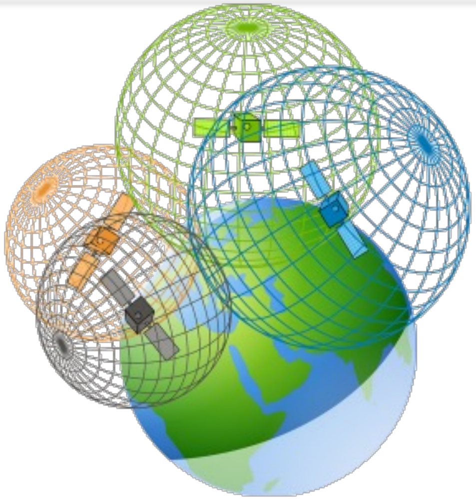


Welcome to EECS 16A!

Designing Information Devices and Systems I

Ana Claudia Arias and Miki Lustig

Fall 2022

Module 2
Lecture 7
Capacitors (Note 16)

Voltage Divider – a Poem

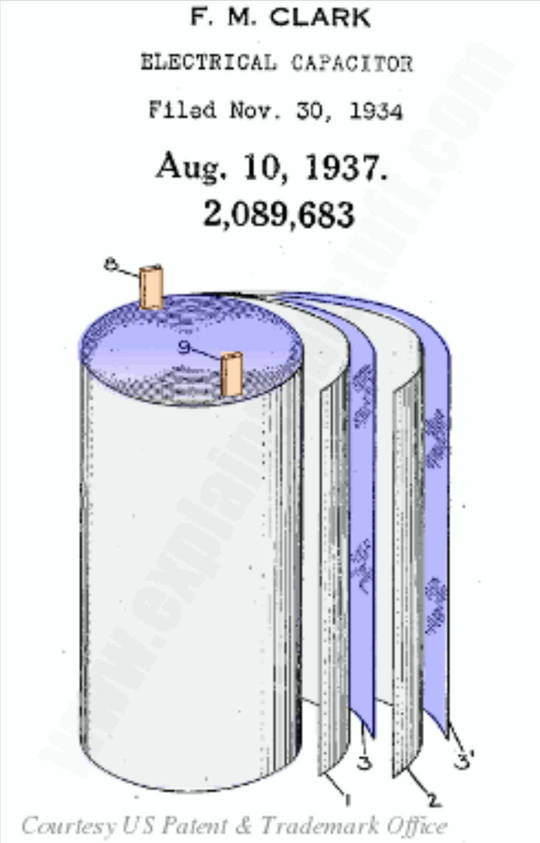
Voltage divider?

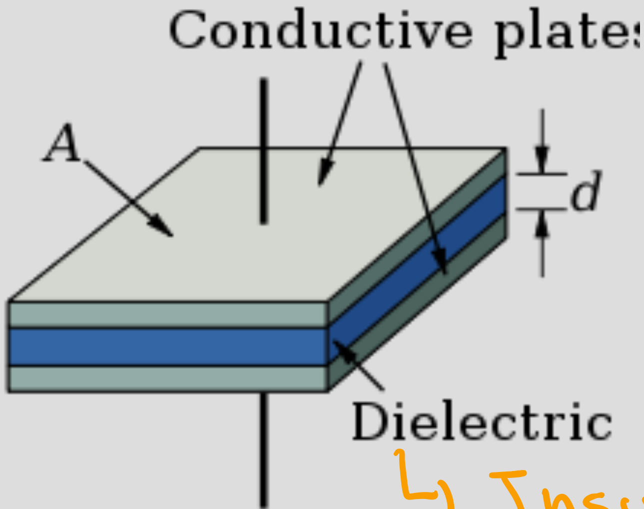
Electrons like their space.
They force others away.
Ah. Such potential!
Overcomes resistance.
But potential fades.

Prof. Satish Rao
03.22.22

Now, Capacitors!

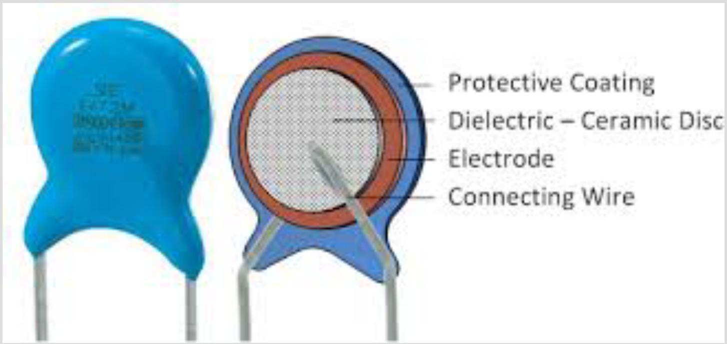
- Charge storage device (like a 'bucket' for charge)

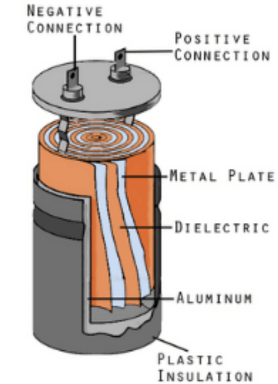


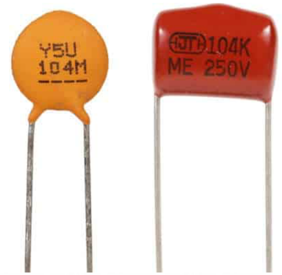


↳ Insulator

↳ Higher Energy is needed to move charge.

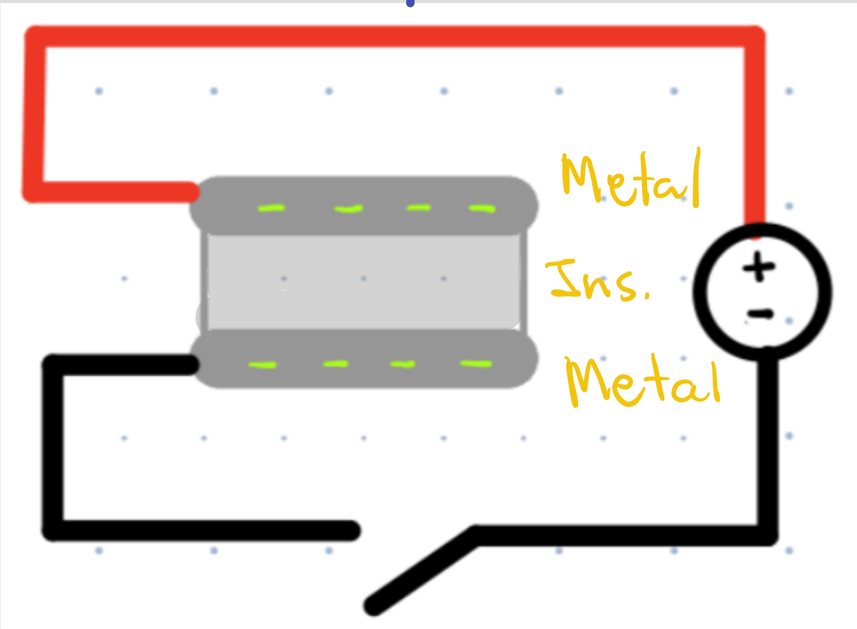






The Physics of a Capacitor

* Energy is needed to move charge.



