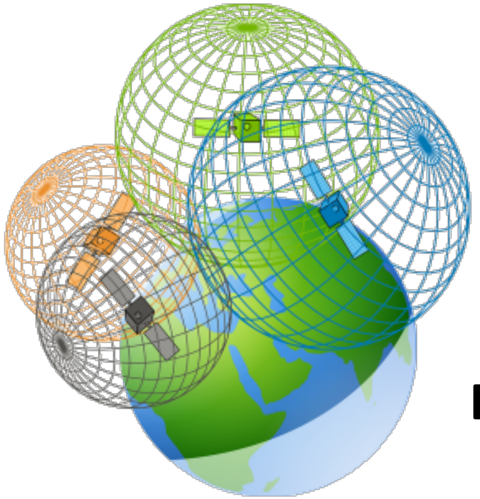


We  CS 16A!

Designing Information Devices and Systems I

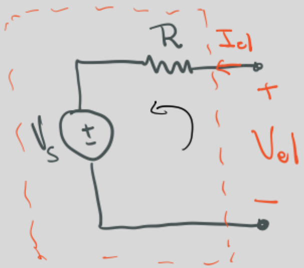
Ana Claudia Arias and Miki Lustig

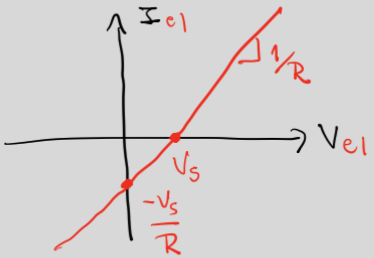
Fall 2022

Module 2
Lecture 6
Thevenin and Norton Equivalent
(Note 15)

Last Class

Equivalence - Example





$$V_{el} = V_s + V_R$$

$$V_{el} = V_s + I_{el} \cdot R$$

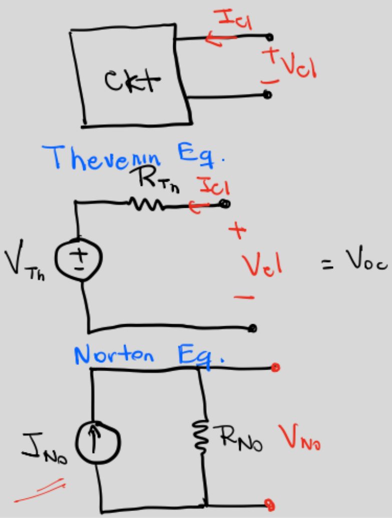
$$I_{el} = \frac{1}{R} V_{el} - \frac{V_s}{R}$$

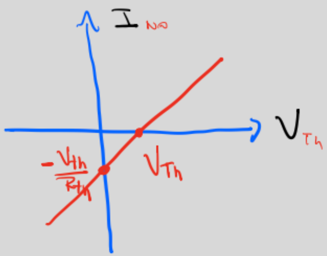
$$I_{el} \cdot R = V_{el} - V_s$$

$$I_{el} = \frac{V_{el}}{R} - \frac{V_s}{R}$$

Two circuits are equivalent if they have the same I-V relationship.

Thevenin and Norton Equivalent





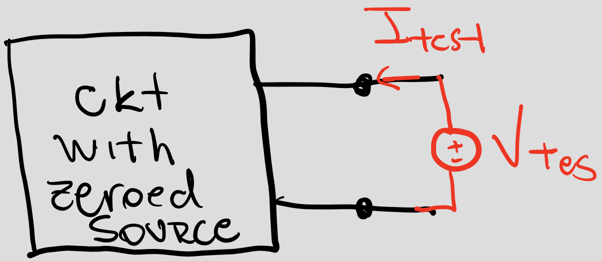
1) Find V_{Th} : Connect open-circuit

— $I = 0$

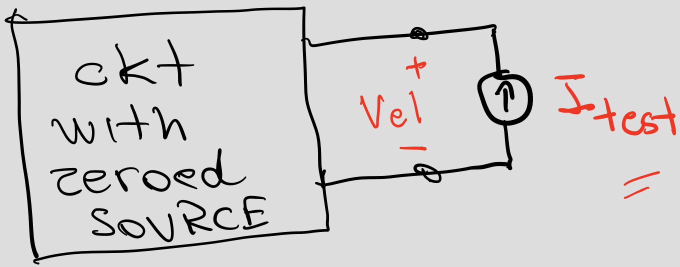
2) Find R_{Th} : Find slope

zero-out independent source

Thevenin and Norton Equivalent

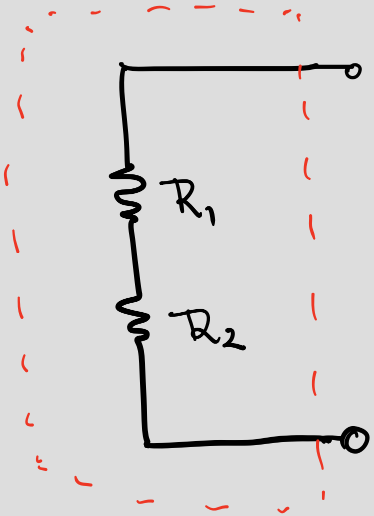


$$R_{Th} = \frac{V_{test}}{I_{test}}$$

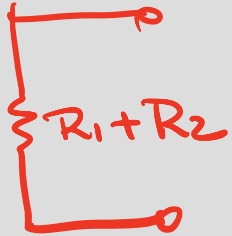


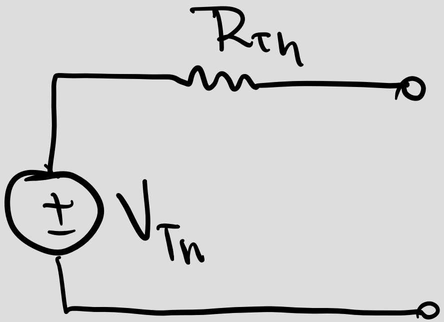
$$R_{No} = \frac{V_{test}}{I_{test}}$$

Practice – Example 1



$\Rightarrow$

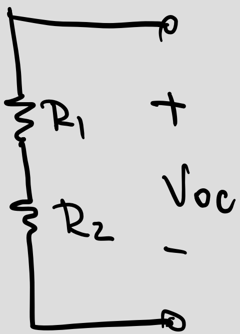




$$R_{Th} = \frac{V_{test}}{I_{test}} = (R_1 + R_2)$$

In series means that the same I flows trough the elements.

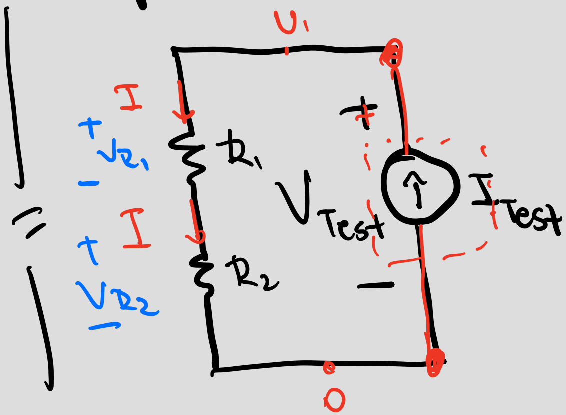
Step 1:



$V_{oc} = 0$

$V_{Th} = 0$

Step 2: No sources



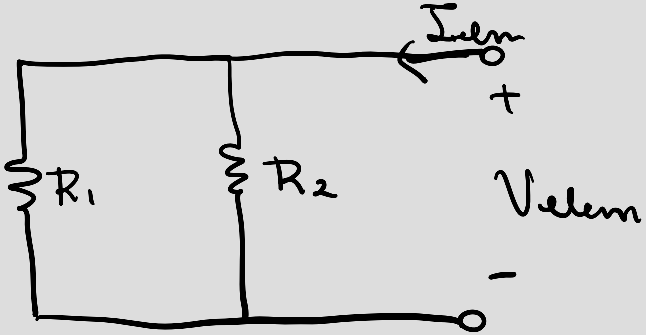
$$V_{TEST} = V_{R1} + V_{R2}$$

$$V_{Test} = IR_1 + IR_2$$

$$= I_{test} R_1 + I_{Test} R_2$$

$$V_{Test} = (R_1 + R_2) \cdot I_{Test}$$

Practice – Example 2

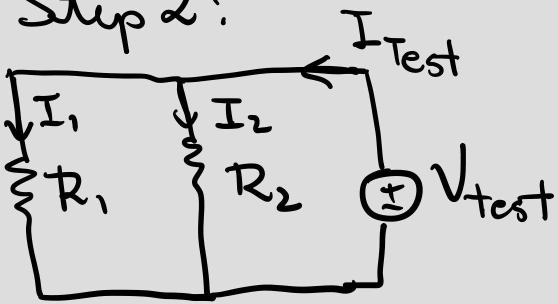


$\Rightarrow$

Step 1

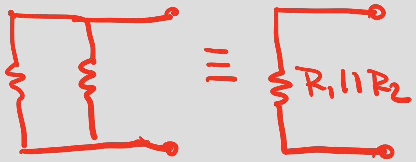
$V_{th} = 0$

Step 2:



$$I_1 = \frac{V_{test}}{R_1}$$

$$I_2 = \frac{V_{test}}{R_2}$$



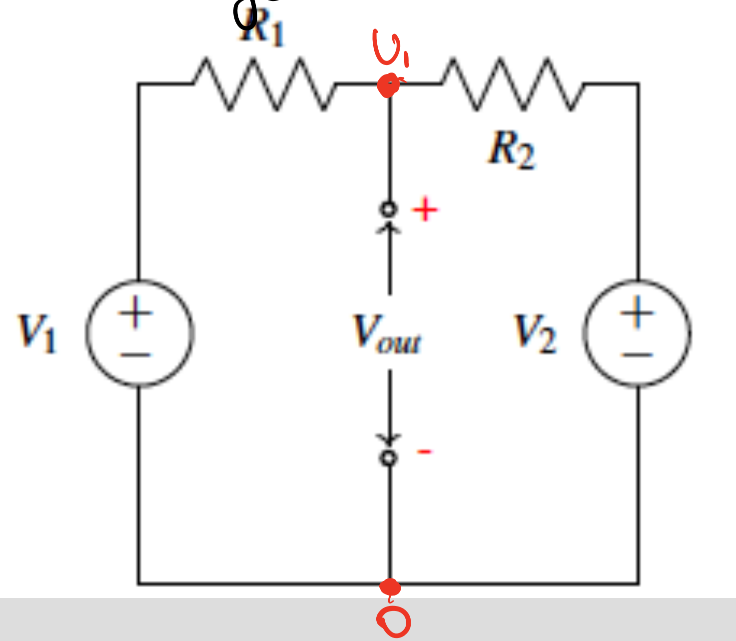
Parallel operator

$$I_{test} = I_1 + I_2 = V_{test}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_{Th} = \frac{V_{test}}{R_{test}} = \frac{\cancel{V_{test}}}{\cancel{V_{test}}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = \frac{R_1R_2}{R_1+R_2} = \boxed{R_1 \| R_2}$$

Circuit Analysis Method – What happens when we have multiple Voltage or Current sources?

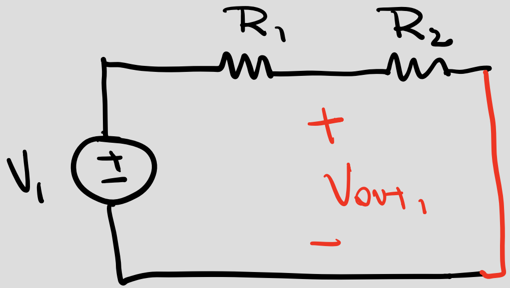
Voltage Summer



$U_1 - 0 = V_{out}$

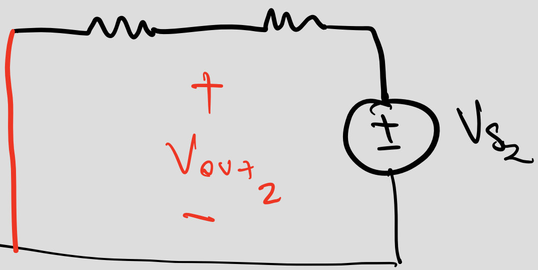
$U_1 = V_{out}$

1 step: Compute a response to V_{s_1} (Set $V_{s_2}=0$)



$$V_{out_1} = \frac{R_2}{R_1+R_2}\cdot V_{s_1}$$

2nd step: Compute a response to V_{s_2}



$$V_{out_2} = \frac{R_1}{R_1+R_2}\cdot V_{s_2}$$

$$V_{out} = V_{out_1} + V_{out_2}$$

$$U_1 = U_{11} + U_{12}$$

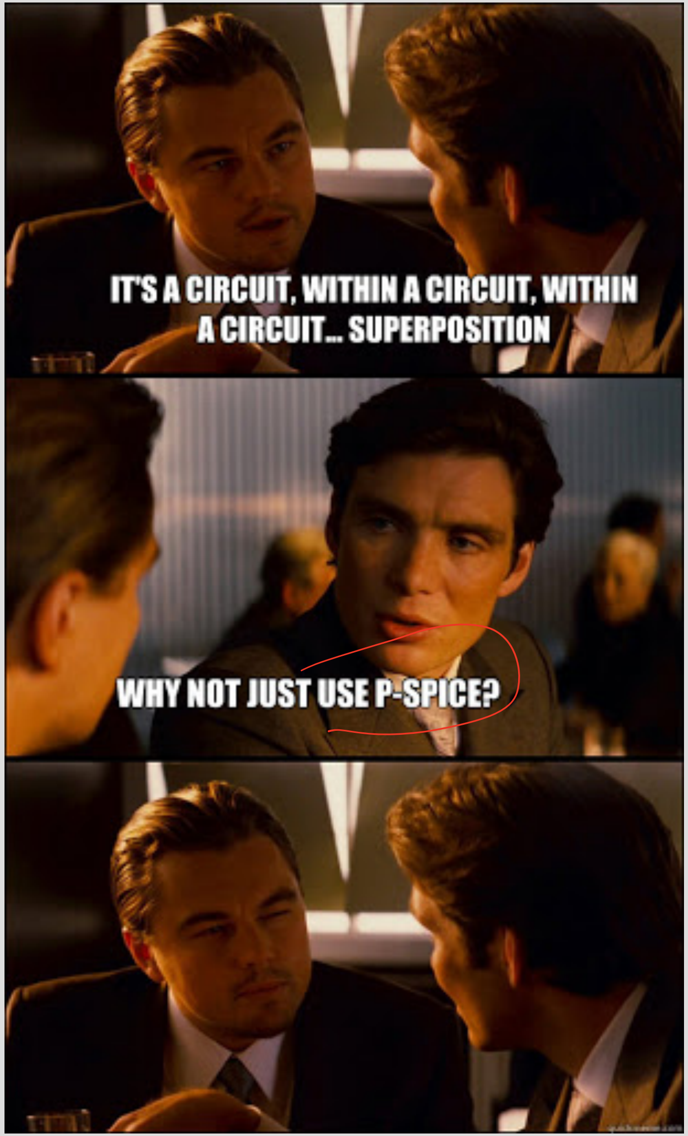
$$U_1 = \frac{R_2}{R_1+R_2}\cdot V_{s_1} + \frac{R_1}{R_1+R_2}\cdot V_{s_2}$$

$\underbrace{\quad}_{\alpha < 1}$ $\underbrace{\quad}_{\beta < 1}$

Superposition

For each independent source k (either voltage source or current source)

- Set all other independent sources to 0
- Voltage source: replace with a wire
- Current source: replace with an open circuit
- Compute the circuit voltages and currents due to this source k
- Compute V_{out} by summing the $V_{out;ks}$ for all k.



