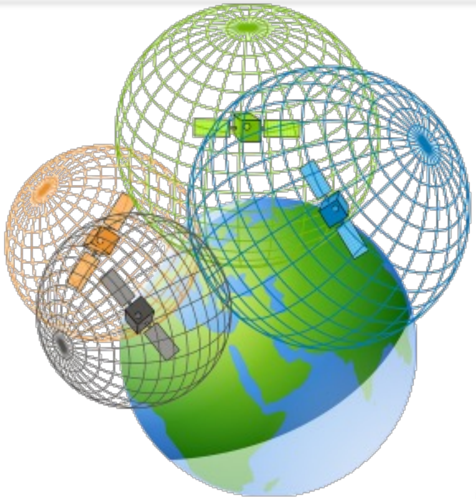


Welcome to EECS 16A!

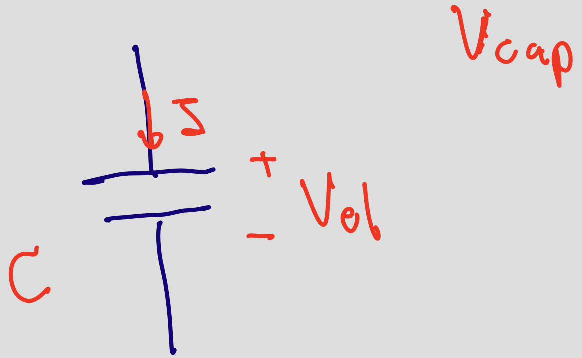
Designing Information Devices and Systems I

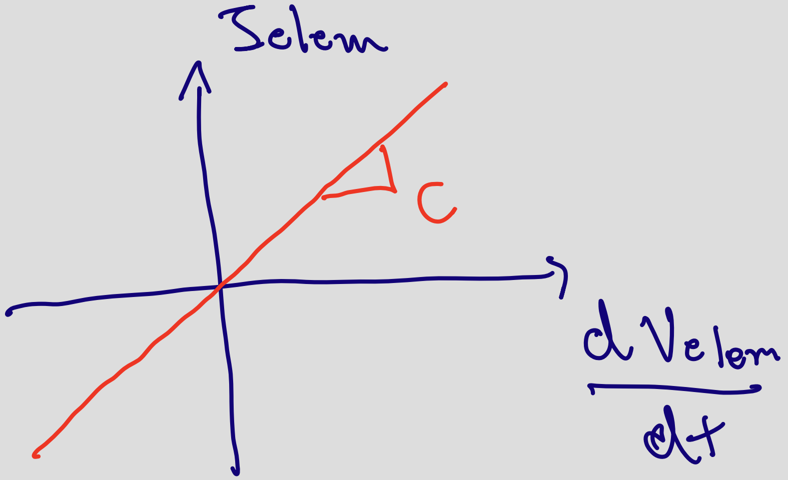
Ana Claudia Arias and Miki Lustig

Fall 2022

Module 2
Lecture 8
Capacitance Modeling and Comparators
(Note 16 and 17)

Last Class





$$Q_{elem} = C \cdot V_{elem}$$

$$[C] = [F]\,[V]$$

↳ Farad

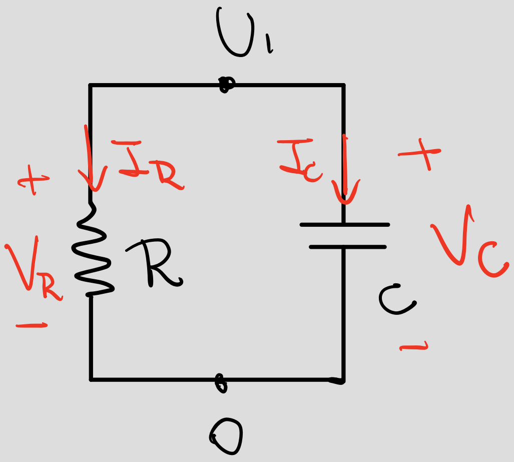
We know : $I_{elem} = \frac{dQ}{dt}$

$$I_{elem} = \frac{d}{dt} C \cdot V_{elem}$$

C = constant over time

$$I_{elem} = C \cdot \frac{dV_{elem}}{dt}$$

Simple Circuit 3



$U_1 = ?$

Steady State: means the Voltages settled.