e^-

→ No current across the capacitor plates

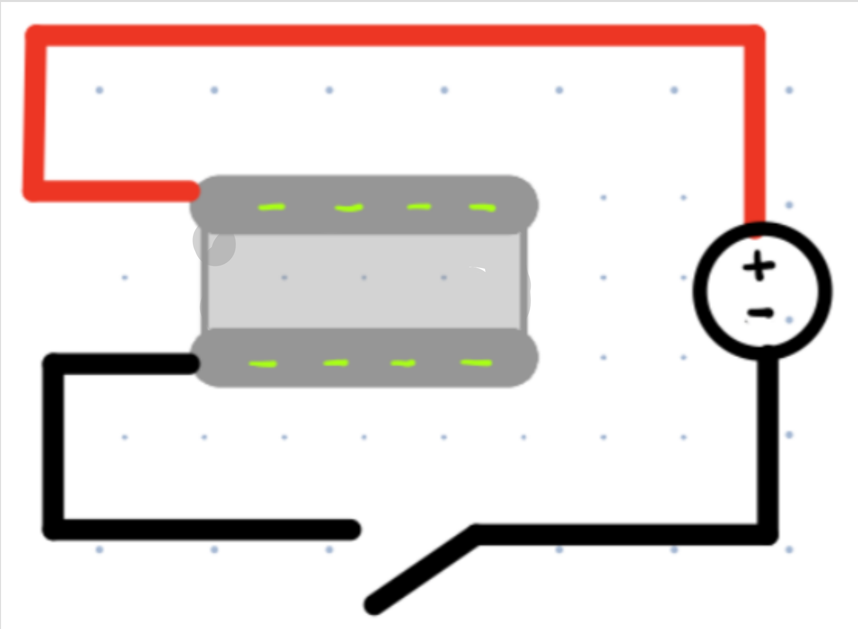
→ Voltage Source provides Energy needed for flow of charges(e^-)

https://www.youtube.com/watch?v=X4EUwTwZ110

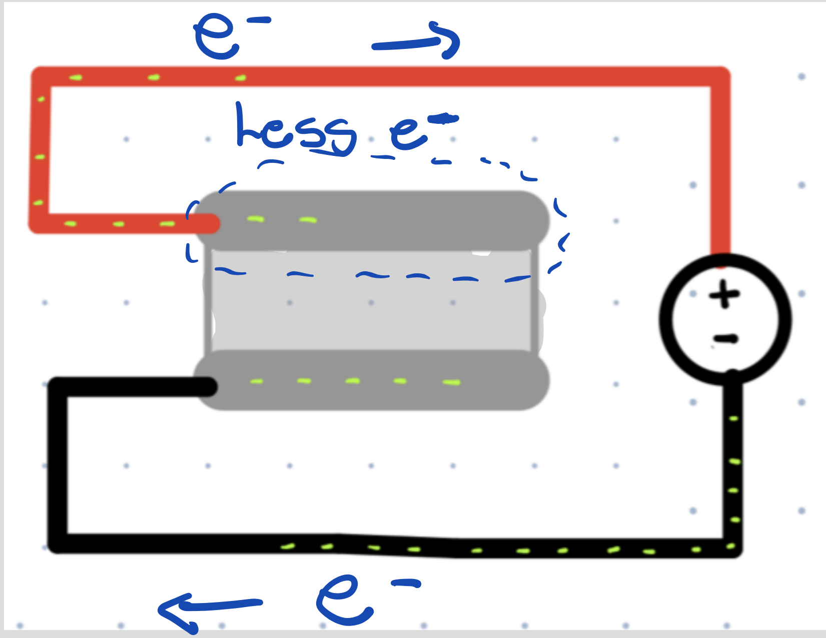
The Physics of a Capacitor

→ Once the switch is ON e^- flow!