Circuit Analysis Method

- Solve circuits for the currents and node potentials
- Set up a matrix problem of the form A $\vec{x} = \vec{b}$

  where

  $\vec{x}$ consists of the unknown currents and potentials

  $\vec{b}$ contains the independent current and voltage sources

  A describes the relationship between them.

Find $\vec{x} = \begin{bmatrix} I_1 \\ \vdots \\ I_m \\ V_1 \\ \vdots \\ V_k \end{bmatrix}$

for a matrix A and some stimulus vector $\vec{b}$

$$\vec{b} = \begin{bmatrix} I_{s1} \\ I_{s1} \\ V_{s1} \\ \vdots \\ V_{s_{m+k-1}} \end{bmatrix}$$

$$A\vec{x} = \vec{b} \Rightarrow \underbrace{\vec{x} = A^{-1}\vec{b}}_{\text{solution}}$$

linear combination of sources

$$I_i = \alpha_1 I_{s_1} + \cdots + \alpha_l I_{s1} + \cdots + \alpha_{m+k} V_{s_{m+k-1}}$$

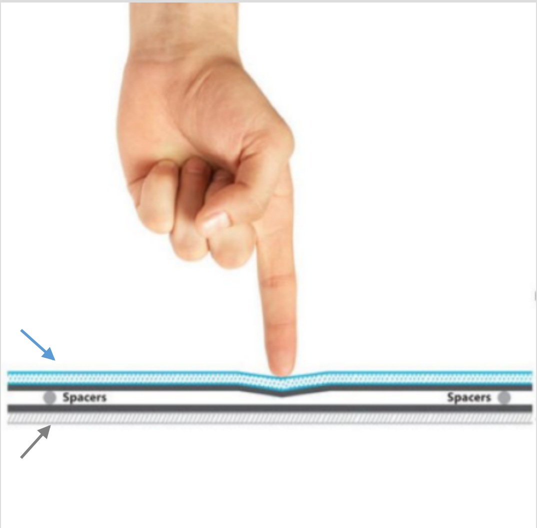
$$V_j = \beta_1 I_s + \cdots \cdots + \beta_{m+k} V_{s_{m+k-1}}$$

$$I_i = \underbrace{I_{i,1}}_{\alpha I_{s1}} + \cdots + \underbrace{I_{i,1}}_{\alpha_1 I_{s1}} + \cdots + \underbrace{I_{i,m+k}}_{\alpha_{m+k} V_{s_{m+k-1}}}$$

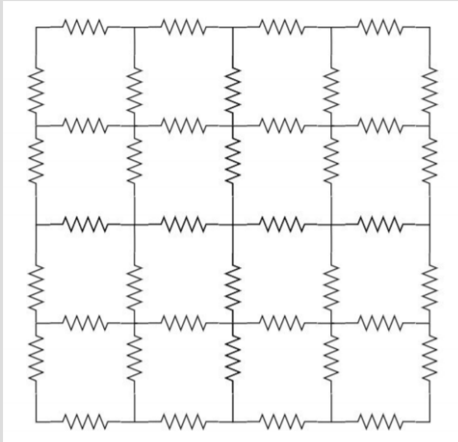
Can calculate I_i by nulling other souces!

Now that we understand 2D resistive touchscreen, let's change it!

resistive sheet



resistive sheet

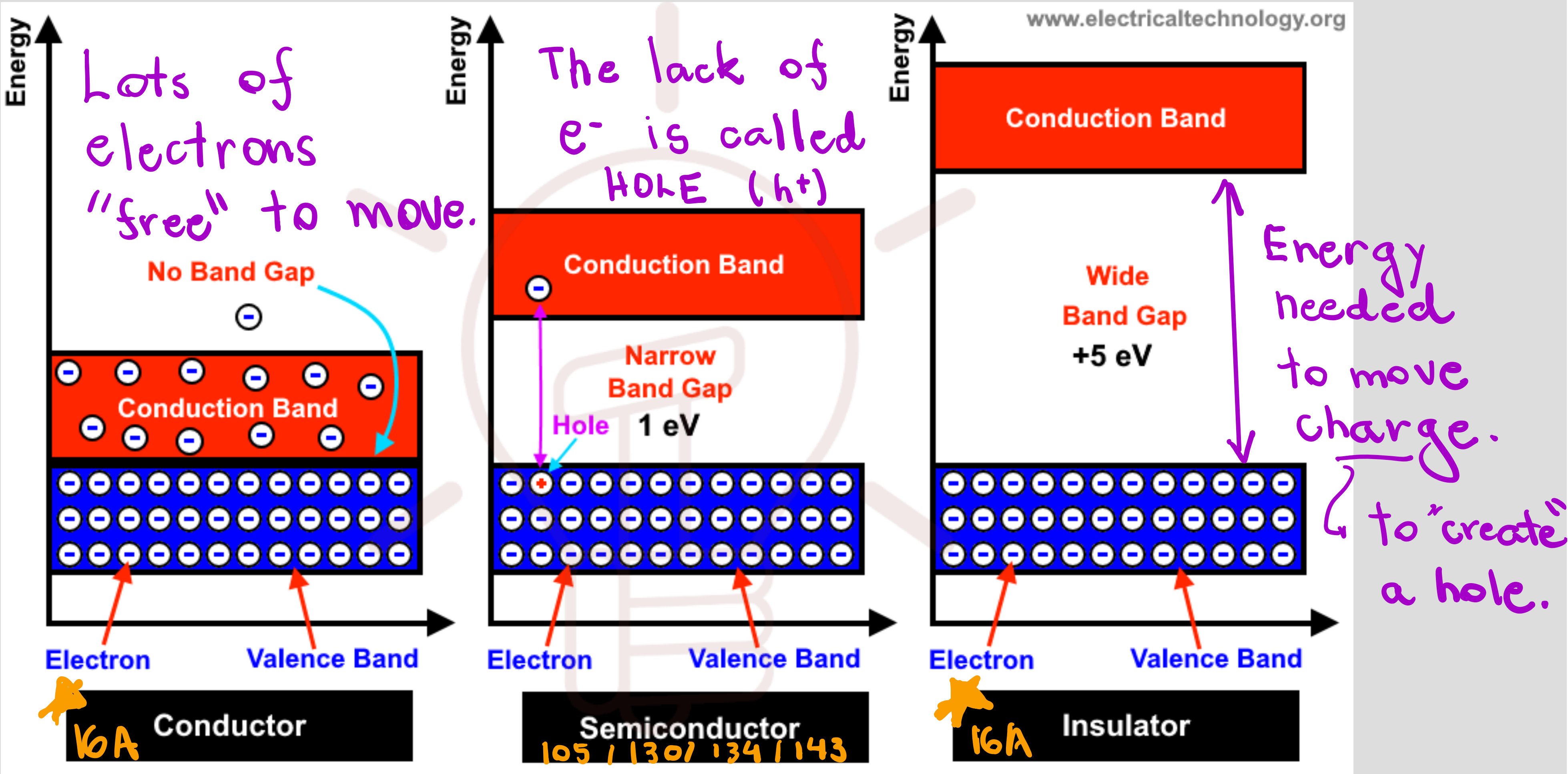


Circuit model for each resistive sheet is a grid of resistors

real-world touchscreens are usually capacitive, not resistive:

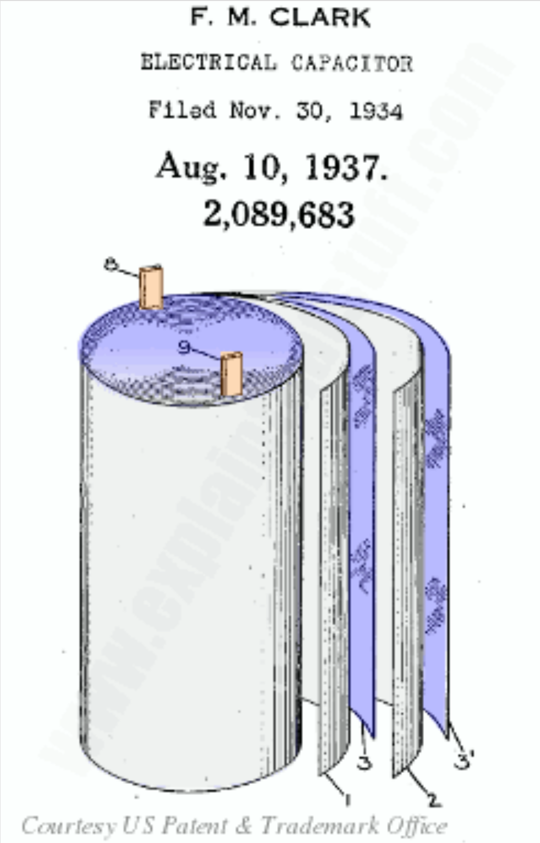
- **don't need to be flexible**
- **multi-touch is easier**
- **more sensitive**
- **increased contrast on screen**

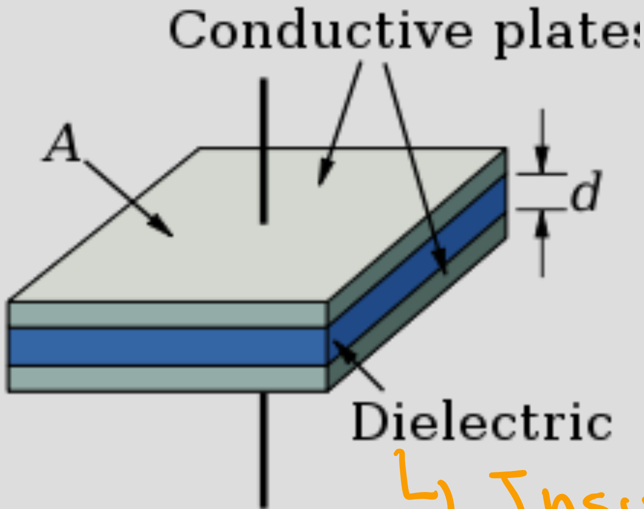
Second: a tiny bit of Solid-State Physics



Now, Capacitors!

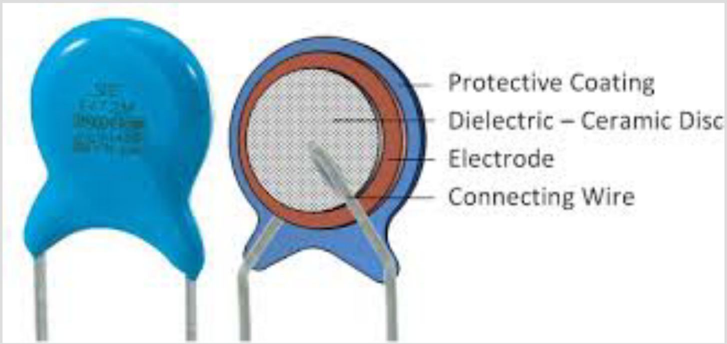
- Charge storage device (like a ‘bucket’ for charge)

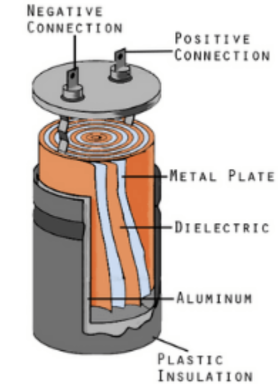


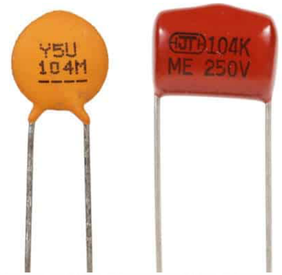


↳ Insulator

↳ Higher Energy is needed to move charge.

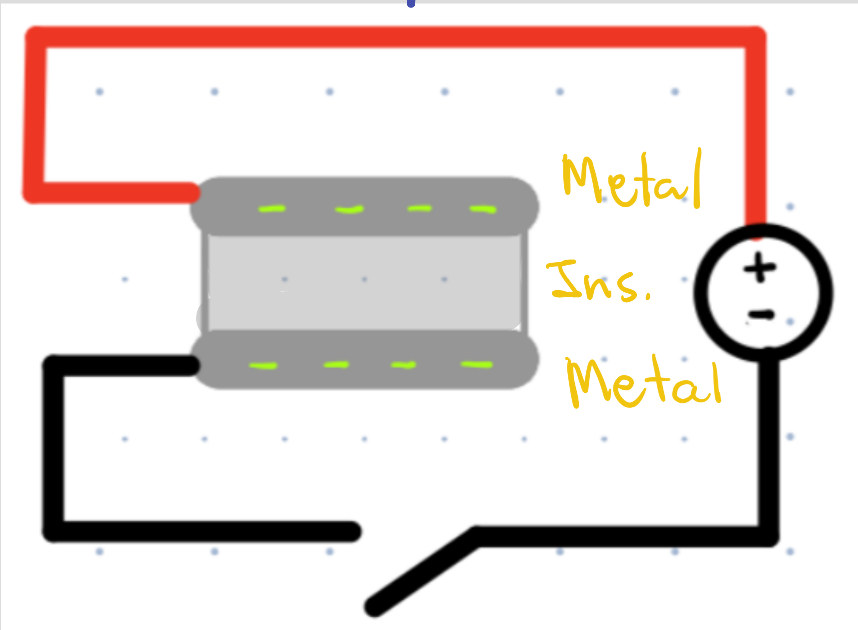






The Physics of a Capacitor

* Energy is needed to move charge.



