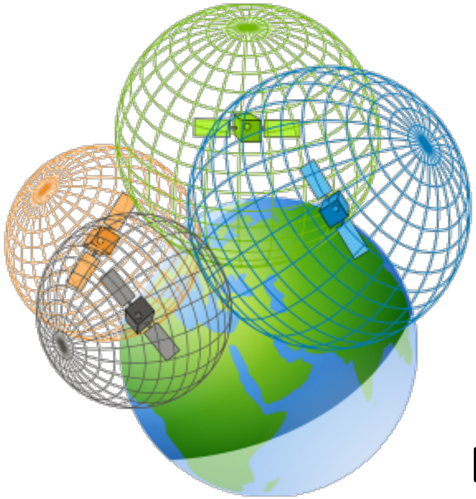
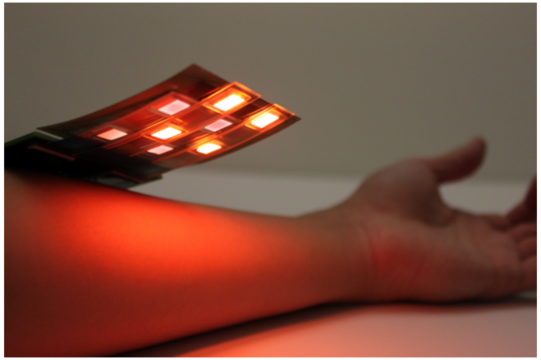




Welcome to EECS 16A!

Designing Information Devices and Systems I

Ana Claudia Arias and Miki Lustig

Fall 2022

Module 2
Lecture 3
Power and Voltage/Current Measurements
(Note 13)

Last Lecture

- Solve circuits for the currents and node potentials
- Set up a matrix problem of the form A $\vec{x} = \vec{b}$

  where

  $\vec{x}$ consists of the unknown currents and potentials

  $\vec{b}$ contains the independent current and voltage sources

  A describes the relationship between them.

$A\vec{x} = \vec{b}$

$\vec{x}$: unknowns

$\vec{b}$: knowns / constants

A : knowns / constants

$\vec{x} = \begin{bmatrix} i_1 \\ \vdots \\ i_k \\ v_1 \\ \vdots \\ v_n \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_{k+n} \end{bmatrix}$

k+n

k+n

Rules:

- KVL
- KCL
- Element definitions
- I x V relantionship

Last Lecture

$I_1 + I_3 = 0 \quad (1)$

$-I_1 + I_2 = 0 \quad (2)$

$R_1 I_1 - U_1 + U_2 = 0 \quad (3)$

$R_2 I_2 - U_2 = 0 \quad (4)$

$U_1 = V_S \quad (5)$

$$\underset{A}{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_S \end{bmatrix}}$$

$$I_1 = \frac{V_S}{R_1 + R_2}, \quad I_2 = \frac{V_S}{R_1 + R_2}, \quad I_3 = -\frac{V_S}{R_1 + R_2}$$

$U_1 = V_S \qquad U_2 = ?$

How to think about Energy and Power in circuits?

Current: flow of charges (electrons moving from point A to B inside a material)

$$\boxed{I = \frac{dQ}{dt}}$$

It takes **energy** to move charge from A → B ⇒ Voltage

$$\boxed{V_{AB} = \frac{dE}{dq}}$$

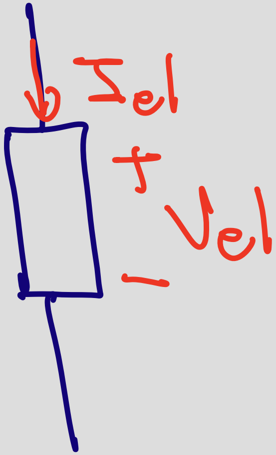
Power: is the rate of change of energy

$$P = \frac{dE}{dt} \cdot \frac{dq}{dt} = V \cdot I$$

$$(V) \cdot (A) = (W)$$

Energy and Power

$$\boxed{P_{el} = V_{el} \cdot I_{el}}$$



if element is a resistor

$$P = V \cdot I = R \cdot I \cdot I = R \cdot I^2 \geq 0$$

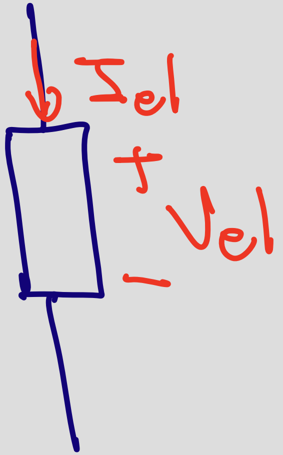
Power dissipated is positive

$$V_{el} = R \cdot I_{el}$$

$$I_{el} = \frac{V_{el}}{R}$$

$$P = V \cdot I = V_{el} \, V_{el}/I = V_{el}^2/I \geq 0$$

Energy and Power

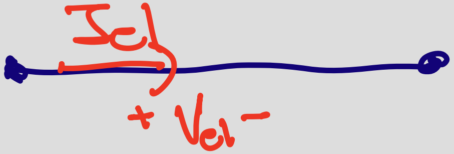


$$\boxed{P_{el} = V_{el} \cdot I_{el}}$$

Open circuit

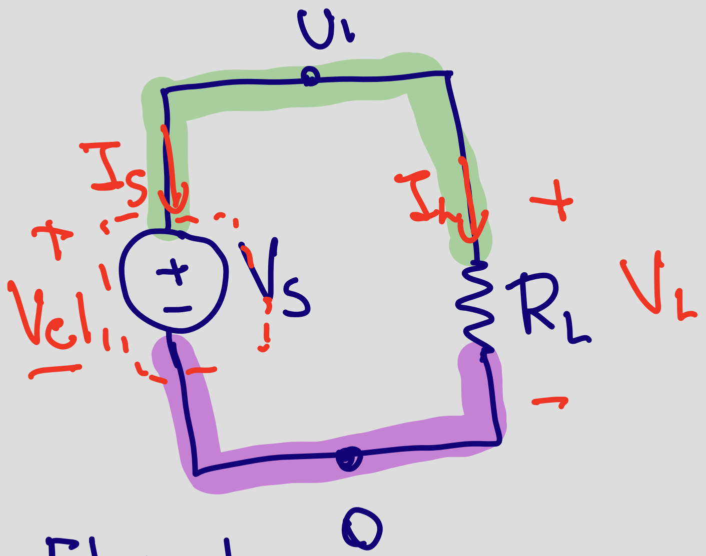
$$P_{el} = V_{el} \cdot \cancel{I_{el}}^{\to 0} = 0$$