Looking for U_1 value when V_c = const. (steady-state)

$$I_c = C \frac{dV_c}{dt} = 0$$

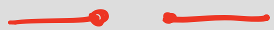
KCL: $I_c + I_R = 0$

$$I_R = 0$$

Ohm's Law: $V_R = I_R R = 0$

Voltage Def: $U_1 - 0 = V_R$

$$\boxed{U_1 = 0}$$

If current is zero ⇒ 

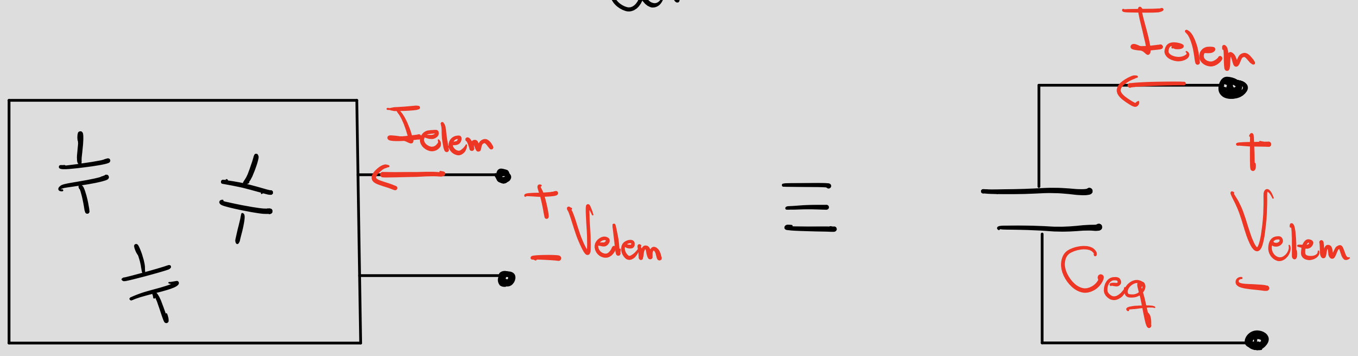
OPEN - CIRCUIT

Equivalent Circuits with Capacitors

* Capacitor-only circuits

~~Step 1 : find V_{th} and I_{No}~~ no source

Step 2 : $C_{eq} = \dfrac{I_{elem}}{\frac{dV_{elem}}{dt}}$



Only if (match $\frac{dV_{elem}}{dt}$)

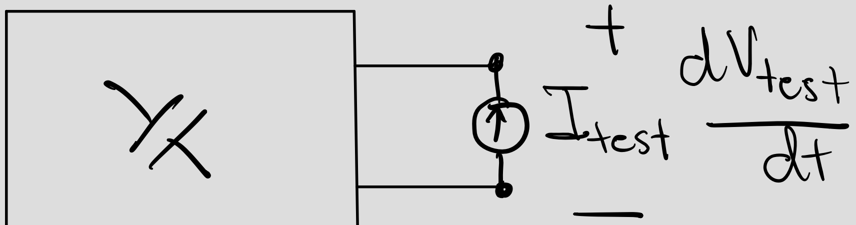
Two Methods:

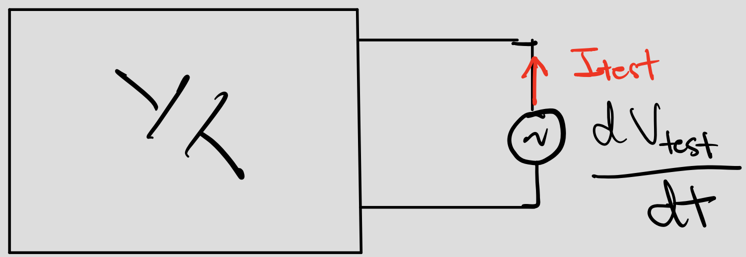
a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}

