Voltage Divider – a Poem

Voltage divider?

Electrons like their space.
They force others away.
Ah. Such potential!
Overcomes resistance.
But potential fades.

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Now, Capacitors!

- Charge storage device (like a ‘bucket’ for charge)
The Physics of a Capacitor

* Energy is needed to move charge.
  
  → No current across the capacitor plates.
  
  → Voltage Source provides Energy needed for slow OS charges (e-)

https://www.youtube.com/watch?v=X4EUwTwZ110
The Physics of a Capacitor

Once the switch is ON e⁻ flow!

https://www.youtube.com/watch?v=X4EUwTwZ110
The Physics of a Capacitor

Lack of electrons means holes! $h^+$

Potential difference between the two plates!
The Physics of a Capacitor

Every Capacitor can be charged up to a fixed Voltage.

The capacitor will charge a "load" until the charges on the plate are equalized. (No change in V)

https://www.youtube.com/watch?v=X4EUwTwZ110
Circuit Model: IV relationship

Capacitor Symbol

\[ Q_{\text{elem}} = C \cdot V_{\text{elem}} \]
\[ [C] \quad [F] \quad [V] \]
(Farad)

We know:
\[ I_{\text{elem}} = \frac{d Q_{\text{elem}}}{dt} \]
\[ I_{\text{elem}} = \frac{d}{dt} (C \cdot V_{\text{elem}}) \]

\[ C = \text{constant over time} \]

\[ I_{\text{elem}} = C \cdot \frac{d V_{\text{elem}}}{dt} \]

Can use 7-step same analysis.
Capacitance

\[ C = \varepsilon \frac{A}{d} \]

\[ [F] = \left[ \frac{F}{m} \right] \left[ \frac{m^2}{m} \right] \]

Depends on:
- Materials: Permittivity
  \[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]
  \[ \varepsilon = \varepsilon_0 \varepsilon_r \]
- Geometry of Conductors

Conductive plates

Symbol:

Capacitance: \( C \)
Units: Farads [F]

IV equation:
\[ I = C \cdot \frac{dV}{dt} \]
Simple Circuit 1

\[ I_S = C \frac{dU_i}{dt} \times dt \]

**KCL:** \( I_S = I_C \)

**Element Def.:**

\[ I_C = C \cdot \frac{dV_c}{dt} \]

**Voltage Def.:**

\[ U_i - 0 = V_c \]

\[ \int_0^t I_S dt = \int C \cdot dU_i \]

\[ I_S \cdot t = C \cdot (U_i(t) - U_i(0)) \]

\[ U_i(t) = \frac{I_S}{C} + U_i(0) \]
Simple Circuit 2

\[ \begin{align*}
V_1 - 0 &= V_s \quad \text{Voltage Def.} \\
V_1 - 0 &= V_c \\
V_s &= V_c \\
I_c &= C \frac{dV_c}{dt} \quad \text{(capacitor Def.)} \\
I_c &= C \frac{dV_c}{dt} = C \frac{dV_s}{dt}^{10} = 0
\end{align*} \]

Current in a capacitor is zero when a constant voltage source is across it.

*Hint*: We like zeros...they make our lives easier!
Simple Circuit 3

Looking for $U_i$ value when

$V_c = \text{const. (steady-state)}$

$I_c = C \frac{dV_c}{dt} = 0$

KCL: $I_c + I_R = 0$

$I_R = 0$

Ohm's law: $V_R = I_R R = 0$

Voltage Def: $U_i - 0 = V_R$

$U_i = 0$

Steady State: means the Voltages Settled.