wire



$$P_{el} = \cancel{V_{el}}^{\to 0} \cdot I_{el} = 0$$

Example



Elem 1

$P_S = I_S \cdot V_{el_1}$ (def.)

$\boxed{P_S = I_S \cdot V_S}$

* Conservation of Energy

KCL: $I_L + I_S = 0$

KVL: $U_1 - 0 = V_L$

$U_1 - 0 = V_{el_1} = V_S$

Power: Elem L

$P_L = I_L \cdot V_L$ (def.)

$P_L = (-I_S) \cdot V_L$

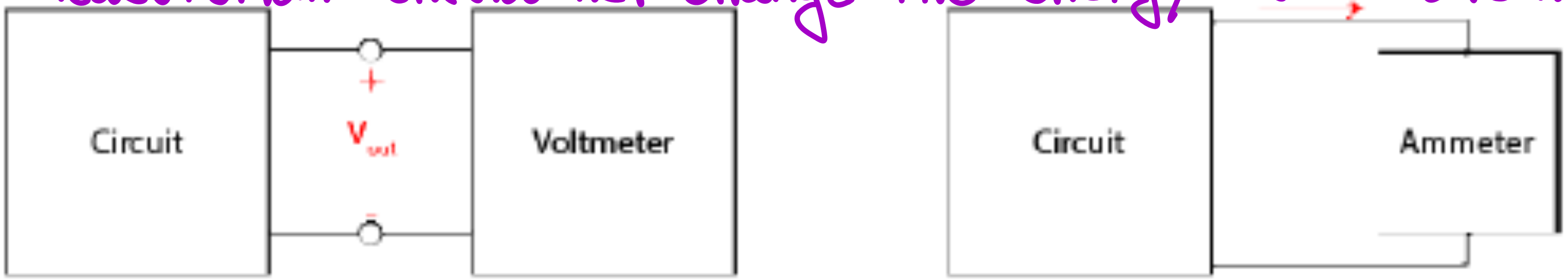
$\boxed{P_L = (-I_S) \cdot V_S}$

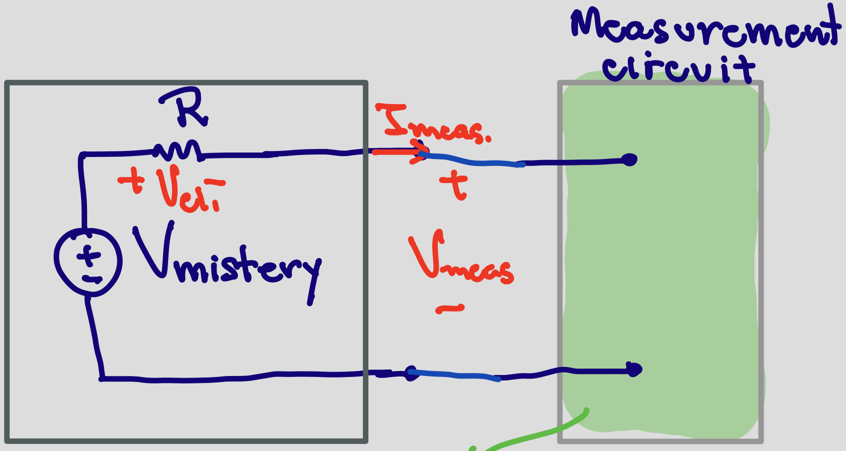
* $P_L = -P_S$

$P_L + P_S = 0$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit





Must behave as an open-circuit

Goal : $V_{meas} = V_{mistery}$

$V_{el_1} = I_{meas} \cdot R$

KVL : $V_{mistery} - V_{el_1} - V_{mea} = 0$

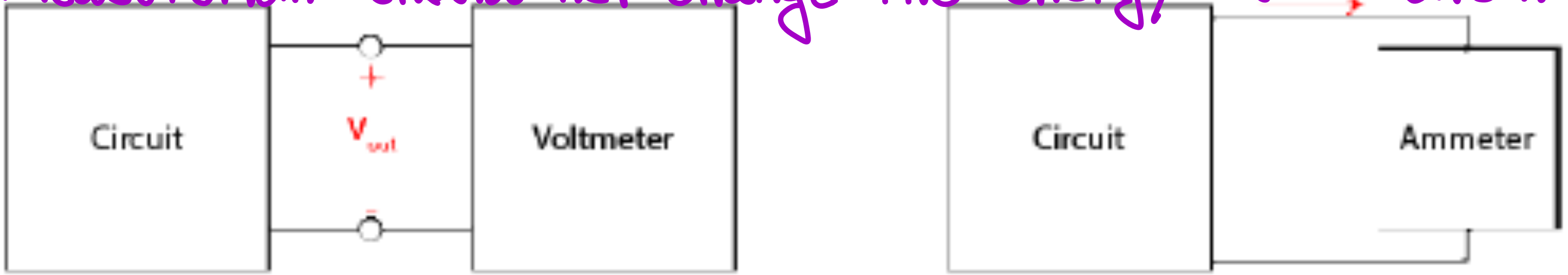
$V_{mistery} = V_{el_1} + V_{meas}$

$V_{mistery} = \cancel{I_{mea}}^{\,0} \cdot R + V_{meas}$

$V_{mistery} = V_{meas} \Big|_{\text{if } I_{meas} = 0}$

How to measure Voltage and Current?