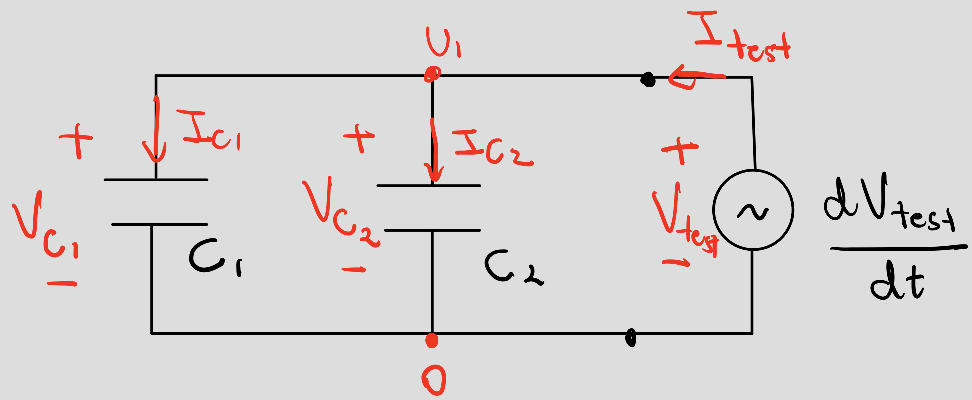
$$= C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

(a)





Example 1



$V_{C_1} = U_1$, $V_{C_2} = U_1$ and

$U_1 = V_{test}$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt}$$

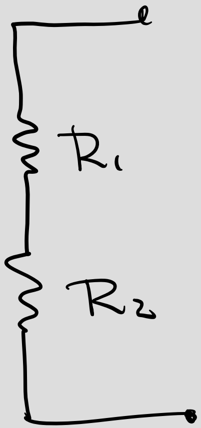
Elem def: $I_{C_1} = C_1 \frac{dV_1}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{test}}{dt}$

Elem def: $I_{C_2} = C_2 \frac{dV_2}{dt} = C_2 \frac{dU_1}{dt} = C_1 \frac{dV_{test}}{dt}$

KCL: $I_{test} = I_{C_1} + I_{C_2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt}$

$$I_{test} = (C_1 + C_2)\frac{dV_{test}}{dt}$$

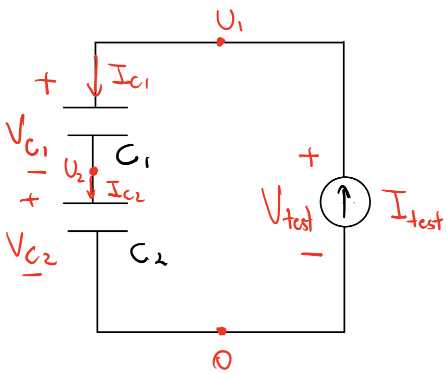
$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2$$



$$\equiv R_{eq} = R_1 + R_2$$

Series

Example 2 : "Capacitors in series"



KCL : $I_{C_1} = I_{C_2} = I_{test}$

Elements :

$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

Voltage Def :

$$V_{C2} = U_2 - 0$$

$$V_{C1} = U_1 - U_2$$

$$V_{test} = U_1 - 0$$

For V_{C2}:

$$I_{C_2} = C_2 \frac{dV_{C2}}{dt}$$

$$I_{test} = C_2 \frac{dU_2}{dt} \Rightarrow \frac{dU_2}{dt} = \frac{I_{test}}{C_2}$$

For V_{C_1}

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$\frac{dV_1}{dt} = \frac{I_C}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1}$$