t_0



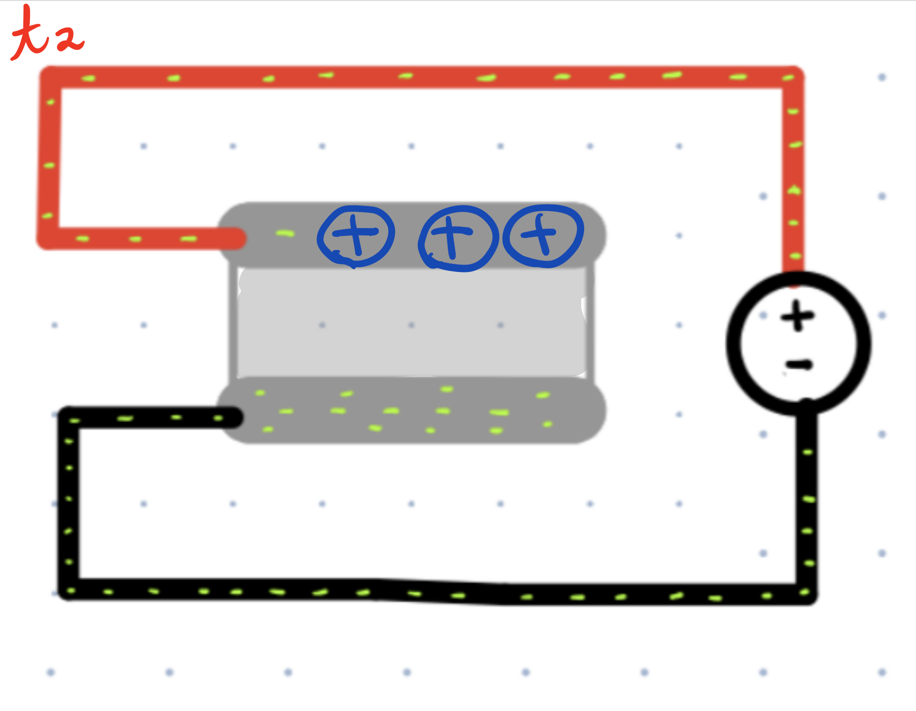
t_1

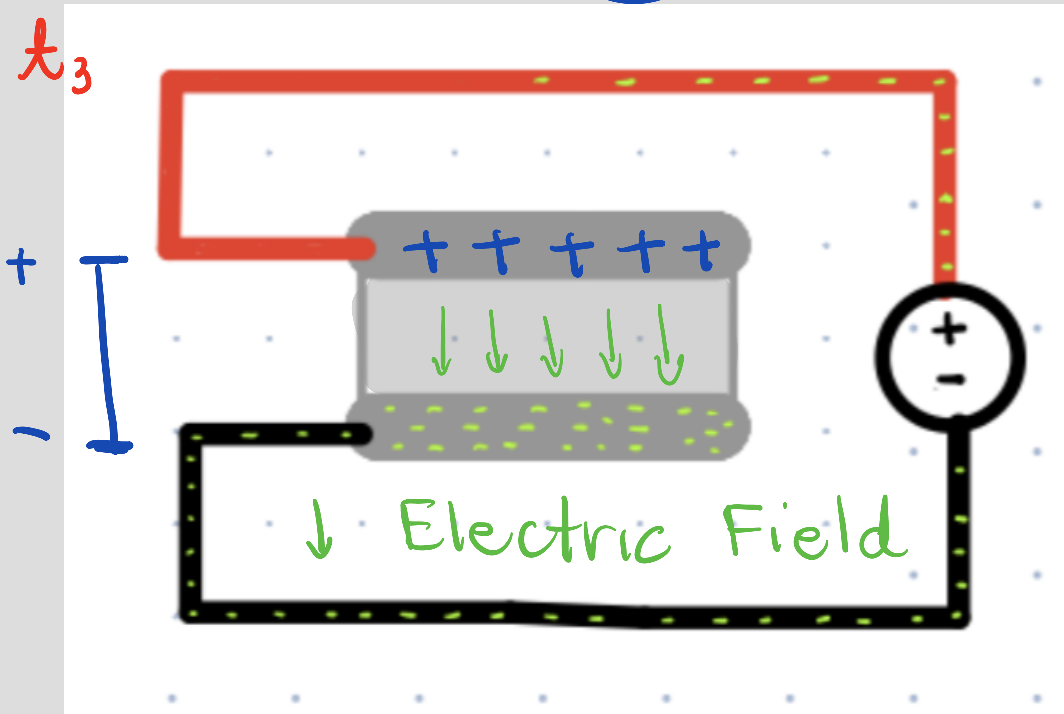


https://www.youtube.com/watch?v=X4EUwTwZ110

The Physics of a Capacitor

Lack of electrons means holes! h^+

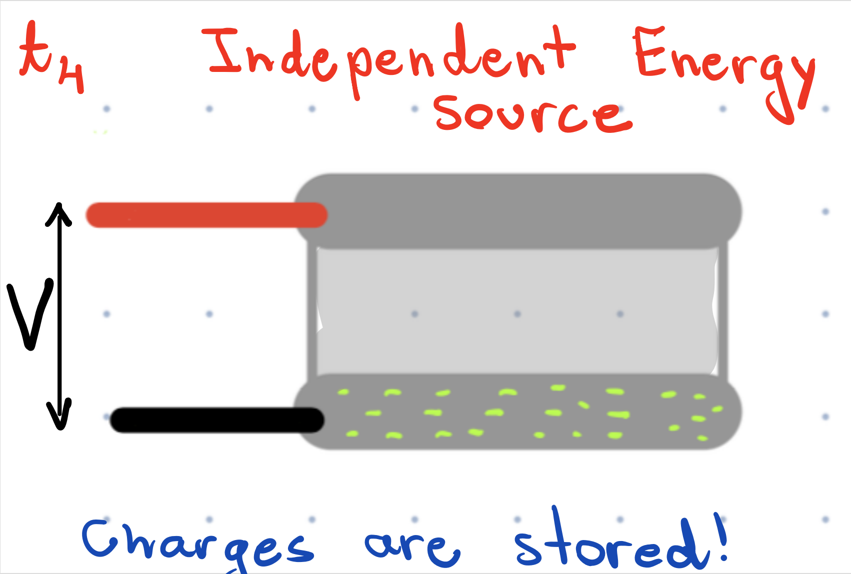


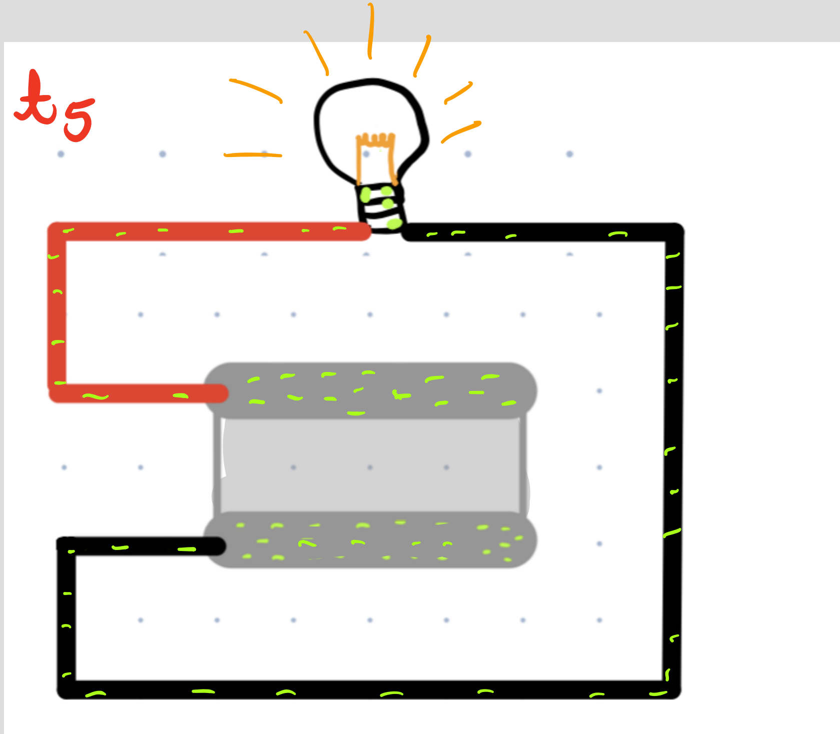


Potential difference between the two plates! $\}$ V

https://www.youtube.com/watch?v=X4EUwTwZ110

The Physics of a Capacitor





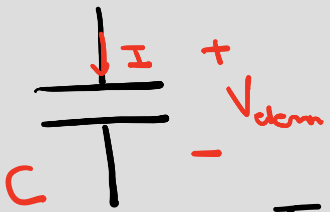
Every Capacitor can be charged up to a fixed Voltage.

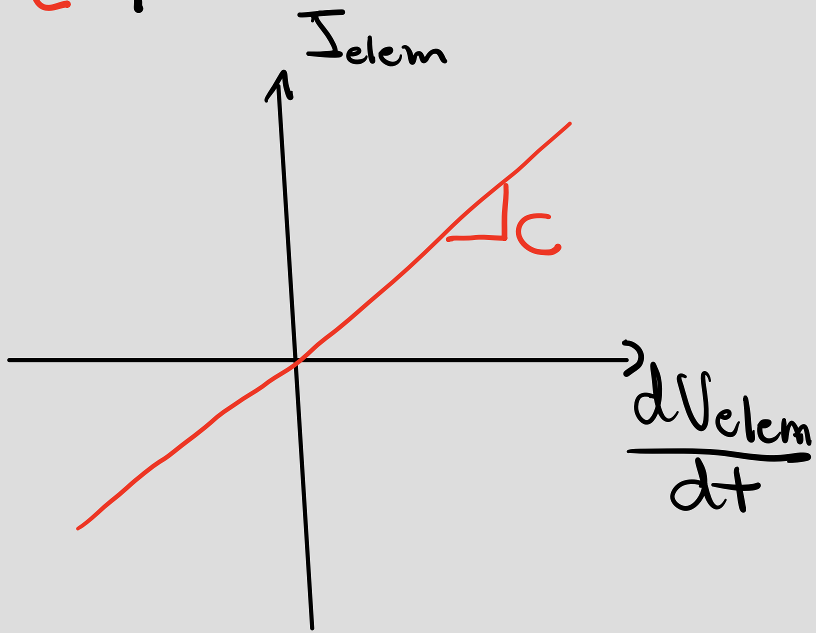
The capacitor will charge a "load" until the charges on the plate are equalized. (No change in V)

https://www.youtube.com/watch?v=X4EUwTwZ110

Circuit Model: IV relationship

Capacitor Symbol





$$Q_{elem} = C \cdot V_{elem}$$

[C]　[F]　[V]

(Farad)

We know : $I_{elem} = \frac{d\,Q_{elem}}{dt}$

$$I_{elem} = \frac{d}{dt} C \cdot V_{elem}$$

C = constant over time

$$\boxed{I_{elem} = C \cdot \frac{d\,V_{elem}}{dt}}$$

↳ Can use the same 7-step analysis.