Measurement should not change the energy of the circuit



Task/Goal:

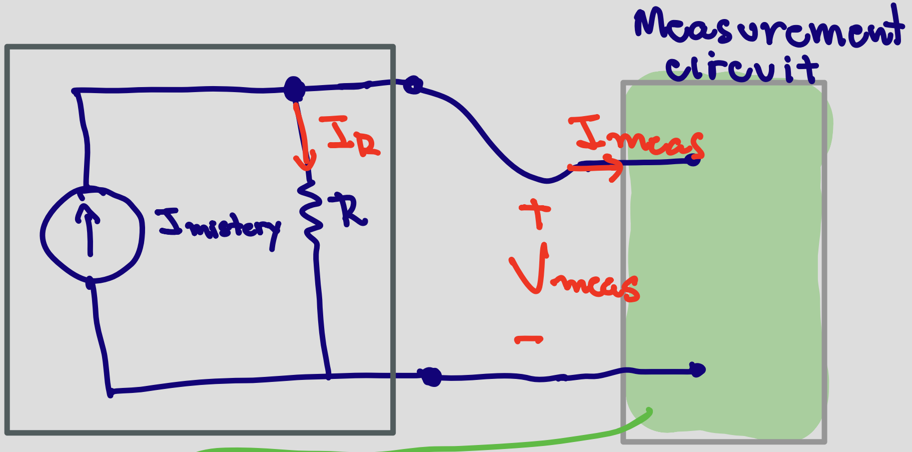
Measure $I_{mistery}$

KCL: $I_{mistery} = I_R + I_{meas}$

$I_{mistery} = I_{meas} \Big|_{\text{if } I_R = 0}$

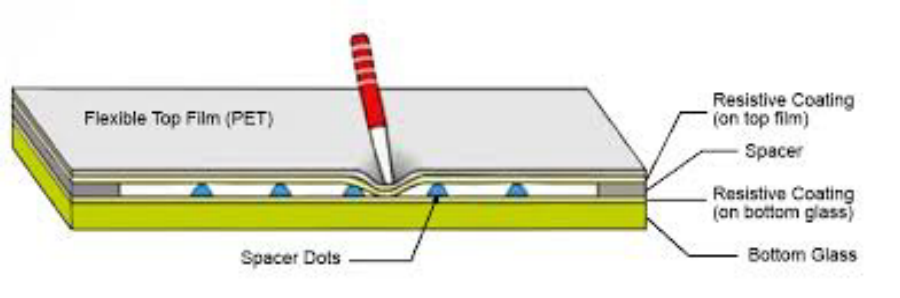
$I_R = \dfrac{V_{meas}}{R}$

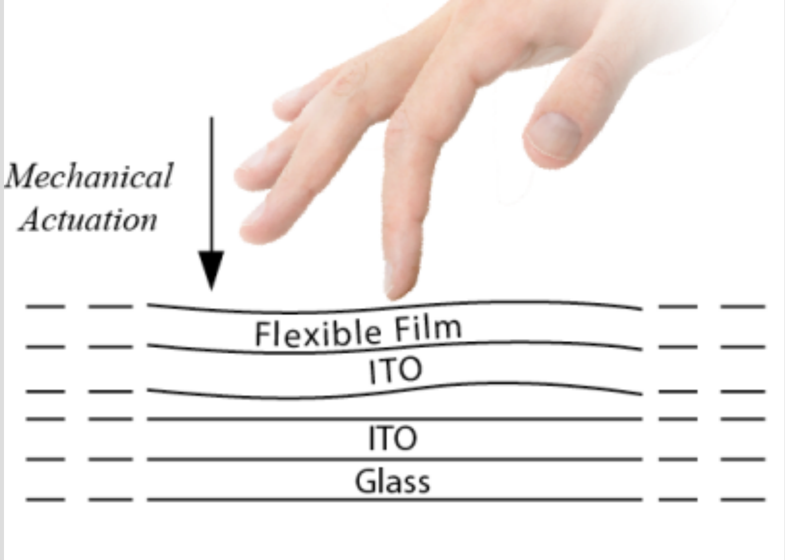
$I_R = 0$; is $V_{meas} = 0$



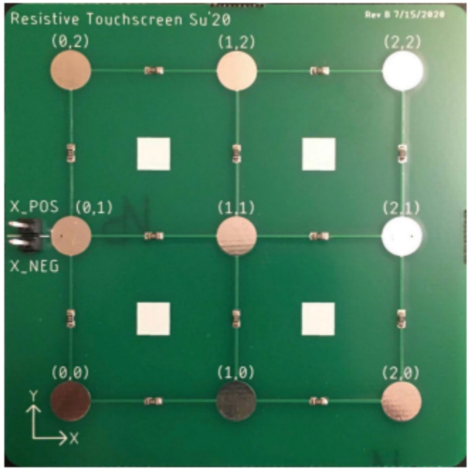
Must behave as a wire

Resistive Touch Screen

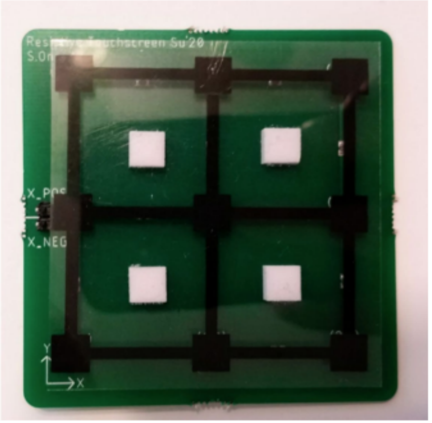




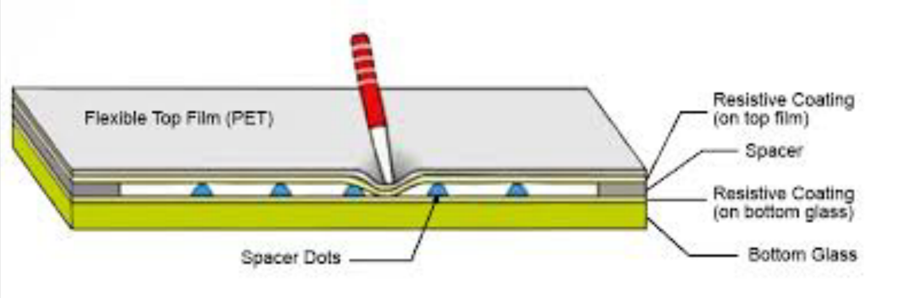
Bottom Layer: Resistive Layer