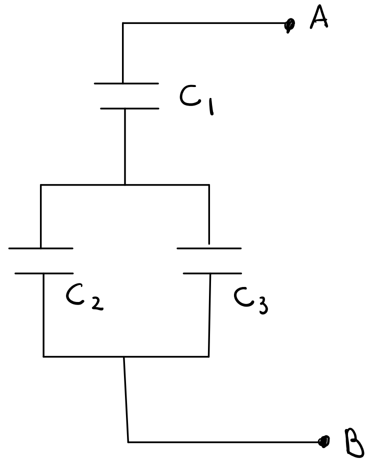
$$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$$

$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt} = I_{test}\left(\frac{1}{C_2} + \frac{1}{C_1}\right)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \| C_2$$

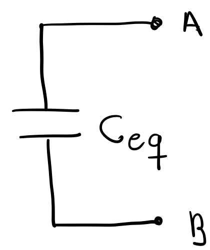
$C_{eq} = C_1 \| C_2$ ($\|$ – parallel mathematical operator)

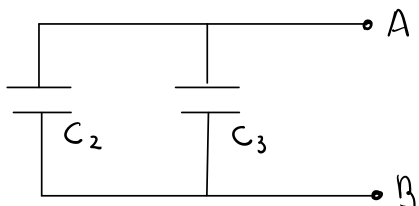
Example 3



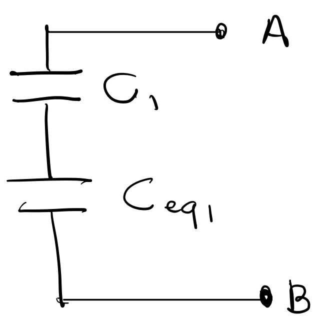
$$C_{eq} = C_1 \parallel (C_2 + C_3)$$

