3 \\
1 & 2 \\
1 & -1 \\
-1 & 1 \\
\end{bmatrix}
\]

\[+ \alpha \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\]
Rank

- $A \in \mathbb{R}^{N \times M}$, $\text{Rank} \{ A \} = \dim \{ \text{Span} \{ A \} \}$

- $\text{Rank} \{ A \} = \dim \{ \text{Span} \{ A \} \} \leq \min(M, N)$

- $\text{Rank} = L$, mean the matrix $A \in \mathbb{R}^{N \times M}$ has $L$ independent rows&columns

- $\text{Rank} \{ A \} + \dim \{ \text{Null} \{ A \} \} = M$
Equivalent Statements

• Matrix $A$ is **invertible**
• $A\vec{x} = \vec{b}$ has a unique solution
• $A$ has linearly independent columns ($A$ is **full rank**)
• $A$ has a **trivial nullspace**
• The **determinant** of $A$ is not zero
The Determinant

• For $A \in \mathbb{R}^{2 \times 2}$

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

When $\det(A) \neq 0$, $A$ is invertible

Recall:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
Interpretation of Determinant of a Matrix in $\mathbb{R}^{2 \times 2}$

- Area of a parallelogram

$$\text{det}(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ Area} \neq 0$$
Interpretation of Determinant of a Matrix in $\mathbb{R}^{2 \times 2}$

• Area of a parallelogram

\[
\begin{vmatrix}
1 & 0 \\
0 & 1 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
1 & 1 \\
1 & -1 \\
\end{vmatrix}
\]

\[
\det(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc
\]
Interpretation of Determinant of a Matrix in $\mathbb{R}^{2\times2}$

- **Area of a parallelogram**

  
  
  $\text{det}(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

  
  
  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Area} \neq 0$

  
  
  $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{Area} \neq 0$

  
  
  $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \quad \text{Area} = 0 \quad \text{det}(A) = 1 \cdot 2 - 1 \cdot 2 = 0$
Interpretation of Determinant of a Matrix in $\mathbb{R}^{2\times2}$

• Area of a parallelogram

$$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
Interpretation of Determinant of a Matrix in $\mathbb{R}^{2\times2}$

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Interpretation of Determinant of a Matrix in $\mathbb{R}^{2\times2}$

- Area of a parallelogram

$\det(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

$(c + d)(a + b)$

- $bc \times 2$
- $\frac{1}{2}ac \times 2$
- $\frac{1}{2}bd \times 2$
The area of the shape can be calculated as follows:

\[
\text{area} = (c + d)(a + b) - 2bc - ac - bd
\]

Expanding the expression:

\[
= ca + cb + da + db - 2bc - ac - bd
\]

Simplifying further:

\[
= ad - bc
\]
Determinant in $\mathbb{R}^3$

$$\det\left( \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \right) = \begin{vmatrix} a_x & e & f \\ d_x & g & i \\ h_x & j & k \end{vmatrix} - \begin{vmatrix} b_x & e & f \\ d_x & g & i \\ h_x & j & k \end{vmatrix} + \begin{vmatrix} d_x & e & f \\ b_x & g & i \\ h_x & j & k \end{vmatrix}$$
PageRank

• Ranks websites based on how many high-ranked pages link to them
PageRank

From

To

1/3

1/3

1/2

1/2

1/2

1/2

1/1

1/1

1/1

1/1
PageRank

<table>
<thead>
<tr>
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<th>1/3</th>
<th>1/3</th>
<th>1/3</th>
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</table>

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<thead>
<tr>
<th>From</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Graph:

- From person A to person B: 1/2
- From person B to person C: 1/3
- From person C to person D: 1/3
- From person D to person E: 1/2
- From person E to person D: 1/2
- From person D to person A: 1/3

Matrix:

\[
\begin{pmatrix}
0 & 1/3 & 1/3 & 1/3 \\
1/3 & 0 & 1/3 & 1/3 \\
1/3 & 1/3 & 0 & 1/3 \\
1/3 & 1/3 & 1/3 & 0 \\
\end{pmatrix}
\]
PageRank

\[
\begin{bmatrix}
0 & \frac{1}{2} \\
\frac{1}{3} & 0 \\
\frac{1}{3} & 0 \\
\frac{1}{3} & \frac{1}{2}
\end{bmatrix}
\]

\[
\begin{array}{c}
\frac{1}{2} \\
\frac{1}{3} \\
\frac{1}{2} \\
1
\end{array}
\]