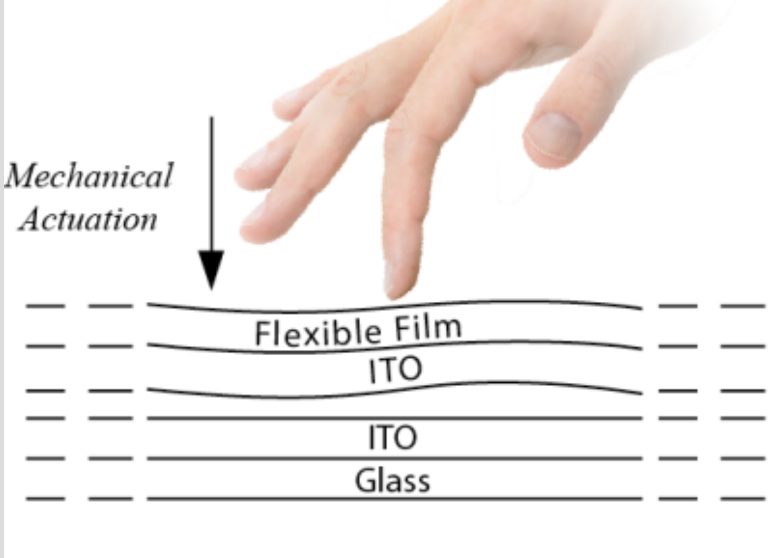


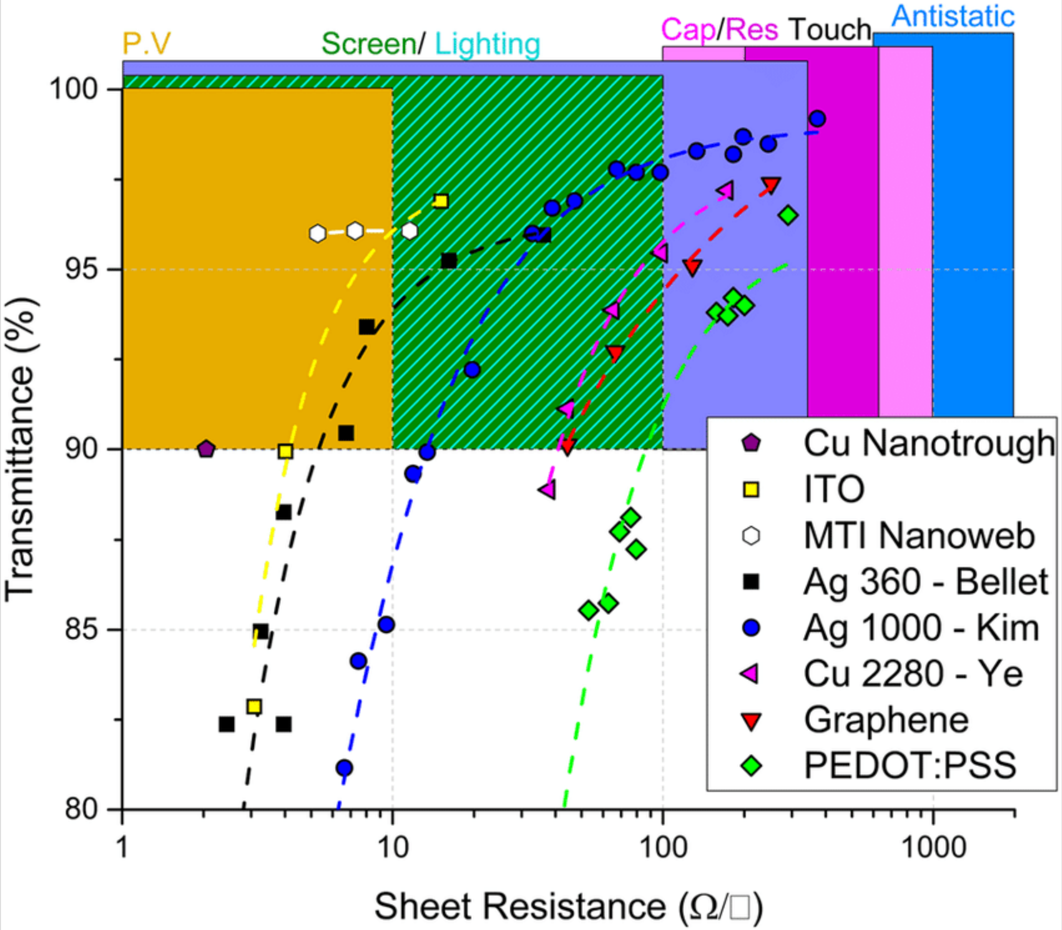
Top Layer: Flexible Resistive Layer



Resistive Touch Screen



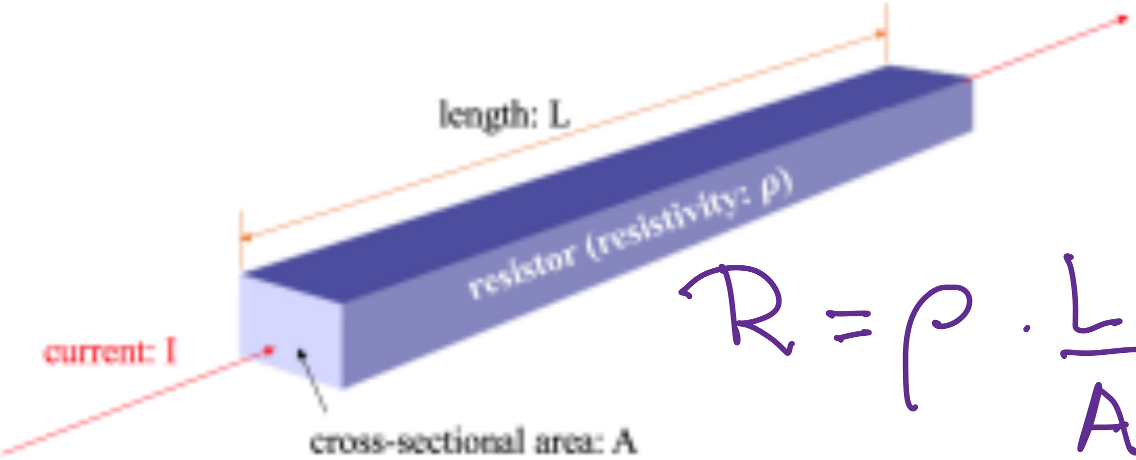




https://doi.org/10.1038/s41598-019-47777-2

Resistance, Resistivity, Conductivity – Properties of Materials

Material	Electrical characteristics	
	Electrical Resistivity (Ω x cm)	Electrical Conductivity (Ω^{-1} x cm^{-1})
Cu	0.034 x10^{-5}	29 x 10^{5}
Fe	32.54 x 10^{-5}	0.031 x 10^{5}
Ag	0.36 x 10^{-5}	2.8 x 10^{5}
Al	0.03 x 10^{-5}	33.3 x 10^{5}
Ni	0.046 x 10^{-5}	21.7 x 10^{5}