If current is zero $\Rightarrow$ OPEN-CIRCUIT
Equivalent Circuits with Capacitors

* Capacitor-only circuits

Step 1: Find $V_{in}$ and $I_{no}$  no source

Step 2: $C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}}$

\[\begin{array}{l}
\text{I}_{elem} \\
\text{+} \\
\text{+} \\
\text{-} \\
\text{-} \\
\text{I}_{elem}
\end{array}\]  \equiv  \begin{array}{l}
\text{I}_{elem} \\
\text{+} \\
\text{+} \\
\text{-} \\
\text{-} \\
\text{C}_{eq}
\end{array}

\text{Only if } (\text{match } \frac{dV_{elem}}{dt})
Two Methods:

a) Apply $I_{\text{test}}$ and measure $\frac{dV_{\text{test}}}{dt}$

b) Apply $\frac{dV_{\text{test}}}{dt}$ and measure $I_{\text{test}}$

$$C_{\text{eq}} = \frac{I_{\text{test}}}{\frac{dV_{\text{test}}}{dt}}$$

(a)
Example 1

\[ V_{\text{c}_1} = U_1, \quad V_{\text{c}_2} = U_1 \quad \text{and} \quad U_1 = V_{\text{test}} \]

\[ \frac{\text{d}U_1}{\text{d}t} = \frac{\text{d}V_{\text{test}}}{\text{d}t} \]

**Elem def.** \[ I_{\text{c}_1} = C_1 \frac{\text{d}V_1}{\text{d}t} = C_1 \frac{\text{d}U_1}{\text{d}t} = C_1 \frac{\text{d}V_{\text{test}}}{\text{d}t} \]

**Elem def.** \[ I_{\text{c}_2} = C_2 \frac{\text{d}V_2}{\text{d}t} = C_2 \frac{\text{d}U_1}{\text{d}t} = C_1 \frac{\text{d}V_{\text{test}}}{\text{d}t} \]

**KCL:** \[ I_{\text{test}} = I_{\text{c}_1} + I_{\text{c}_2} = C_1 \frac{\text{d}V_{\text{test}}}{\text{d}t} + C_2 \frac{\text{d}V_{\text{test}}}{\text{d}t} \]
\[ I_{\text{test}} = (C_1 + C_2) \frac{dV_{\text{test}}}{dt} \]

\[ C_{\text{eq}} = \frac{I_{\text{test}}}{dV_{\text{test}}} = C_1 + C_2 \]

\[ R_1 \parallel R_2 \quad \text{Series} \]

\[ R_{\text{eq}} = R_1 + R_2 \]
Example 2: "Capacitors in series"

\[ \text{KCL: } I_{c_1} = I_{c_2} = I_{\text{test}} \]

\[ \text{Elements: } \]
\[ I_{c_2} = C_2 \frac{dV_{c_2}}{dt} \]
\[ I_{c_1} = C_1 \frac{dV_{c_1}}{dt} \]

\[ \text{Voltage Def: } \]
\[ V_{c_2} = U_2 - 0 \]
\[ V_{c_1} = U_1 - U_2 \]
\[ V_{\text{test}} = U_1 - 0 \]

For \( V_{c_2} \):
\[ I_{c_2} = C_2 \frac{dV_{c_2}}{dt} \]
\[ I_{\text{test}} = C_2 \frac{dU_2}{dt} = \frac{dU_2}{dt} = \frac{I_{\text{test}}}{C_2} \]

For \( V_{c_1} \):
\[ I_{c_1} = C_1 \frac{dV_{c_1}}{dt} \]
\[ \frac{dV_{c_1}}{dt} = \frac{I_{c_1}}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{\text{test}}}{C_1} \]
\[ \frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{\text{test}}}{C_1} = \frac{I_{\text{test}}}{C_2} + \frac{I_{\text{test}}}{C_1} \]
\[ \frac{dU_1}{dt} = \frac{dV_{\text{test}}}{dt} = I_{\text{test}} \left( \frac{1}{C_2} + \frac{1}{C_1} \right) \]

\[ C_{eq} = \frac{I_{\text{test}}}{\frac{dV_{\text{test}}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_{1,\parallel C_2} \]

\[ C_{eq} = C_{1,\parallel C_2} \quad (\text{11 - parallel mathematical operator}) \]
Example 3

\[ C_{eq} = \frac{C_1}{1} \left( \frac{1}{C_2} + \frac{1}{C_3} \right) \]

\[ = \frac{C_1}{C_2 + C_3} \]

\[ \Rightarrow C_{eq} = C_2 + C_3 \]

\[ C_{eq} = \frac{C_1}{1} \left( \frac{1}{C_{eq_1}} \right) \]
Capacitive Touchscreen – Model without touch

\[ C_0 = \varepsilon \cdot \frac{A}{d} \]
Capacitive Touchscreen – Model with touch

When there is a touch, it makes a capacitor!