Capacitance

$$C = \varepsilon \frac{A}{d}$$

$$[F] = \left[\frac{F}{m}\right]\left[\frac{m^2}{m}\right]$$

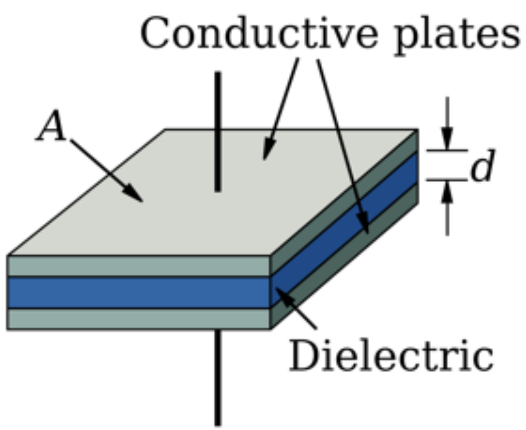
Depends on:

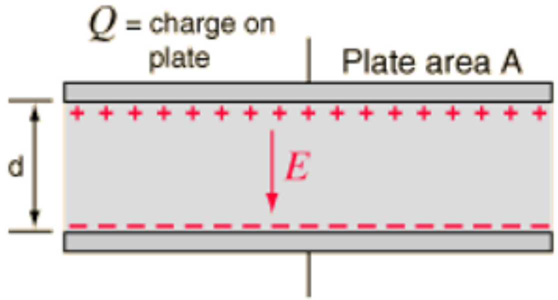
- Materials : ε permitivity

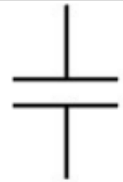
  $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m

  $\varepsilon = \varepsilon_0 \varepsilon_r$

- Geometry of Conductors



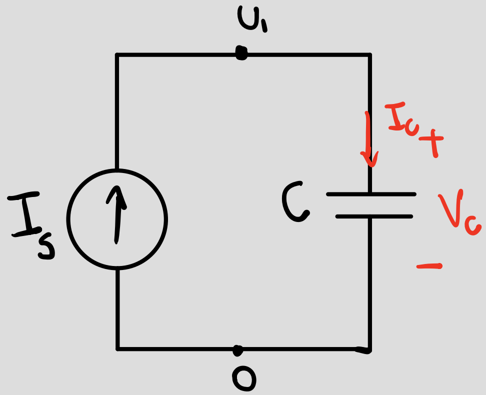


Symbol: 

Capacitance: C Units: Farads [F]

IV equation: $I = C \cdot \frac{dV}{dt}$

Simple Circuit 1



KCL : $\underline{I_s = I_c}$

Element Def.:

$$\underline{I_c} = C \cdot \frac{dV_c}{dt}$$

Voltage Def:

$$U_1 - 0 = V_c$$

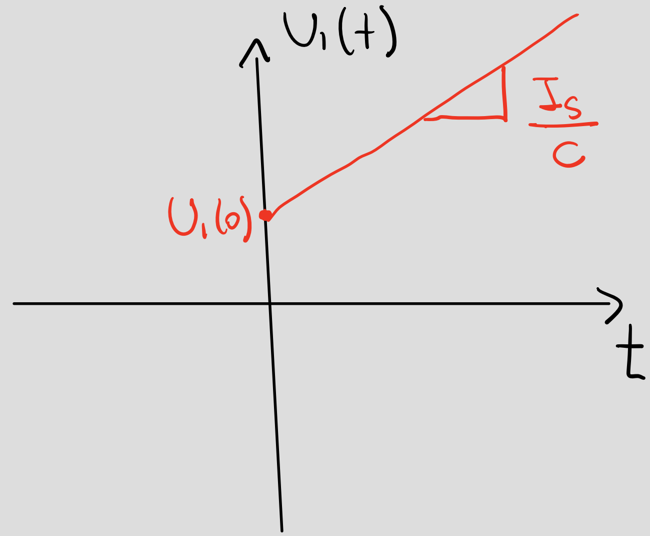
$$\boxed{I_s = C\,\frac{dU_1}{dt}} \times dt$$

$$I_s \cdot dt = C\, dU_1$$

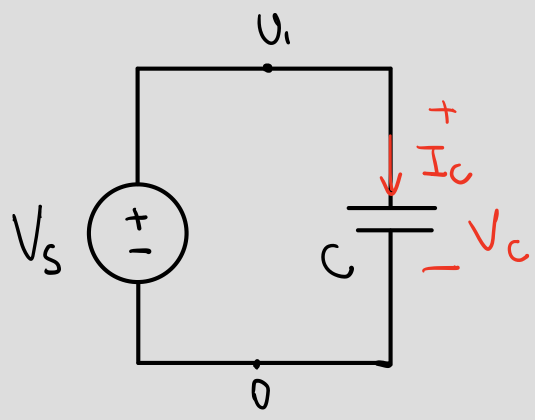
$$\int_0^t I_s dt = \int_{U_1(0)}^{U_1(t)} C \cdot dU_1$$

$$I_s t = C \cdot \left(U_1(t) - U_1(0)\right)$$

$$U_1(t) = \frac{I_s}{C} \cdot t + U_1(0)$$



Simple Circuit 2



$$\left.\begin{array}{l} v_1 - 0 = V_s \\ v_1 - 0 = V_c \end{array}\right\} \text{Voltage Def.}$$

$$V_s = V_c$$

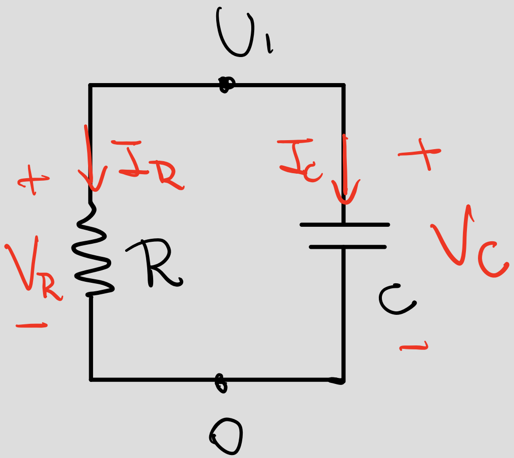
$$I_c = C \frac{dV_c}{dt} \quad (\text{capacitor Def.})$$

$$I_c = C \frac{dV_c}{dt} = C \cdot \frac{\cancel{dV_s}^{\,0}}{\cancel{dt}} = 0$$

Current in a capacitor is zero when a constant Voltage source is across it.

Hint: We like zeros ... they make our lives easier!

Simple Circuit 3



$U_1 = ?$

Steady State: means the Voltages settled.

Looking for U_1 value when $V_c = \text{const.}$ (steady-state)