Cu-Fe	33.37 x 10^{-5}	0.030 x 10^{5}
Cu-Ag	2.71 x 10^{-5}	0.37 x 10^{5}
Al-Ni	0.564 x 10^{-5}	1.77 x 10^{5}



$$R = \rho \cdot \frac{L}{A}$$

Note 12

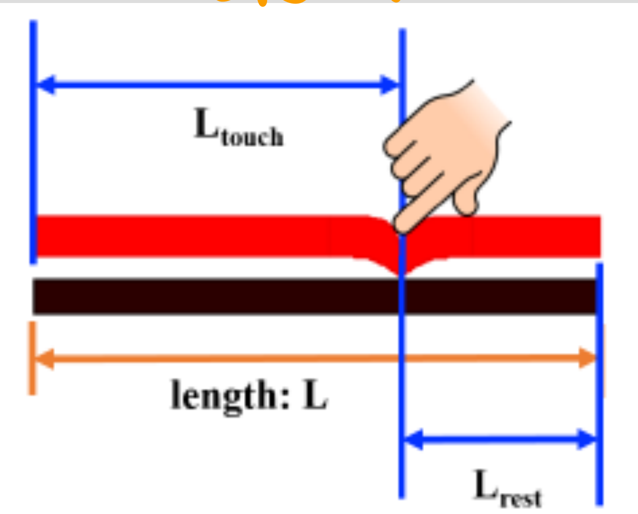
- Longer the wire → the more E is lost
- Wide wires → lower resistance
- Wire properties depend on materials choice.

ρ = resistivity (property of materials)

$\frac{L}{A}$: geometric parameters (property of the wire)

Resistive Touch Screen

Problem: to find the location of touch.



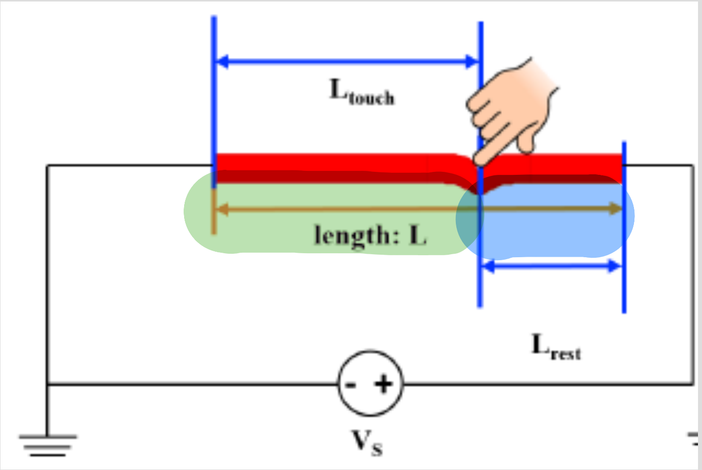
Go from mechanical to electrical quantity.