From \quad To

- From 1 to 1: \frac{1}{3}
- From 1 to 2: \frac{1}{3}
- From 1 to 3: \frac{1}{2}
- From 2 to 1: \frac{1}{2}
- From 2 to 3: \frac{1}{2}
- From 3 to 1: \frac{1}{2}
- From 3 to 2: \frac{1}{2}
- From 3 to 3: 1
PageRank

From  

To  

<table>
<thead>
<tr>
<th></th>
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<td>0</td>
<td>1/3</td>
<td>1/2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Diagram:

- From left to right: 0 → 1/2 → 0 → 0 → 1/3 → 0 → 0 → 1/3 → 1/3 → 1/3
- From top to bottom: 1/2 → 1/3 → 1/3 → 1/3 → 1/3 → 1/2 → 1/2 → 1/2 → 1/2 → 1/2
<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>0</th>
<th>1/2</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td></td>
<td></td>
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<td>1/2</td>
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<td>1/3</td>
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<td>1/2</td>
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<tr>
<td>1/3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/2</td>
</tr>
</tbody>
</table>

PageRank

From User to User:

- User 1: 1/3 to User 2, 1/2 to User 3, 1/3 to User 4
- User 2: 1/3 to User 3, 1/2 to User 1
- User 3: 1/3 to User 2, 1/2 to User 4
- User 4: 1/2 to User 3
PageRank

\[ \vec{x}(t+1) = \begin{bmatrix} 0 & \frac{1}{12} & 0 & 0 \\ \frac{1}{12} & 0 & \frac{1}{12} & 0 \\ \frac{1}{12} & 0 & \frac{1}{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{x}(t) \]

\[ \vec{x}(t) \Rightarrow \text{Page ranking} \]

\[ \vec{x}(0) = \begin{bmatrix} \frac{1}{14} \\ \frac{1}{14} \\ \frac{1}{14} \\ \frac{1}{14} \end{bmatrix} \]

equal ranking
PageRank

\[ \vec{x}(t+1) = \begin{bmatrix} 0 & \frac{1}{12} & 0 & 0 \\ \frac{1}{12} & 0 & 0 & \frac{1}{12} \\ \frac{1}{12} & 0 & \frac{1}{12} & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \vec{x}(t) \]

\[ \vec{x}(t) \Rightarrow \text{Page ranking} \]

\[ \vec{x}(0) = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} \]

equal ranking
Page Rank

\[ x(t+1) = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 1 & 0 \end{bmatrix} x(t) \]

\[ x(t) \Rightarrow \text{Page ranking} \]

\[ x(0) = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} \]

\[ x(1) = \begin{bmatrix} 0.125 \\ 0.208 \\ 0.208 \\ 0.458 \end{bmatrix} \]

equal ranking
Page Rank

\[
\begin{bmatrix}
0.12 & 0.24 & 0.24 & 0.4 \\
0 & 1/3 & 0 & 0 \\
1/3 & 0 & 0 & 1/2 \\
1/3 & 1/2 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
0.12 \\
0.24 \\
0.24 \\
0.4
\end{bmatrix}
\]

steady state
Judge me by my PageRank, do you?
General Steady-state solution

\[ \mathbf{x}_{ss} = Q \cdot \mathbf{x}_{ss} \]

\[ Q \cdot \mathbf{x}_{ss} - \mathbf{x}_{ss} = \mathbf{0} \]

\[ (Q-I) \mathbf{x}_{ss} = \mathbf{0} \]

The Null \( Q - I \) is the steady state solution

Find via Gauss elimination!
Eigen Values

We saw an example for a steady-state vector

$$ Q \cdot \vec{x}_{ss} = 1 \cdot \vec{x}_{ss} $$

Direction, and size of the vector did not change!

We will now look at the more general case

$$ Q \cdot \vec{x} = \lambda \cdot \vec{x} $$

In this case, we say that

$\vec{x}$ is an Eigen Vector of $Q$ with Eigen Value $\lambda$

and $\text{span}\{\vec{x}\}$ is the associated Eigen-space
Eigen Values

\[ Q \cdot \vec{x} = \lambda \cdot \vec{x} \]

What happens if, 
\[ \lambda = 1 \] ? 
\[ \lambda > 1 \] ? 
\[ \lambda < 1 \] ?
Eigen Values and Eigen Vectors

• Definition: Let $Q \in \mathbb{R}^{N \times N}$ be a square matrix, and $\lambda \in \mathbb{R}$ if $\exists \vec{x} \neq \vec{0}$ such that $Q \vec{x} = \lambda \vec{x}$,

then $\lambda$ is an eigenvalue of $Q$, $\vec{x}$ is an eigenvector and $\text{Null}(Q - \lambda I)$ is its eigenspace.