$$I_c = C \cancel{\frac{dV_c}{dt}}^{\,0} = 0$$

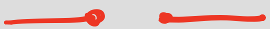
KCL: $\cancel{I_c}^{\,0} + I_R = 0$

$$I_R = 0$$

Ohm's Law: $V_R = \cancel{I_R}^{\,0} R = 0$

Voltage Def: $U_1 - 0 = \cancel{V_R}^{\,0}$

$$\boxed{U_1 = 0}$$

If current is zero $\Rightarrow$ 

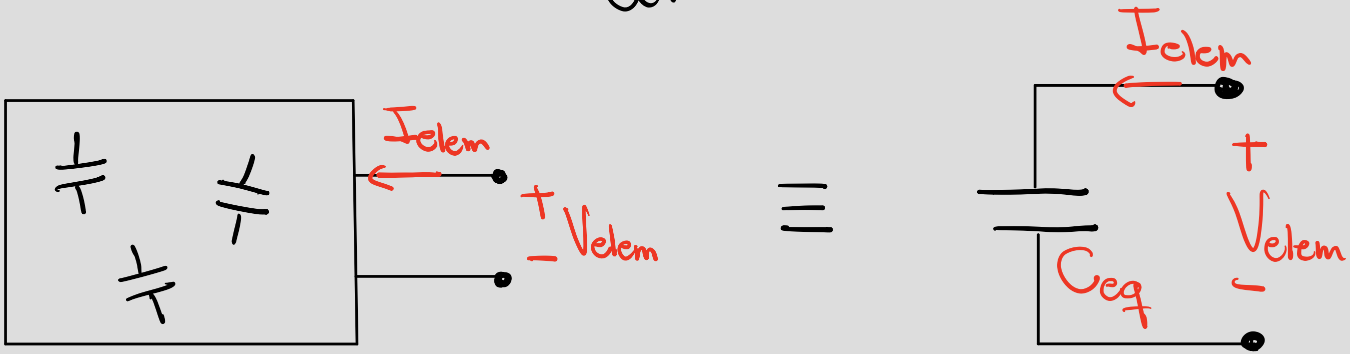
OPEN - CIRCUIT

Equivalent Circuits with Capacitors

* Capacitor - only circuits

~~Step 1 : find V_{th} and I_{No}~~ no source

Step 2 : $C_{eq} = \dfrac{I_{elem}}{\frac{d V_{elem}}{dt}}$



Only if

$\left(\text{match } \dfrac{dV_{elem}}{dt} \right)$

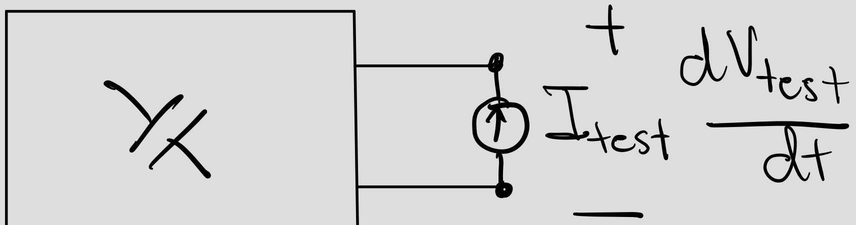
Two Methods:

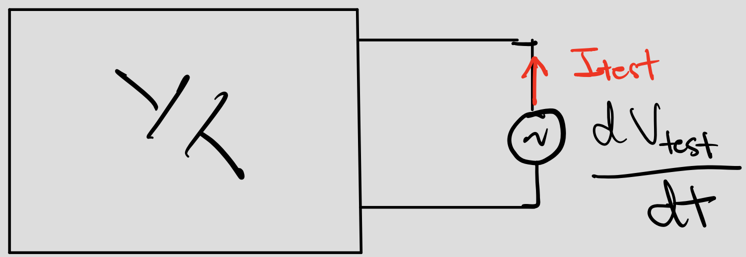
a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}

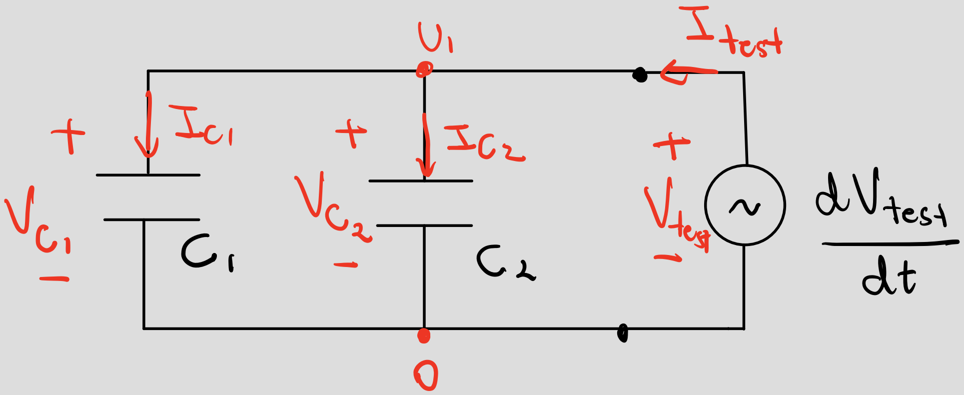
$$= C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

(a)





Example 1



$V_{C_1} = U_1$, $V_{C_2} = U_1$ and

$U_1 = V_{test}$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt}$$

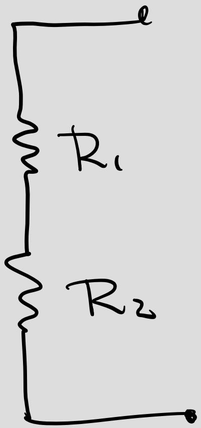
Elem def: $I_{C_1} = C_1 \frac{dV_1}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{test}}{dt}$

Elem def: $I_{C_2} = C_2 \frac{dV_2}{dt} = C_2 \frac{dU_1}{dt} = C_1 \frac{dV_{test}}{dt}$

KCL : $I_{test} = I_{C_1} + I_{C_2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt}$

$$I_{test} = (C_1 + C_2)\frac{dV_{test}}{dt}$$

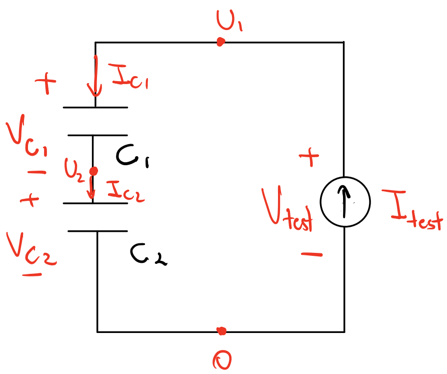
$$\boxed{C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2}$$



$$\equiv R_{eq} = R_1 + R_2$$

Series

Example 2 : "Capacitors in series"



KCL : $I_{C_1} = I_{C_2} = I_{test}$

Elements :

$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

Voltage Def :

$$V_{C_2} = U_2 - 0$$

$$V_{C_1} = U_1 - U_2$$

$$V_{test} = U_1 - 0$$

For V_{C_2}:

$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$I_{test} = C_2 \frac{dU_2}{dt} \Rightarrow \frac{dU_2}{dt} = \frac{I_{test}}{C_2}$$

For V_{C_1}

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$\frac{dV_1}{dt} = \frac{I_C}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1}$$

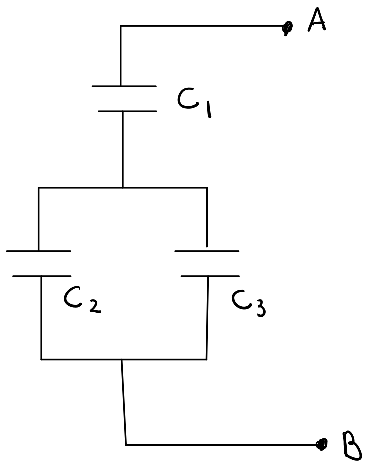
$$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt} = I_{test}\left(\frac{1}{C_2} + \frac{1}{C_1}\right)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \parallel C_2$$

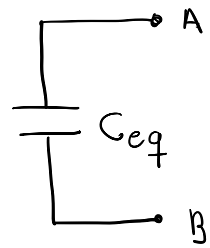
$C_{eq} = C_1 \parallel C_2$ ($\parallel$ – parallel mathematical operator)