e^-

→ No current across the capacitor plates

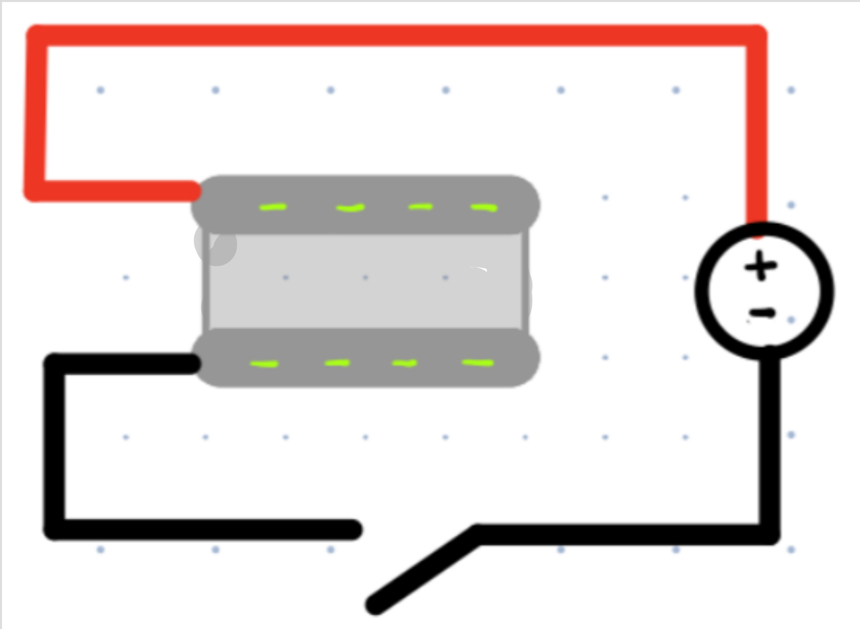
→ Voltage Source provides Energy needed for flow of charges(e^-)

https://www.youtube.com/watch?v=X4EUwTwZ110

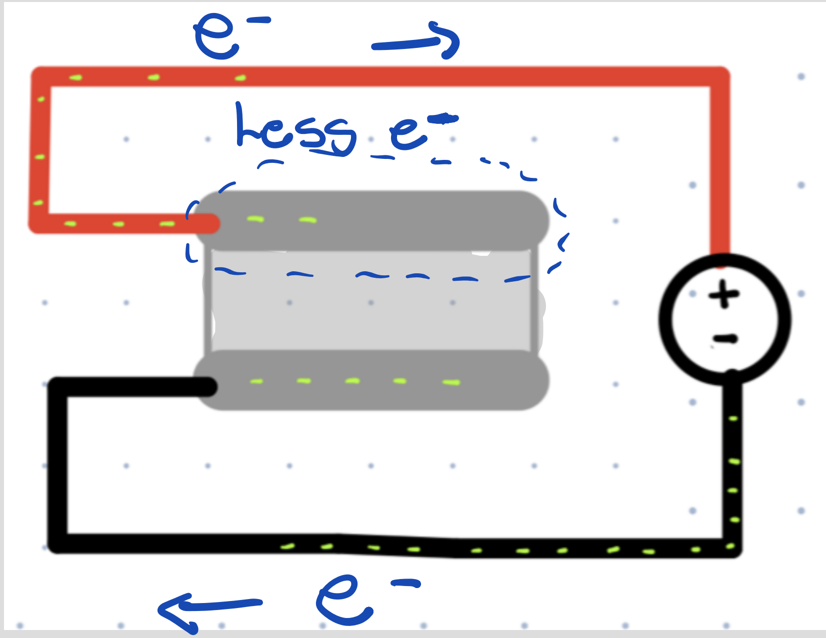
The Physics of a Capacitor

→ Once the switch is ON e^- flow!