**In general $\lambda \in \mathbb{C}$**
Computing eigenvalues and vectors via determinant

Consider:

\[ Q = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \]

we want to find \( \lambda, \vec{x} \) such that \( Q \vec{x} = \lambda \vec{x} \)

\[ Q \vec{x} - \lambda \vec{x} = \vec{0} \]

\[ (Q - \lambda I) \vec{x} = \vec{0} \]

Find \( \vec{x} \in \text{Null}(Q - \lambda I) \):

\[ Q - \lambda I = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1/2 - \lambda & 0 \\ 1/2 & 1 - \lambda \end{bmatrix} \]

1. Find \( \lambda \)
2. Find \( \vec{x} \)
Computing eigenvalues and vectors via determinant

Find $\vec{x} \in \text{Null}(Q - \lambda I)$:

$$Q - \lambda I = \begin{bmatrix} 2 \lambda & 0 & 0 \\ 0 & 2 \lambda & 0 \\ 0 & 0 & -\lambda \end{bmatrix}$$

Find $\lambda$ that results in a non-trivial null space

$$\det(Q - \lambda I) = 0$$

$$(1/2 - \lambda)(1 - \lambda) - (0) \cdot 1/2 = 0$$

$$(1/2 - \lambda)(1 - \lambda) = 0$$

$\lambda_1 = 1/2, \lambda_2 = 1$$
Computing eigenvalues and vectors via determinant

Find $\vec{x} \in \text{Null}(Q - \lambda I)$:

$Q - \lambda I = \begin{bmatrix} \frac{1}{2} - \lambda & 0 \\ \frac{1}{2} & 1 - \lambda \end{bmatrix}$

$\lambda_1 = 1/2$

$\begin{bmatrix} 1/2 - 1/2 & 0 \\ 1/2 & 1 - 1/2 \end{bmatrix} \vec{x} = 0$

$\begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix} \vec{x} = 0$

$\begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \Rightarrow x_1 = -x_2$

$\begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \Rightarrow \vec{x}_1 \in \text{span}\{[1\ 0]^T\}$
Computing eigenvalues and vectors via determinant

Find $\vec{x} \in \text{Null}(Q - \lambda I)$:

\[
\begin{bmatrix}
\frac{1}{2} - \frac{1}{2} & 0 \\
\frac{1}{2} & 1 - \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = 0
\]

$\lambda_1 = \frac{1}{2}$

$\begin{bmatrix}
0 & 0 \\
\frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = 0$

$\begin{bmatrix}
\frac{1}{2} & \frac{1}{2} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = 0$

$x_1 = -x_2$

$x_1 \in \text{span}\{[-1]\}$

$\lambda_2 = 1$

$\begin{bmatrix}
\frac{1}{2} - 1 & 0 \\
\frac{1}{2} & 1 - 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = 0$

$\begin{bmatrix}
-\frac{1}{2} & 0 \\
\frac{1}{2} & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = 0$

$x_1 = x_2$

$x_2 \in \text{span}\{[1]\}$

$\lambda_1 = \frac{1}{2}, \lambda_2 = 1$
The matrix $Q$ has the Eigen-vector

$$\vec{v} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Associated with eigenvalue $\lambda_1 = 1/2$

$$Q\vec{v} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1/2 \cdot 2 + 0(-2) \\ 1/2 \cdot 2 + 1(-2) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Q\vec{v} = 1/2 \vec{v}$$
The matrix $Q$ has the Eigenvector $\mathbf{v}$ associated with eigenvalue $\lambda_1 = 1/2$

$$\mathbf{v} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \frac{1}{2} \cdot 2 + 0(-2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Q\mathbf{v} = \frac{1}{2} \mathbf{v}$$

and, has the Eigenvector $\mathbf{u}$ associated with eigenvalue $\lambda_2 = 1$

$$\mathbf{u} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{1}{2} \cdot 0 + 1 \cdot 2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$Q\mathbf{u} = \mathbf{u}$$
How well do you function in a matrixed environment?

* So long as my eigenvalue is always 1, just fine.
Matrix transformations

What does the matrix do?
What is the A matrix?
What are its eigenvectors?
What are its eigenvalues?
Matrix transformations

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What is the A matrix?
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What are its eigenvalues?
Matrix transformations

What does the matrix do?

What is the A matrix?

What are its eigenvectors?

What are its eigenvalues?
Matrix transformations

For a matrix that flips (reflects) vectors along a line:

What is the A matrix?

What are its eigenvectors?

What are its eigenvalues?