Want to measure $\frac{L_{touch}}{L}$

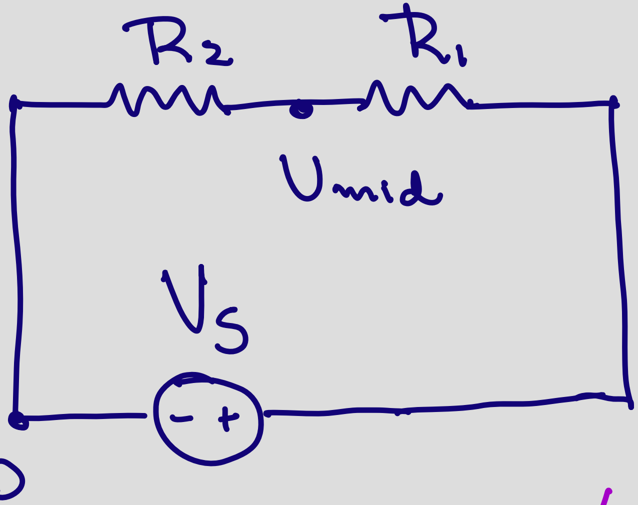
L_{touch} is unknown

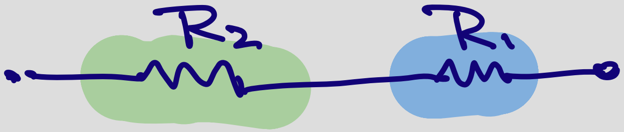
Resistive Touch Screen – First model

$U_{mid} = ?$



$\Rightarrow$





$$U_{mid} = \frac{R_2}{R_2+R_1} \cdot V_s$$

(Voltage Divider)*

$$R_1 = \rho \cdot \frac{L_{rest}}{A}$$

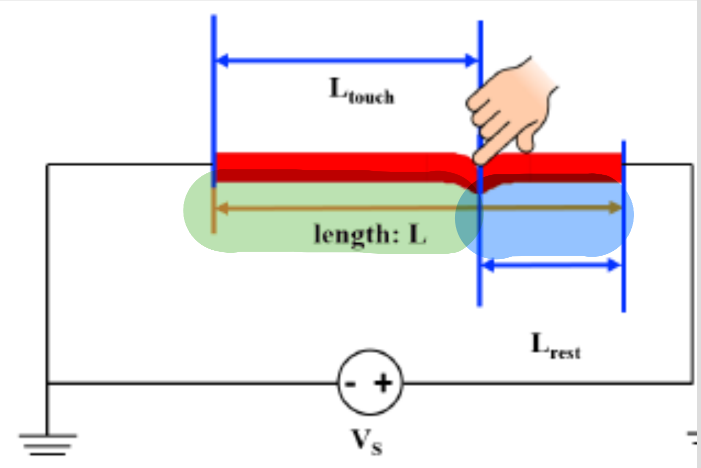
$$R_2 = \rho \cdot \frac{L_{touch}}{A}$$

$$U_{mid} = \frac{\cancel{\rho} \cdot L_{touch}/\cancel{A}}{\cancel{\rho} \cdot \frac{L_{touch}}{\cancel{A}} + \cancel{\rho} \frac{L_{rest}}{\cancel{A}}} \cdot V_s$$

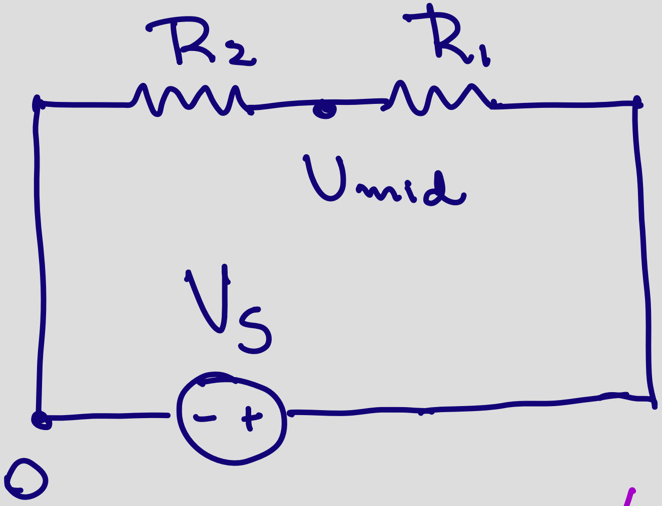
$$U_{mid} = \frac{L_{touch}}{L_{touch} + L_{res}} \cdot V_s = \frac{L_{touch}}{L} \cdot V_s$$

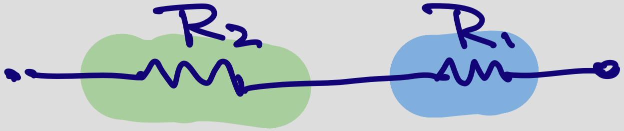
Resistive Touch Screen – First model

$U_{mid} = ?$



$\Rightarrow$





$$U_{mid} = \frac{R_2}{R_2 + R_1} \cdot V_s \quad \left(\text{Voltage Divider}\right)^*$$

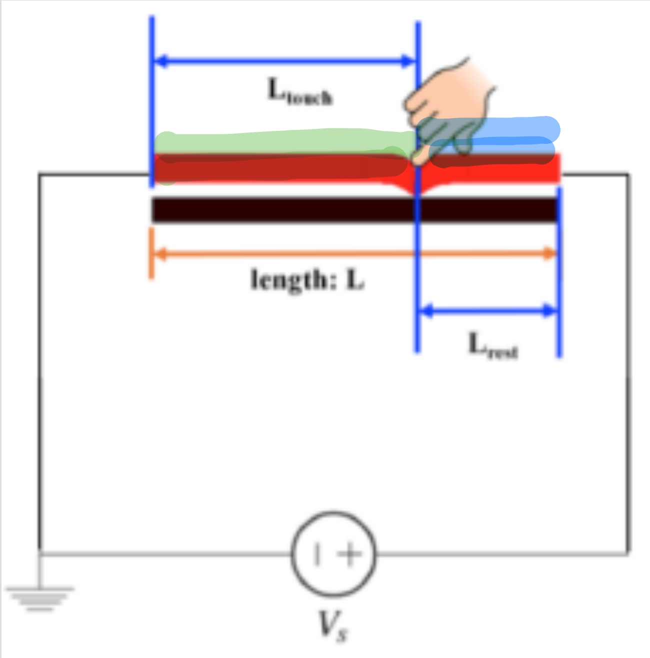
$$R_1 = \rho \cdot \frac{L_{rest}}{A}$$

$$R_2 = \rho \cdot \frac{L_{touch}}{A}$$

$$U_{mid} = \frac{\cancel{\rho} \cdot L_{touch}/\cancel{A}}{\cancel{\rho} \cdot \frac{L_{touch}}{\cancel{A}} + \cancel{\rho} \frac{L_{rest}}{\cancel{A}}} \cdot V_s$$

$$U_{mid} = \frac{L_{touch}}{L_{touch} + L_{res}} \cdot V_s = \frac{L_{touch}}{L} \cdot V_s$$