- finger
- dielectric
- conductive plate

Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture
when no touch:

\[
\begin{align*}
\text{e} & \quad \frac{1}{C_0} \\
\text{g} & \quad \frac{1}{C_0}
\end{align*}
\]

with touch:

\[
\begin{align*}
\text{e} & \quad \frac{1}{C_0 + \frac{C_1 C_2}{C_1 + C_2}} \\
\text{g} & \quad \frac{1}{C_0 + \frac{C_1 C_2}{C_1 + C_2}}
\end{align*}
\]

We only have access to nodes e and g, not f

Redraw to focus on terminals (nodes) e and g

Equivalent capacitance for \( C_1 \) in series with \( C_2 \)

Equivalent capacitance for \( C_0 \) in parallel to \( \frac{C_1 C_2}{C_1 + C_2} \)
2D View – How do we measure Capacitance?

Problem: We don’t have a capacitance meter!

We want to measure capacitance here.

\[ \frac{C_1C_2}{C_1 + C_2} = C_{\Delta} \quad \text{(change)} \]

This capacitor goes away with no touch.

We will try ideas to get to a final model.
Measuring Capacitance Models – Attempt #1

If there is touch: \( V_c = V_s \)

If there is no touch: \( V_c = V_s \)

\( V_{out} \) does not change!

Bad idea! \( \times \)

Assume starts out discharged:

\( V_{out}(t=0) = 0 \)

\( I_s = C_{eq} \frac{dV_{out}(t)}{dt} \)

\( \Rightarrow V_{out}(t) = \int_{0}^{t} \frac{I_s}{C_{eq}} \, dt \)

\( V_{out} = \frac{I_s \, t}{C_{eq}} \)

\( \Rightarrow C_{eq} = \frac{I_s}{dV_{out}(t)} \)

Very hard to make current sources! \( \times \)
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

- 1st: Close both switches. We want to charge $C_{\text{ref}}$ and measure $V_{\text{out}}$ as $C_{\text{ref}}$ discharges. If both closed - nothing happens. ! Attempt #1

Phase 1: Close $S_1$, Open $S_2$

$q = C_{\text{eq}} \cdot V_s$ accumulates on capacitor plates.
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Phase 2: close $S_2$, open $S_1$

- There is a path for charge to move.
- $C_{eq}$ can provide the energy needed for current.

Charge will split between $C_{eq}$ and $C_{ref}$

“charge sharing”

So close! But we don’t know initial $C_{ref}$. 
Measuring Capacitance Models – Attempt #3 – known initial condition

Phase 1: $S_3$ closed, $S_1$ closed, $S_2$ open
- $C_{eq}$ discharges $V_{out} \rightarrow 0$
- $q = C_{eq} \cdot V_{out} = 0$

Phase 2: $S_1$ open, $S_2$ closed, $S_3$ open
- $C_{eq}$ charged
- $q = C_{eq} \cdot V_S$

Use $S_3$ to discharge $C_{ref}$ so we know $C_{ref} = 0$
Measuring Capacitance Models – Attempt #3 – known initial condition

Voltage across $C_{eq}$: $V_{out}$
Voltage across $C_{ref}$: $V_{out}$
Charge in $C_{eq}$: $q_1 = C_{eq} \cdot V_{out}$
Charge in $C_{ref}$: $q_2 = C_{ref} \cdot V_{out}$

Total charge is conserved!

$q_1 (\text{phase 1}) = q_2 (\text{phase 2})$

$C_{eq} \cdot V_s = C_{eq} \cdot V_{out} + C_{ref} \cdot V_{out}$

$V_{out} = \frac{C_{eq} \cdot V_s}{C_{eq} + C_{ref}} \Rightarrow V_{out} \text{ changes when } C_{eq} \text{ changes}!!!