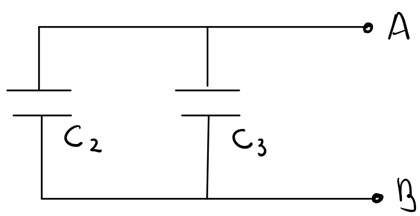
Example 3



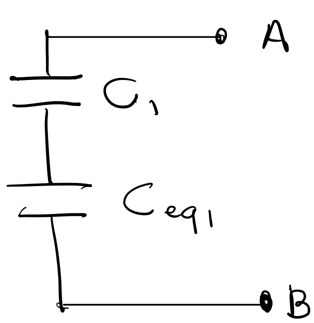
$$C_{eq} = C_1 \,||\, (C_2 + C_3)$$

$\equiv$



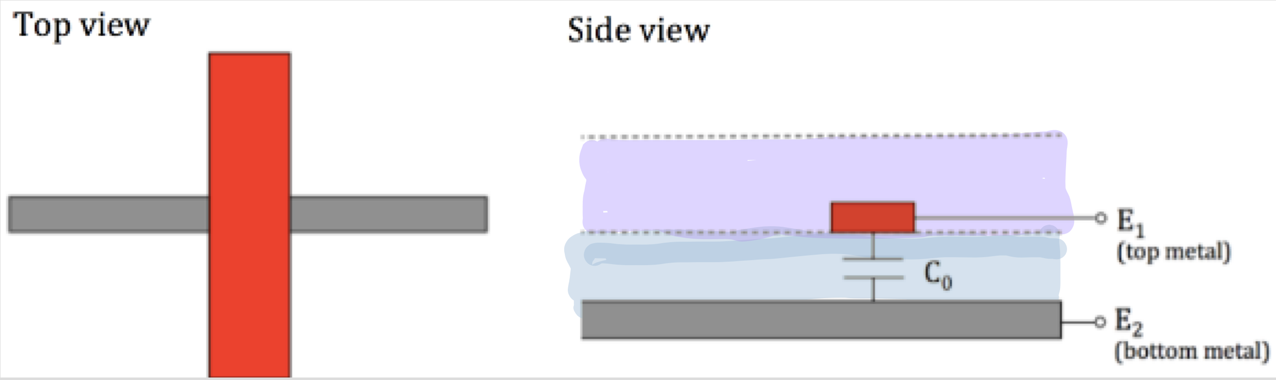


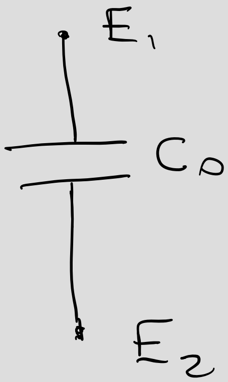
$$\Rightarrow \quad C_{eq_1} = C_2 + C_3$$



$$C_{eq} = C_1 \,||\, C_{eq_1}$$

Capacitive Touchscreen – Model without touch

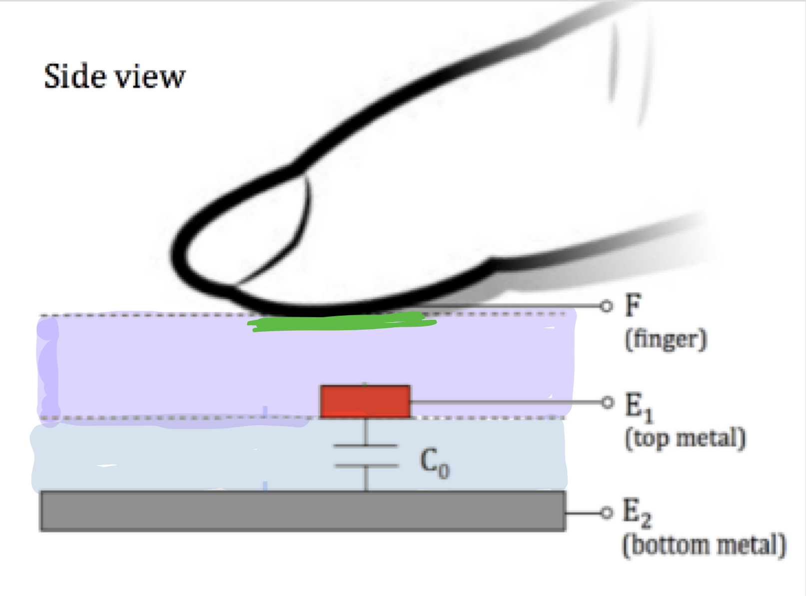


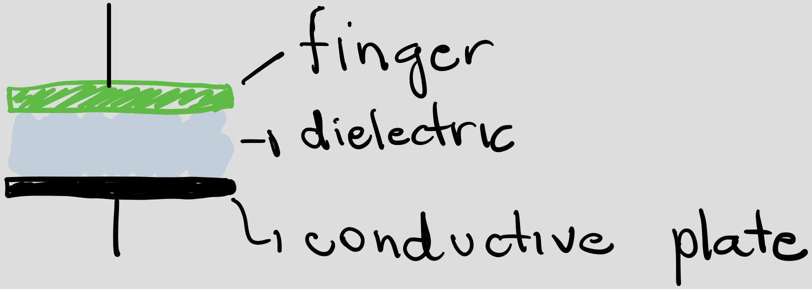


$$C_0 = \varepsilon \cdot \frac{A}{d}$$

Capacitive Touchscreen – Model with touch

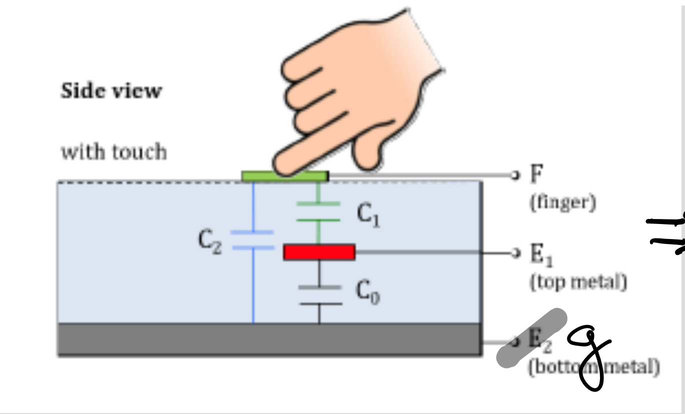
When there is a touch, it makes a capacitor!



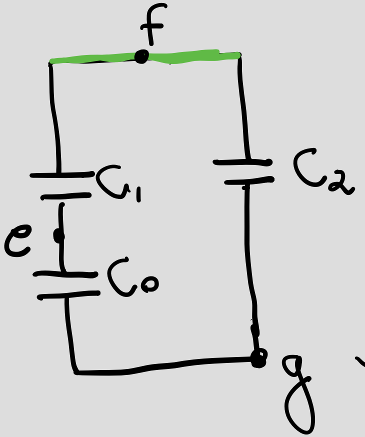


Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture

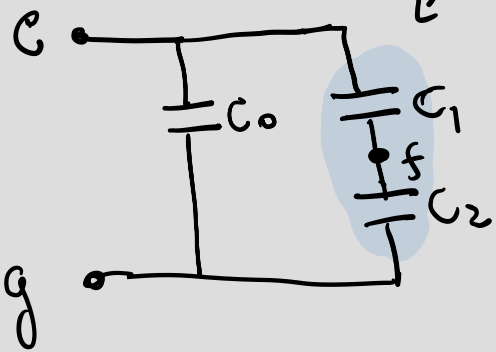


⇒ circuit model

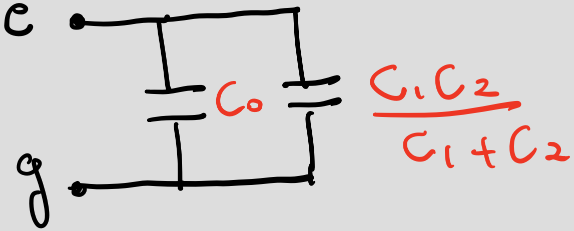


We only have acess to nodes e and g, not f

Redraw to focus on terminals (nodes) e and g

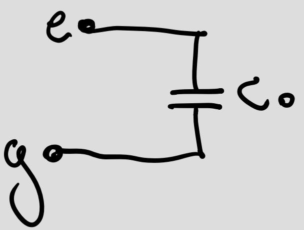


Equivalent capacitance for C_1 in series with C_2

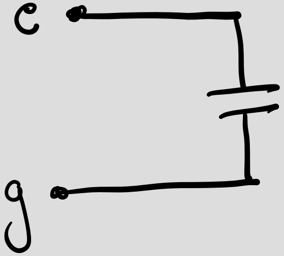


⇒ Equiv. Capacitance for C_0 in parallel to $\frac{C_1C_2}{C_1+C_2}$

when no touch:



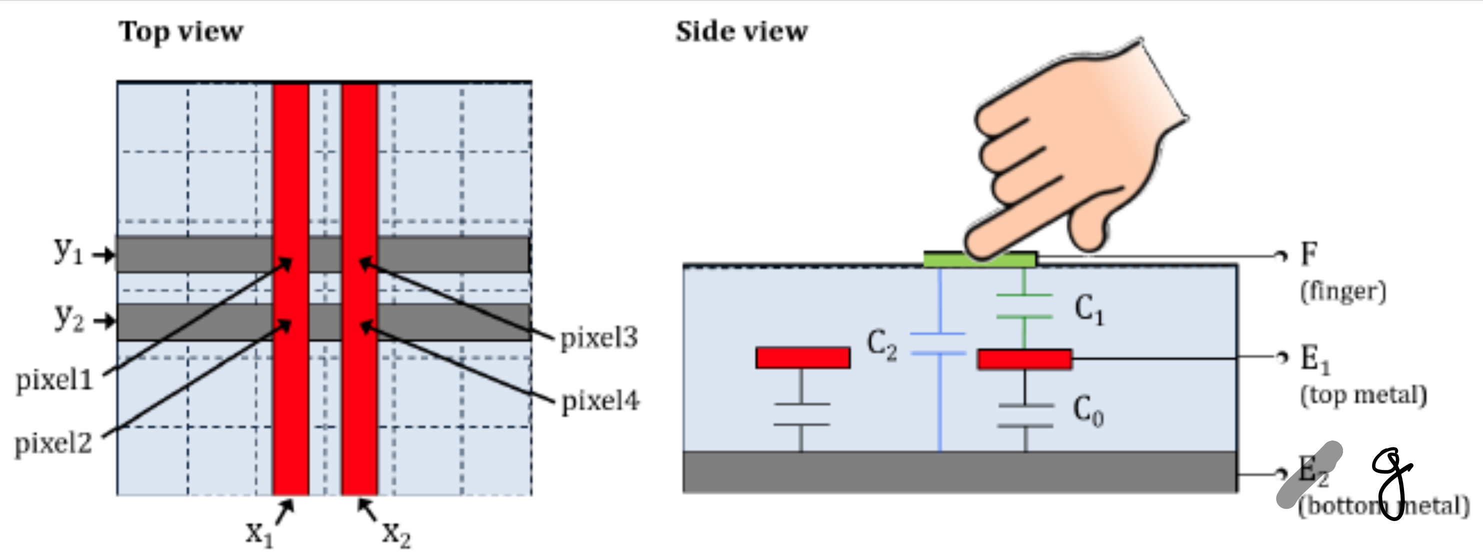
with touch:



$C_0 + \frac{C_1C_2}{C_1+C_2}$

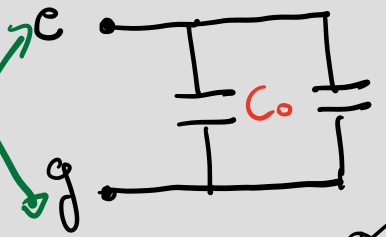
Extra Capacitance due to touch!

2D View – How do we measure Capacitance?



Problem: We don't have a capaci-meter!

We want to measure Capacitance here



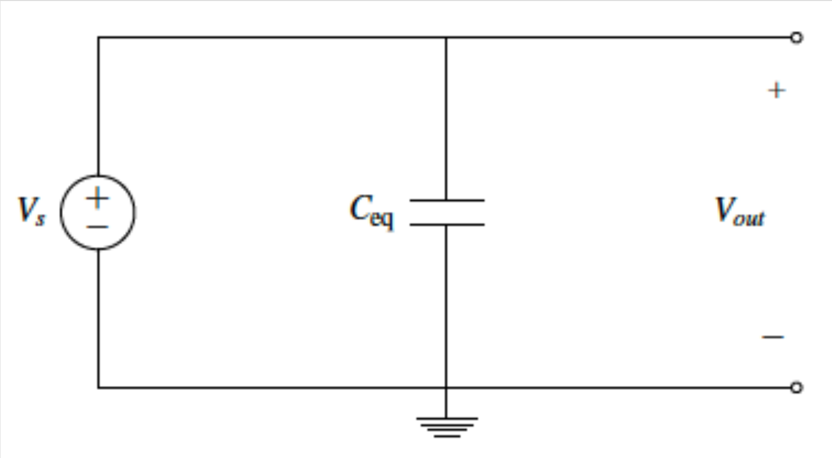
$$\frac{C_1 C_2}{C_1 + C_2} = C_\Delta$$

(change)

This capacitor goes away with no touch

We will try ideas to get to a final model.

Measuring Capacitance Models – Attempt #1

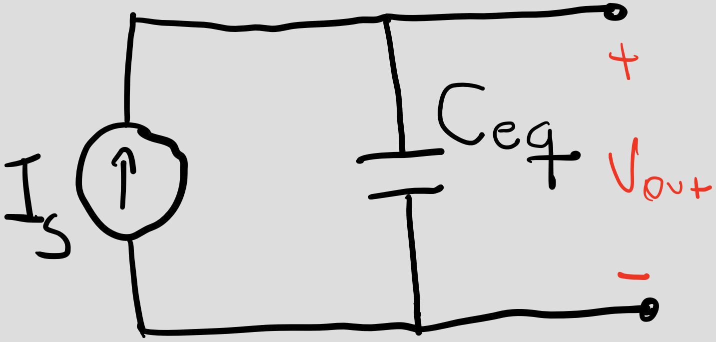


If there is touch: $V_c = V_s$

If there is no touch: $V_c = V_s$

Vout does not change!

Bad idea!



Very hard to make current sources!

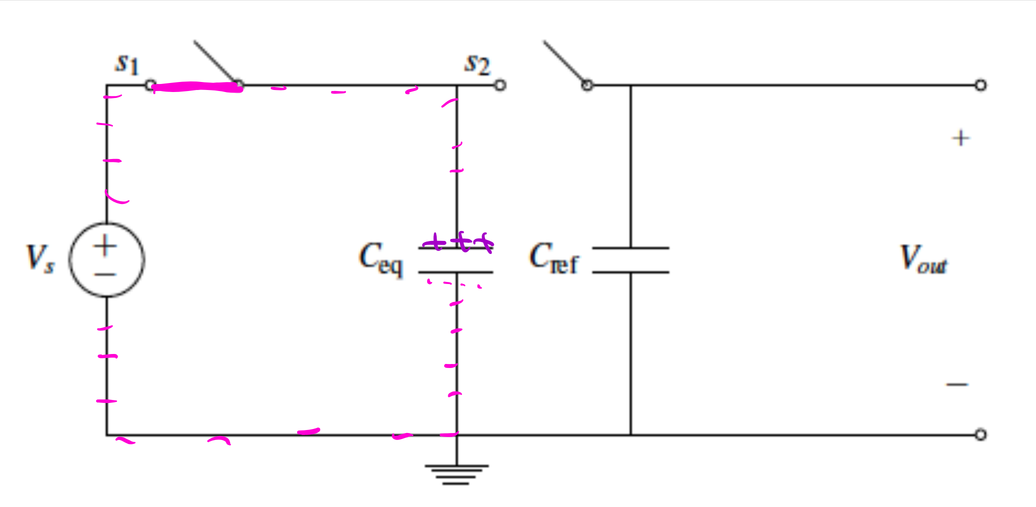
Assume starts out discharged:

$$V_{out}(t=0) = 0$$

$$I_s = C_{eq}\frac{dV_{out}(t)}{dt} \longrightarrow V_{out}(t) = \int_0^t \frac{I_s}{C_{eq}}\, dt$$

$$V_{out} = \frac{I_s \cdot t}{C_{e}} \Rightarrow C_{eq} = \frac{I_s}{\frac{dV_c(t)}{dt}}$$

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor



- 1st _ Close both switches

We want to charge Cref and measure Vout as Cref discharges.