$\equiv$



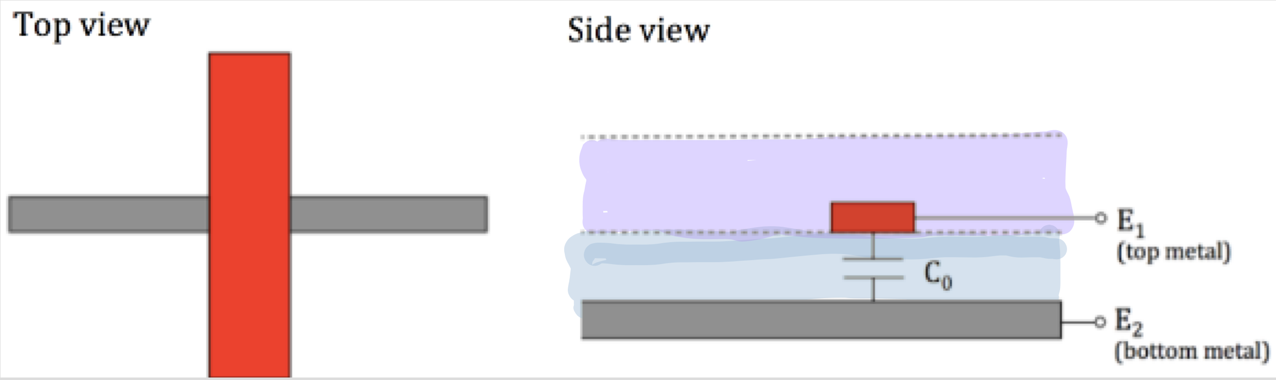


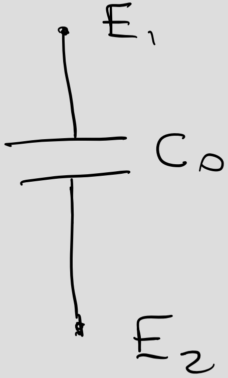
$\Rightarrow$ $C_{eq_1} = C_2 + C_3$



$$C_{eq} = C_1 \parallel C_{eq_1}$$

Capacitive Touchscreen – Model without touch

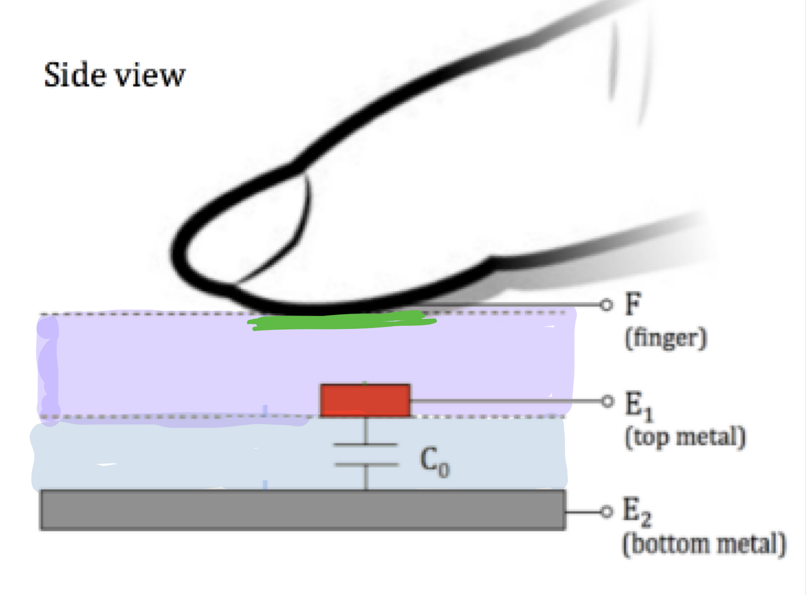


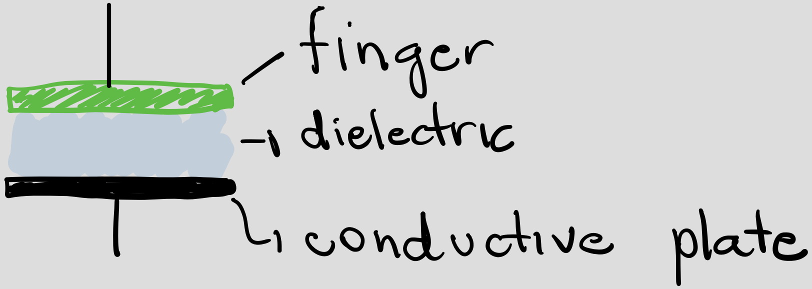


$$C_0 = \varepsilon \cdot \frac{A}{d}$$

Capacitive Touchscreen – Model with touch

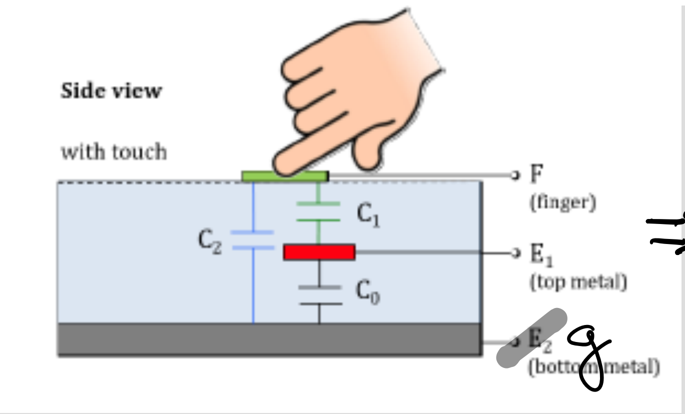
When there is a touch, it makes a capacitor!