Resistive Touch Screen – More realistic model

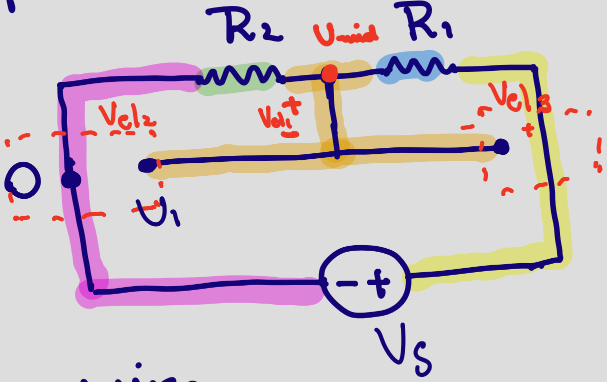


$$U_{mid} = \frac{R_2}{R_1 + R_2} \cdot V_s$$

Voltage Divider

⇒ Model 1

- Add ideal wire to represent bottom plate

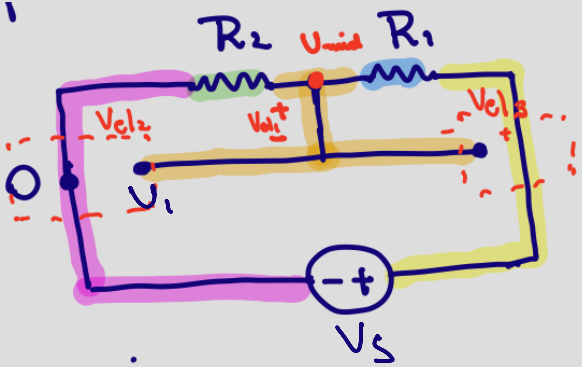


el_1 : wire

el_2: open-circuit (V_{el_2})

el_3: open-circuit (V_{el_3})

Resistive Touch Screen – More realistic model



el_1 : wire
el_2 : open-circuit (V_{el_2})
el_3 : open-circuit (V_{el_3})

Voltage Definition

El_2 ∴ $V_{el_2} = V_1 - 0$

El_1 ∴ $V_{el_1} = U_{mid} - U_1$

KVL

$U_{mid} - 0 = V_{el_2} + V_{el_1}$

$U_{mid} = V_{el_2} + \cancel{V_{el_1}}^{0}$

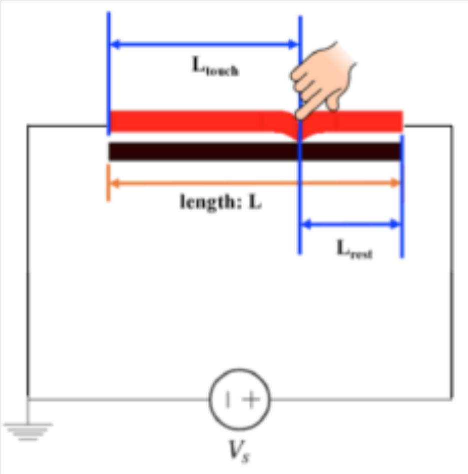
$U_{mid} = \cancel{V_{el_1}}^{0} + U_1$

el_1 is a wire ∴ $V_{el_1} = 0$

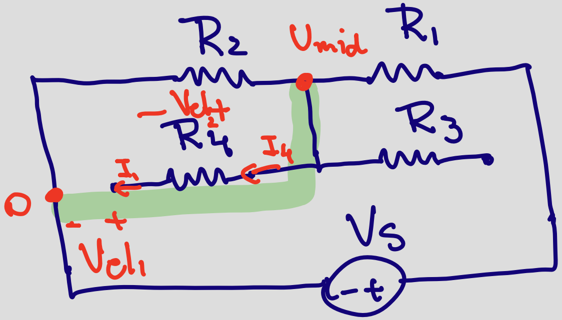
$$\boxed{U_{mid} = U_1}$$

↳ By measuring V_{el_2} we get U_{mid} for any touch

Resistive Touch Screen – More realistic and better model



Model 2 - imperfect conductor (resistor)
(top and botton plates)



KCL for el2

$I_1 = I_4$

$I_1 = 0 \therefore I_4 = 0$

In this model we added:

el1 : open- circuit

el2 : resistor (R_4)

$V_{el_2} = R_4 \cdot I_4$ (Ohm's Law)

KVL

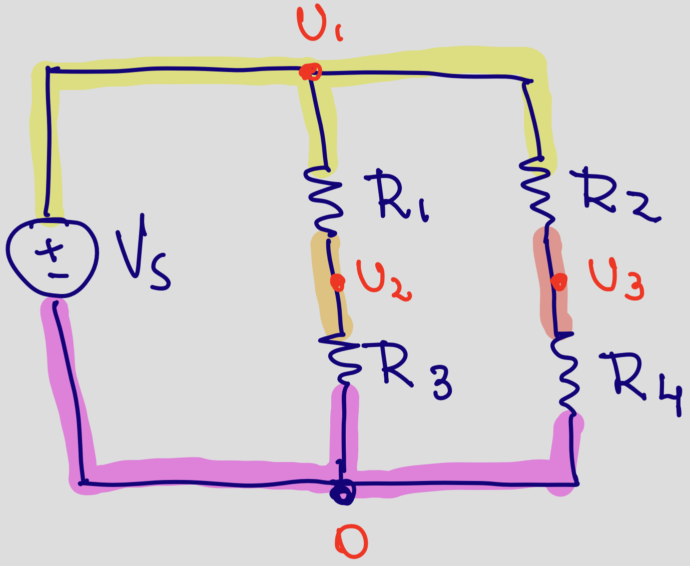
$V_{el_1} + V_{el_2} = U_{mid} - 0$

$V_{el_1} + R_4 \cdot \cancel{I_4}^{\to 0} = U_{mid}$

$\boxed{U_{mid} = V_{el_1}}$

* By measuring Vel1 we get Umid for any Ltouch ;
independently of materials used in botton lane!