If both closed – nothing happens.!. Attempt #1

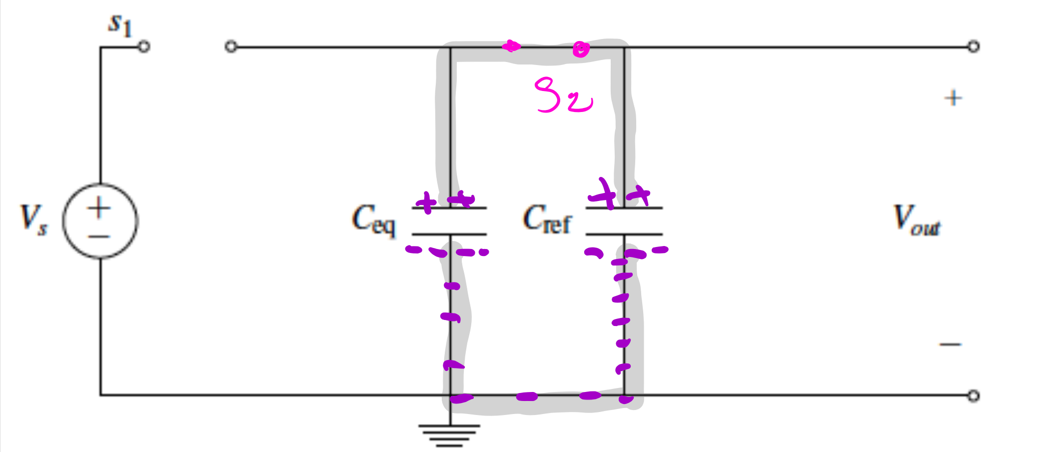
Phase 1 : Close S_1 ; Open S_2

Ceq is charging

$q = C_{eq} \cdot V_s$ accumulates on capacitor plates.

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Phase 2: close S_2, open S_1



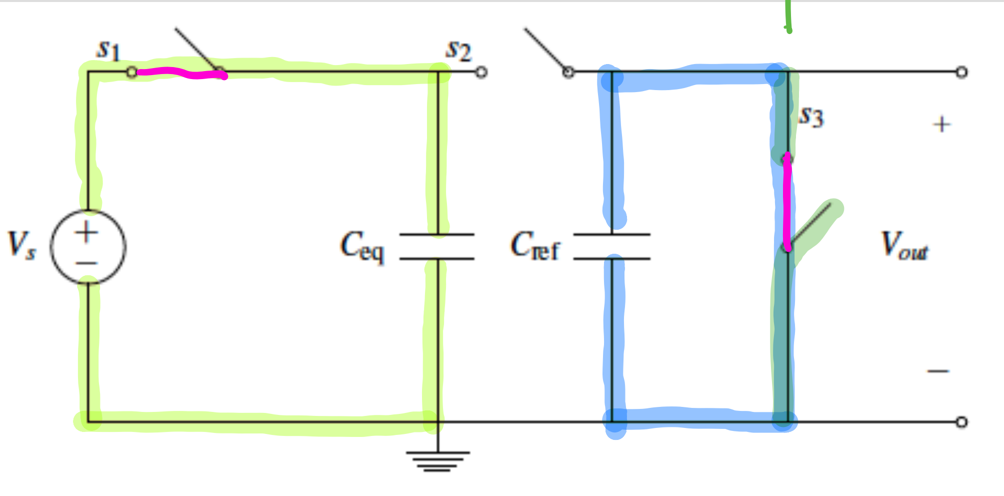
- There is a path for charge to move.
- Ceq can provide the energy needed for current.

Charge will split between Ceq and Cref

"charge sharing"

So close! But we don't know initial Cref.

Measuring Capacitance Models – Attempt #3 – known initial condition



Use S_3 to discharge C_{ref} so we know $\boxed{C_{ref} = 0}$

Phase 1: S_3 closed, S_1 closed, S_2 open

C_{req} discharges $V_{out} \to 0$

$q = C_{req} \cdot V_{out} = 0$ ✓

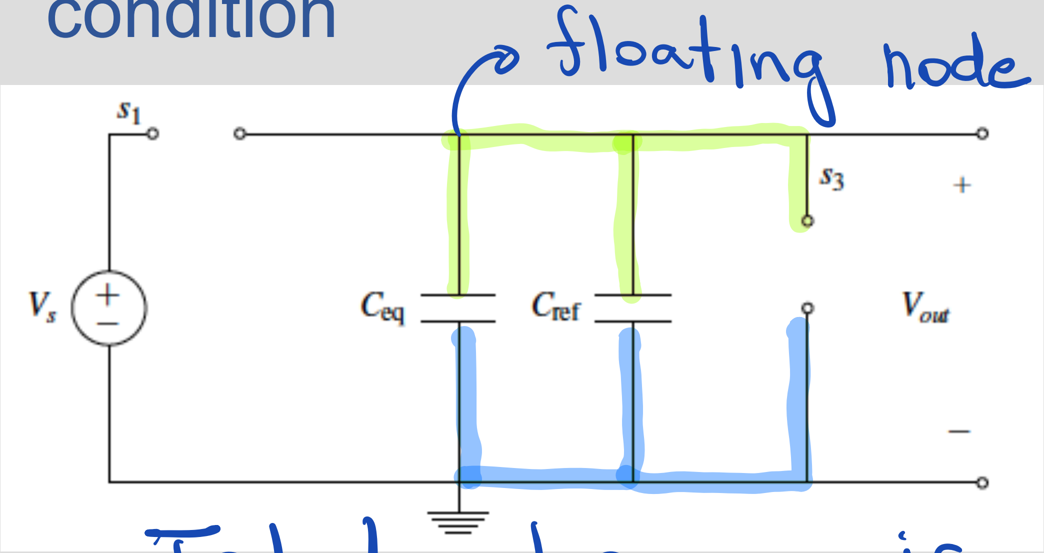
C_{eq} charges

$q = C_{eq} \cdot V_s$

Phase 2: S_1 open, S_2 closed, S_3 open

C_{eq} - charged

Measuring Capacitance Models – Attempt #3 – known initial condition



Voltage across Ceq : Vout
Voltage across Cref : Vout
charge in Ceq: $q_1 = C_{eq} \cdot V_{out}$
charge in Cref: $q_2 = C_{ref} \cdot V_{out}$

Total charge is conserved!

$$q(\text{phase } 1) = q(\text{phase } 2)$$

$$C_{eq} \cdot V_s = C_{eq} V_{out} + C_{ref} \cdot V_{out}$$

$$V_{out} = \frac{C_{eq} V_s}{C_{eq} + C_{ref}}$$

⇒ Vout changes when Ceq changes!!!