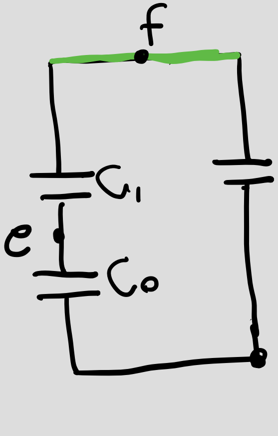


Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture

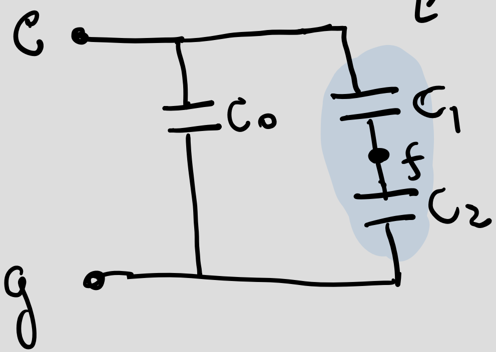


⇒ circuit model

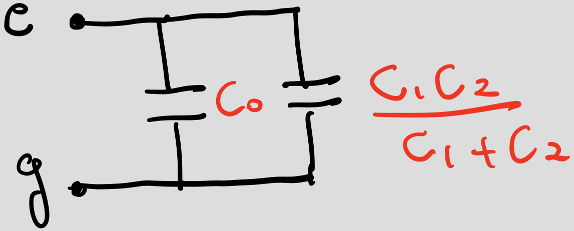


We only have acess to nodes e and g, not f

Redraw to focus on terminals (nodes) e and g

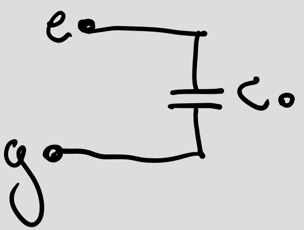


Equivalent capacitance for C_1 in series with C_2

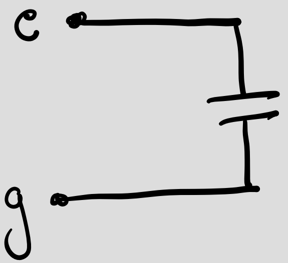


⇒ Equiv. Capacitance for C_0 in parallel to $\frac{C_1 C_2}{C_1+C_2}$

when no touch:



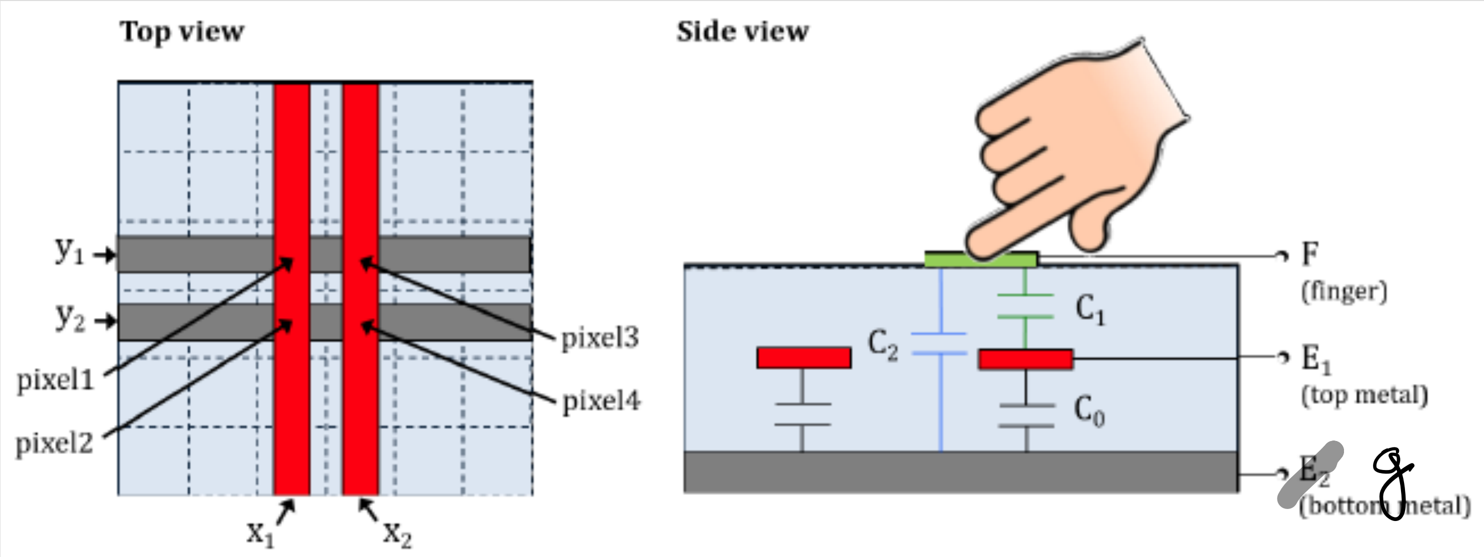
with touch:



$$C_0 + \frac{C_1 C_2}{C_1+C_2}$$

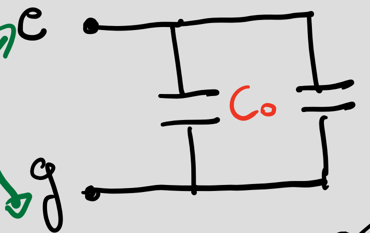
Extra Capacitance due to touch!

2D View – How do we measure Capacitance?



Problem: We don't have a capaci-meter!

We want to measure Capacitance here



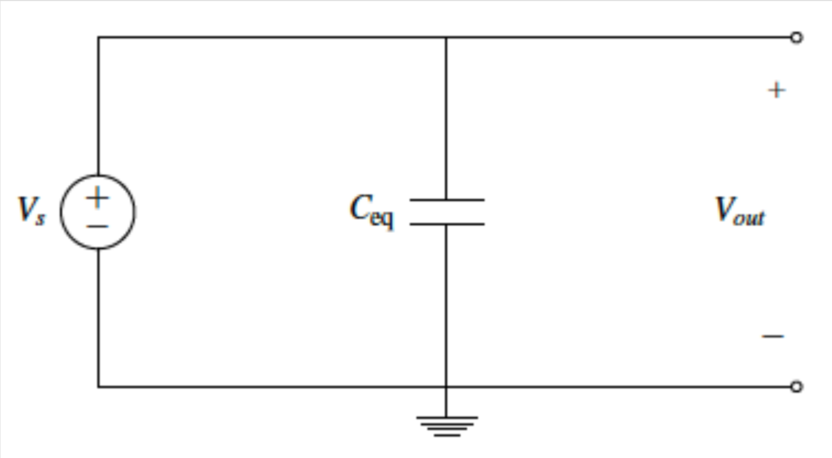
$$\frac{C_1 C_2}{C_1 + C_2} = C_\Delta$$

(change)

This capacitor goes away with no touch

We will try ideas to get to a final model.

Measuring Capacitance Models – Attempt #1

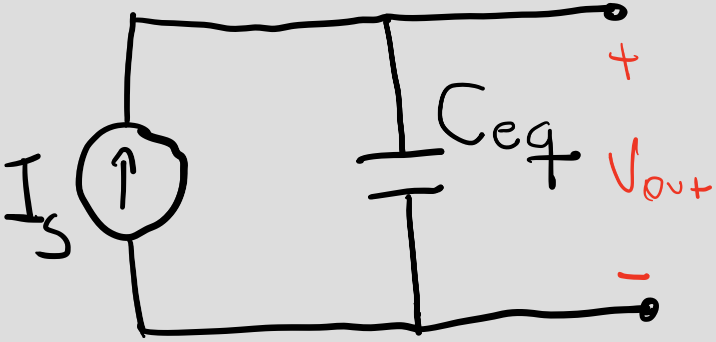


If there is touch: $V_c = V_s$

If there is no touch: $V_c = V_s$

Vout does not change!

Bad idea!



Very hard to make current sources!