An interesting circuit



- What are V_2 and V_3?

$$V_2 = \frac{R_3}{R_1+R_3} \cdot V_s$$

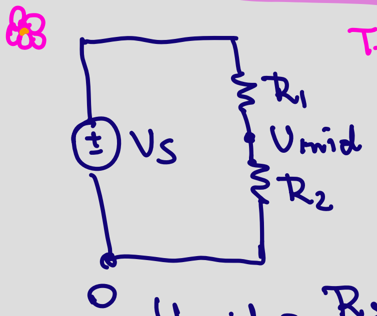
$$V_3 = \frac{R_4}{R_2+R_4} \cdot V_s$$

$$V_2 - 0 = \frac{R_3}{R_1+R_3} \cdot (\cancelto{V_s}{V_1 - 0})$$

$$V_3 - 0 = \frac{R_4}{R_2+R_4} \cdot (\cancelto{V_s}{V_1 - 0})$$

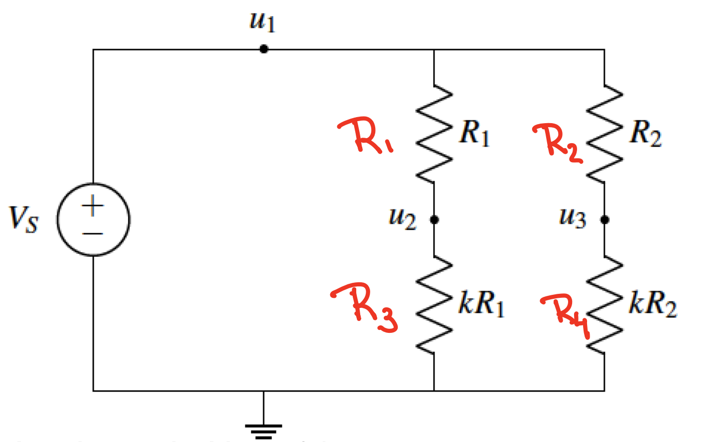
$$V_1 - 0 = V_s$$

Tool box



$$V_{mid} = \frac{R_2}{R_1+R_2} \cdot V_s$$

An interesting circuit



As shown in Note 14

Power supply keeps U in wires equal to Vs regardless of how many branches we have!

$$U_2 = \frac{R_3}{R_1 + R_3} \cdot V_s$$

$$U_3 = \frac{R_4}{R_2 + R_4} \cdot V_s$$

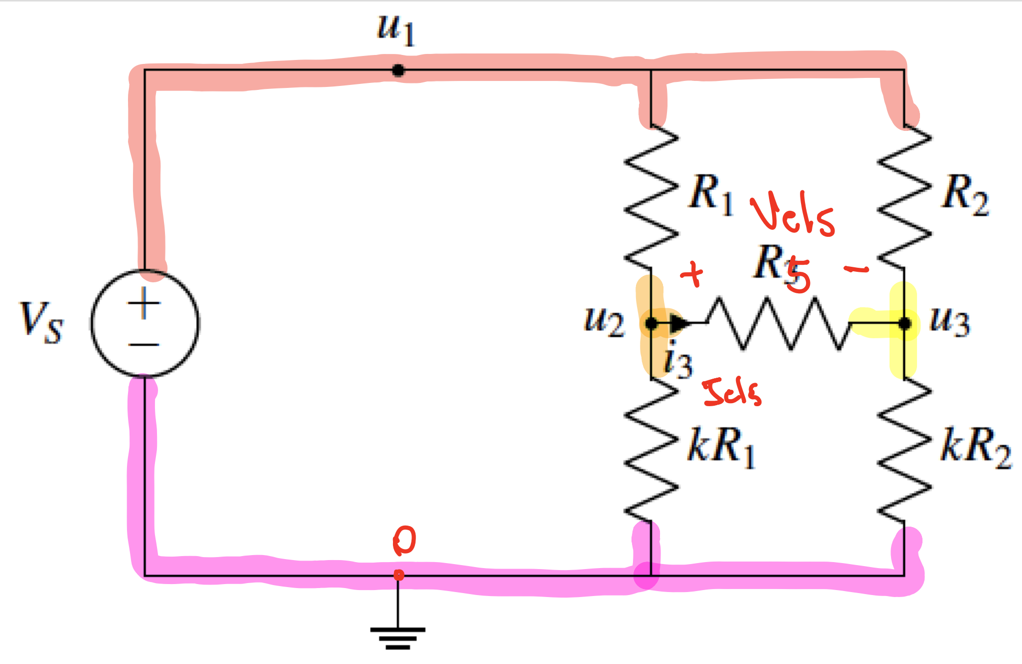
$$U_2 = \frac{k\cancel{R_1}}{\cancel{R_1} + k\cancel{R_1}} \cdot V_s \quad \therefore \quad U_2 = \frac{k}{1+k} V_s$$

$$U_3 = \frac{k\cancel{R_2}}{\cancel{R_2} + k\cancel{R_2}} \cdot V_s \quad \therefore \quad U_3 = \frac{k}{1+k} V_s$$

$$\boxed{U_2 = U_3}$$

* wow!

Let's add on more resistor



$Elem_5$ = resistor (R_5)

$V_{el5} = U_2 - U_3$ (Voltage Def.)

Bold Assumption

$V_{el5} = 0$

if $V_{el5} = 0 \Rightarrow I_{el5} = \frac{\cancel{V_{el5}}^{\to 0}}{R_5} = 0$

if $I_{el5} = 0$

The circuit is the same as the one we already analysed without R_5.

We showed : $U_2 = U_3$

$\boxed{V_{el5} = U_2 - U_3 = 0}$