t_0



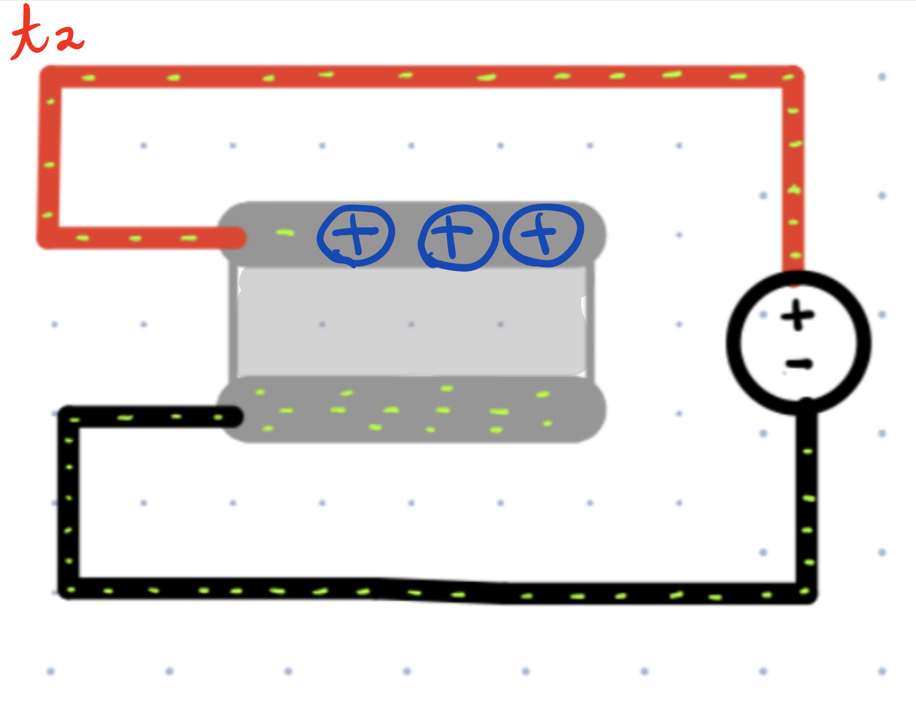
t_1

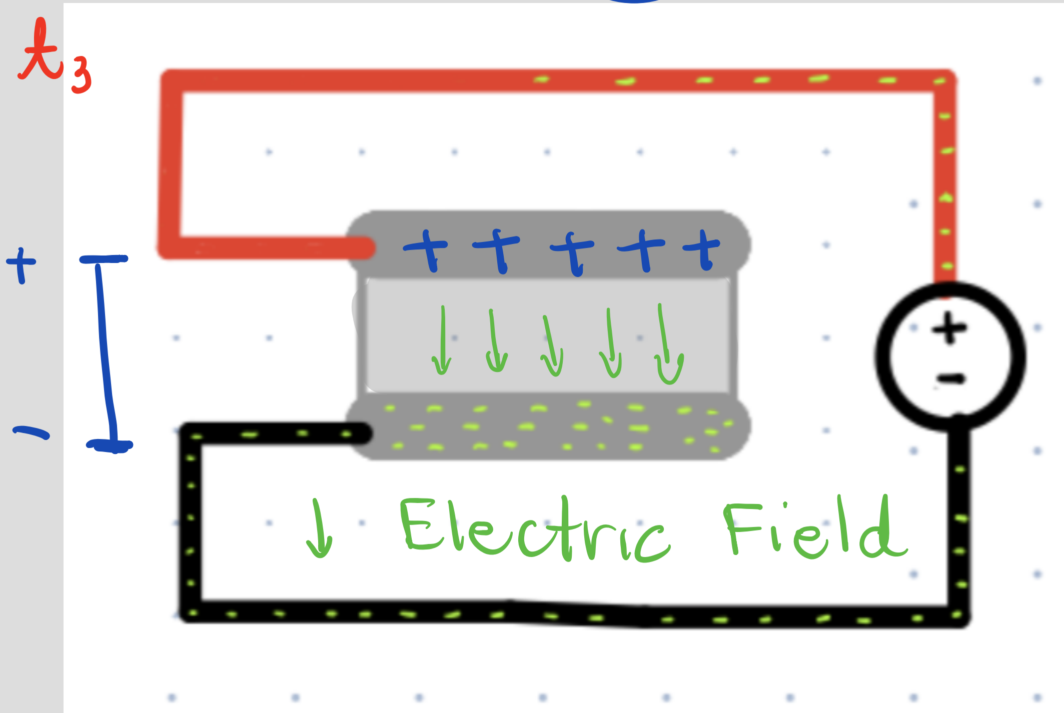


https://www.youtube.com/watch?v=X4EUwTwZ110

The Physics of a Capacitor

Lack of electrons means holes! h^+

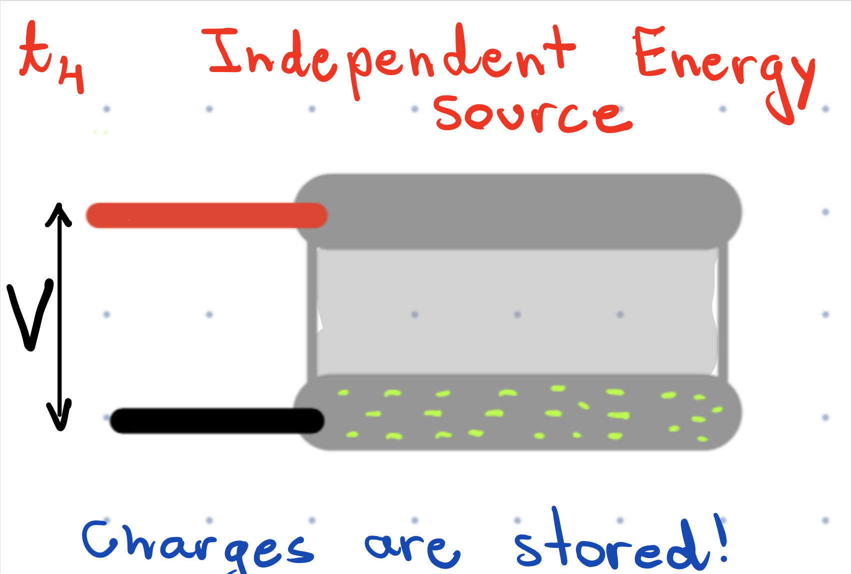


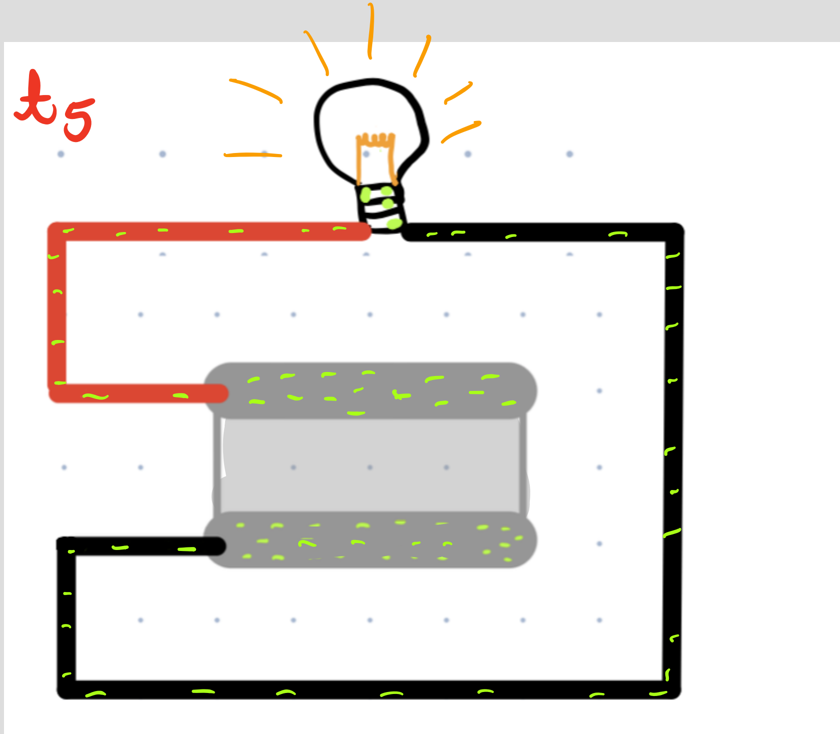


Potential difference between the two plates! } V

https://www.youtube.com/watch?v=X4EUwTwZ110

The Physics of a Capacitor





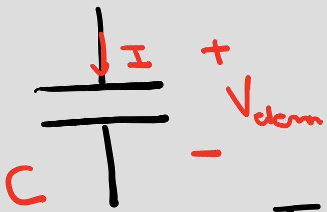
Every Capacitor can be charged up to a fixed Voltage.

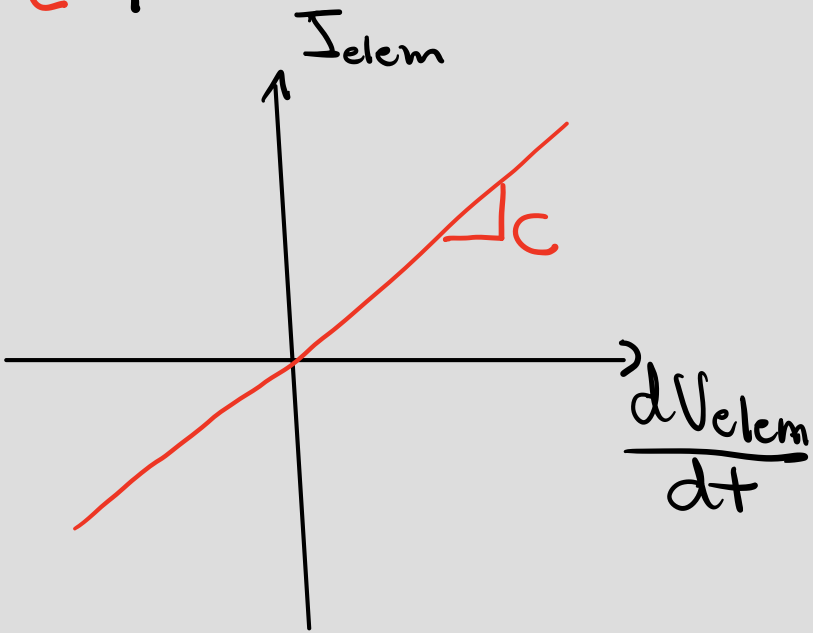
The capacitor will charge a "load" until the charges on the plate are equalized. (No change in V)

https://www.youtube.com/watch?v=X4EUwTwZ110

Circuit Model: IV relationship

Capacitor Symbol





$$Q_{elem} = C \cdot V_{elem}$$

$[C]$ $[F]$ $[V]$

(Farad)

We know : $I_{elem} = \frac{dQ_{elem}}{dt}$

$$I_{elem} = \frac{d}{dt} C \cdot V_{elem}$$

C = constant over time

$$\boxed{I_{elem} = C \cdot \frac{dV_{elem}}{dt}}$$

↳ Can use the same 7-step analysis.

Capacitance

$$C = \varepsilon \frac{A}{d}$$

$$[F] = \left[\frac{F}{m}\right] \left[\frac{m^2}{m}\right]$$

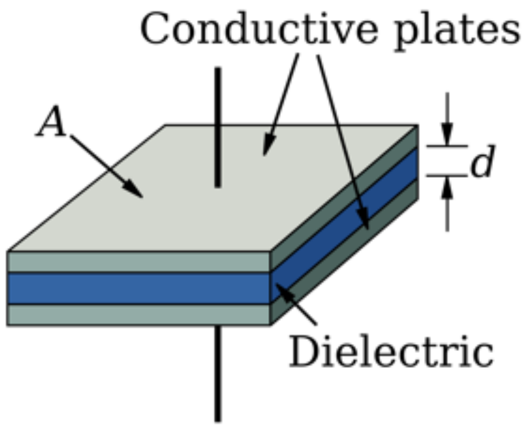
Depends on:

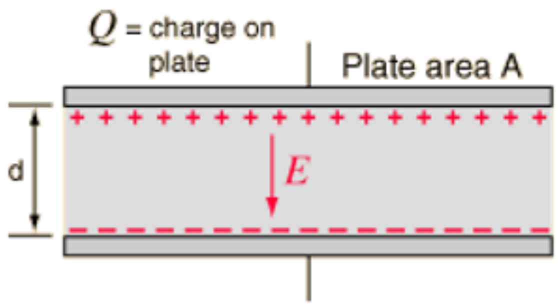
- Materials : ε permitivity

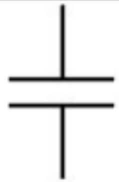
  $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m

  $\varepsilon = \varepsilon_0 \varepsilon_r$

- Geometry of Conductors



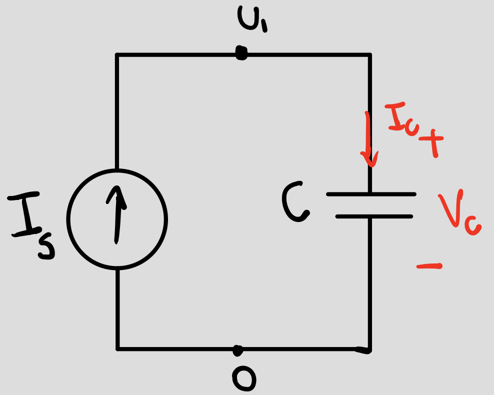


Symbol: 

Capacitance: C Units: Farads [F]

IV equation: $I = C \cdot \frac{dV}{dt}$

Simple Circuit 1



KCL : $I_s = I_c$

Element Def. :

$$I_c = C \cdot \frac{dV_c}{dt}$$

Voltage Def:

$$U_1 - 0 = V_c$$

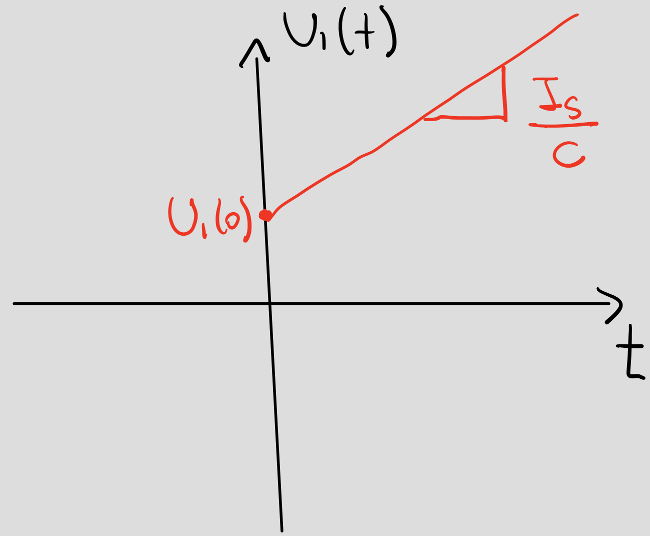
$$\boxed{I_s = C\,\frac{dU_1}{dt}} \times dt$$

$$I_s \cdot dt = C\,dU_1$$

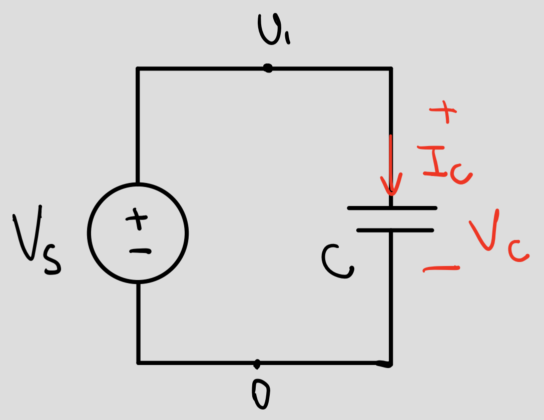
$$\int_0^t I_s\,dt = \int_{U_1(0)}^{U_1(t)} C \cdot dU_1$$

$$I_s\,t = C \cdot \left(U_1(t) - U_1(0)\right)$$

$$U_1(t) = \frac{I_s}{C} \cdot t + U_1(0)$$



Simple Circuit 2



$$\left.\begin{array}{l} V_1 - 0 = V_s \\ V_1 - 0 = V_c \end{array}\right\} \text{ Voltage Def.}$$

$$V_s = V_c$$

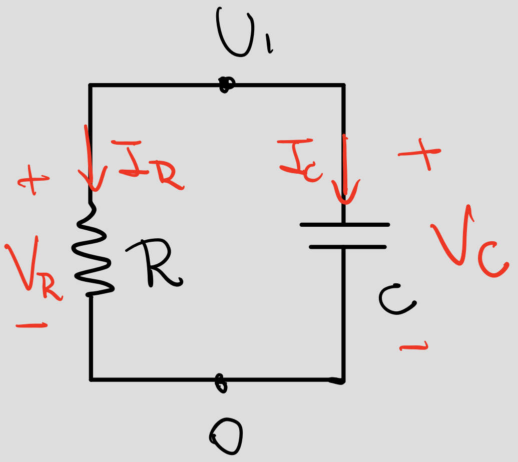
$$I_c = C \frac{dV_c}{dt} \quad (\text{capacitor Def.})$$

$$I_c = C \frac{dV_c}{dt} = C \cdot \frac{d\cancel{V_s}^{\,0}}{\cancel{dt}} = 0$$

Current in a capacitor is zero when a constant Voltage source is across it.

Hint: We like zeros ... they make our lives easier!

Simple Circuit 3



$U_1 = ?$

Steady State: means the Voltages settled.

Looking for U_1 value when V_c = const. (steady-state)

$$I_c = C \frac{\cancel{dV_c}^{\,0}}{\cancel{dt}} = 0$$

KCL: $\cancel{I_c}^{\,0} + I_R = 0$

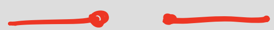
$$I_R = 0$$

Ohm's Law: $V_R = \cancel{I_R}^{\,0} R = 0$

Voltage Def: $U_1 - 0 = \cancel{V_R}^{\,0}$

$$\boxed{U_1 = 0}$$

If current is zero $\Rightarrow$



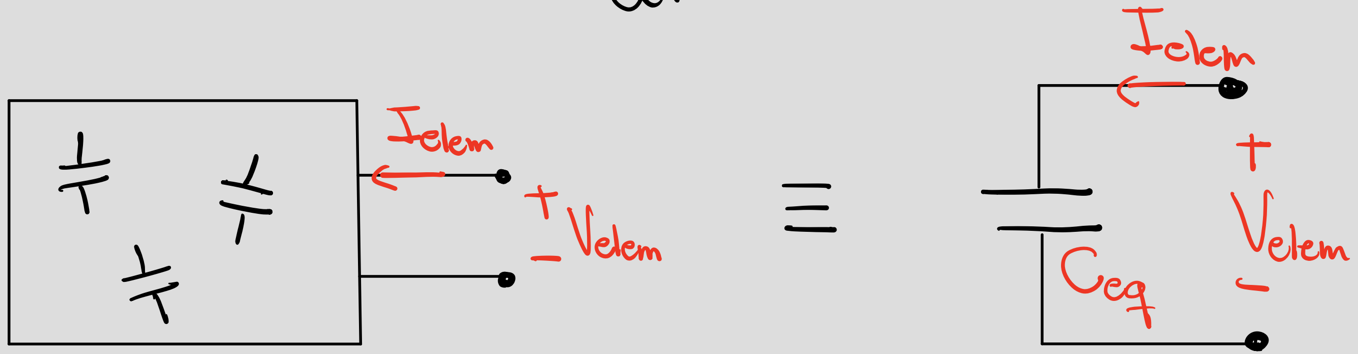
OPEN - CIRCUIT

Equivalent Circuits with Capacitors

* Capacitor-only circuits

~~Step 1 : find V_{th} and I_{No}~~ no source

Step 2 : $C_{eq} = \dfrac{I_{elem}}{\dfrac{dV_{elem}}{dt}}$



Only if

$\left(\text{match } \dfrac{dV_{elem}}{dt} \right)$

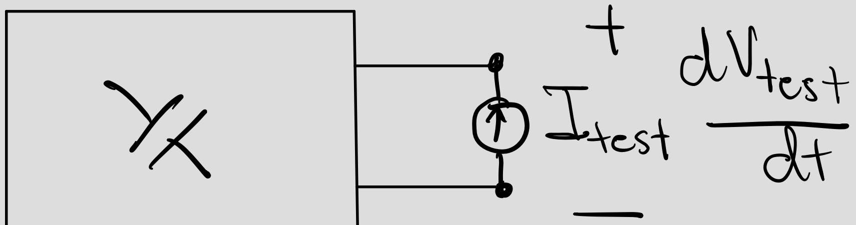
Two Methods:

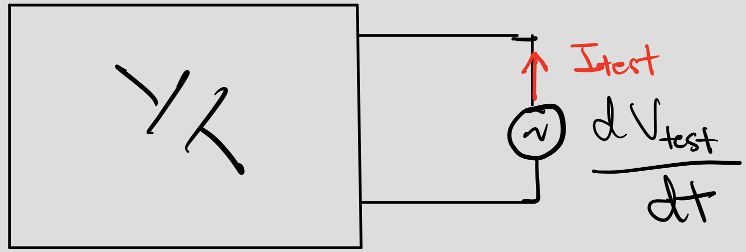
a) Apply I_{test} and measure $\dfrac{dV_{test}}{dt}$

b) Apply $\dfrac{dV_{test}}{dt}$ and measure I_{test}

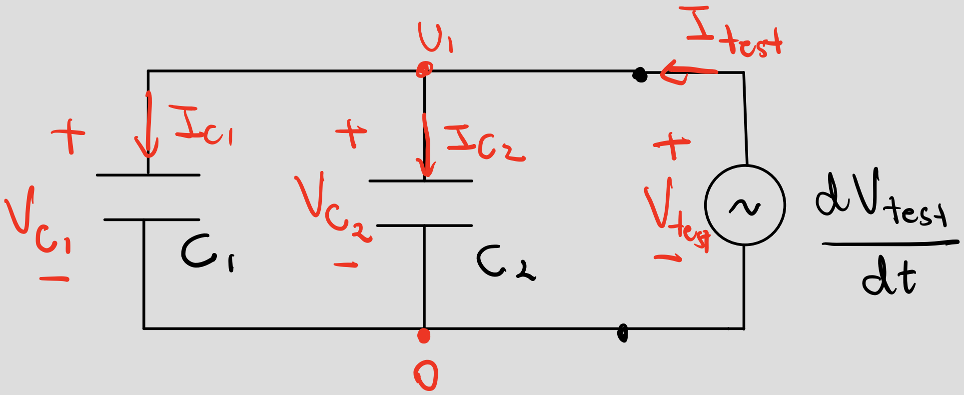
$$= C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

(a)





Example 1



$$V_{C_1} = U_1 \;,\; V_{C_2} = U_1 \quad \text{and}$$

$$U_1 = V_{test}$$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt}$$

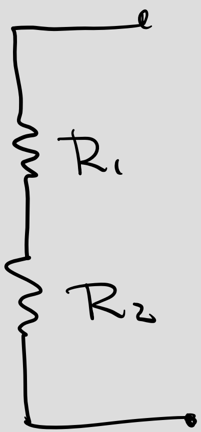
Elem def: $I_{C_1} = C_1 \dfrac{dV_1}{dt} = C_1 \dfrac{dU_1}{dt} = C_1 \dfrac{dV_{test}}{dt}$

Elem def: $I_{C_2} = C_2 \dfrac{dV_2}{dt} = C_2 \dfrac{dU_1}{dt} = C_1 \dfrac{dV_{test}}{dt}$

KCL: $I_{test} = I_{C_1} + I_{C_2} = C_1 \dfrac{dV_{test}}{dt} + C_2 \dfrac{dV_{test}}{dt}$

$$I_{test} = (C_1 + C_2)\,\frac{dV_{test}}{dt}$$

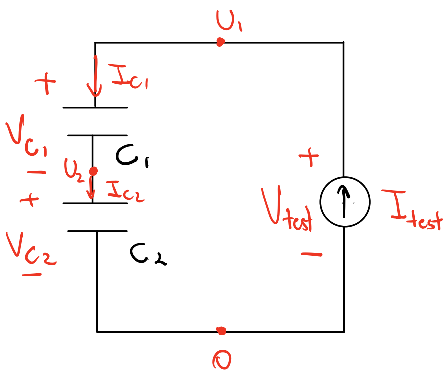
$$\boxed{C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2}$$



$$\equiv R_{eq} = R_1 + R_2$$

Series

Example 2 : "Capacitors in series"



KCL : $I_{C_1} = I_{C_2} = I_{test}$

Elements :

$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

Voltage Def :

$$V_{C2} = U_2 - 0$$

$$V_{C_1} = U_1 - U_2$$

$$V_{test} = U_1 - 0$$

For V_{C2}:

$$I_{C_2} = C_2 \frac{dV_{C2}}{dt}$$

$$I_{test} = C_2 \frac{dU_2}{dt} \equiv \frac{dU_2}{dt} = \frac{I_{test}}{C_2}$$

For V_{C_1}

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$\frac{dV_1}{dt} = \frac{I_C}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1}$$

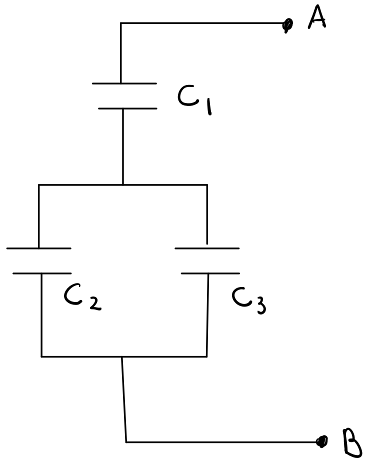
$$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt} = I_{test}\left(\frac{1}{C_2} + \frac{1}{C_1}\right)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \| C_2$$

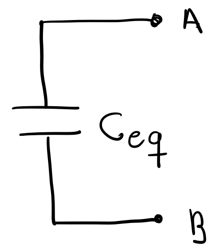
$C_{eq} = C_1 \| C_2$ ($\|$ – parallel mathematical operator)

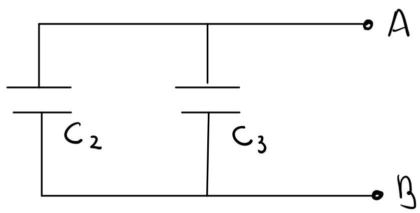
Example 3



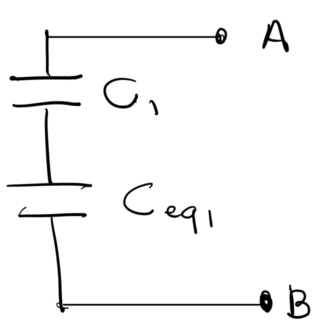
$$C_{eq} = C_1 \parallel (C_2 + C_3)$$

$\equiv$





$\Rightarrow$ $C_{eq_1} = C_2 + C_3$



$$C_{eq} = C_1 \parallel C_{eq_1}$$