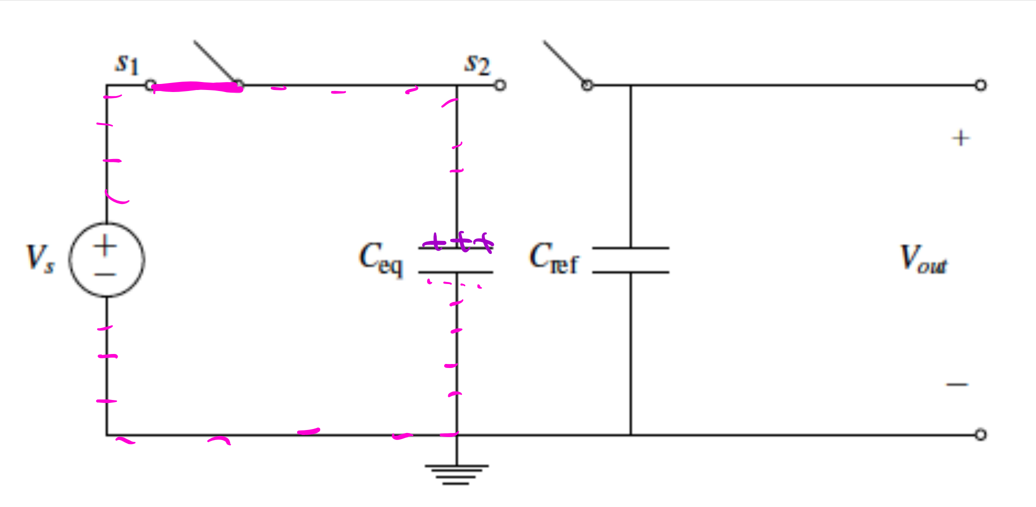
Assume starts out discharged:

$V_{out}(t=0) = 0$

$I_s = C_{eq} \frac{dV_{out}(t)}{dt} \rightarrow V_{out}(t) = \int_0^t \frac{I_s}{C_{eq}} \cdot dt$

$V_{out} = \frac{I_s \cdot t}{C_{e}} \Rightarrow C_{eq} = \frac{I_s}{\frac{dV_c(t)}{dt}}$

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor



- 1st _ Close both switches

We want to charge Cref and measure Vout as Cref discharges.

If both closed – nothing happens.! Attempt #1 ✗

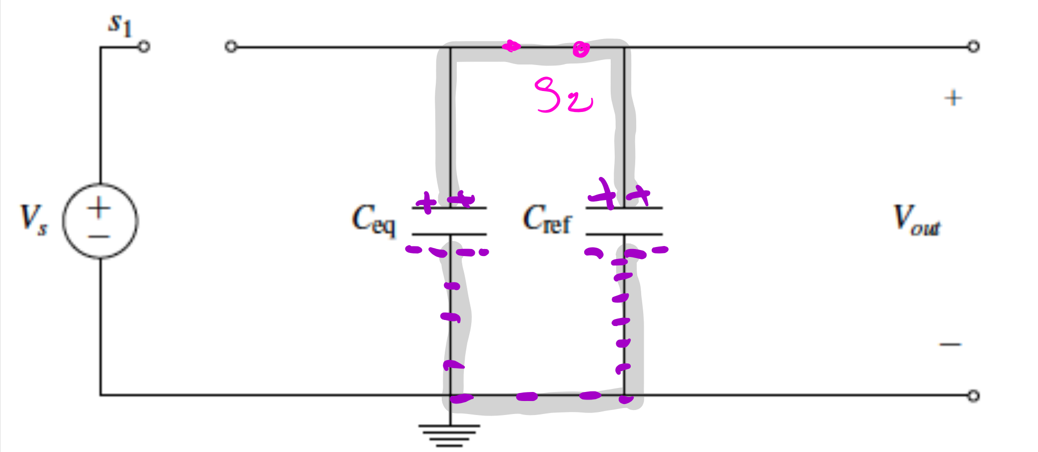
Phase 1: Close S_1; Open S_2

Ceq is charging

$q = C_{eq} \cdot V_s$ accumulates on capacitor plates.

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Phase 2: close S_2, open S_1



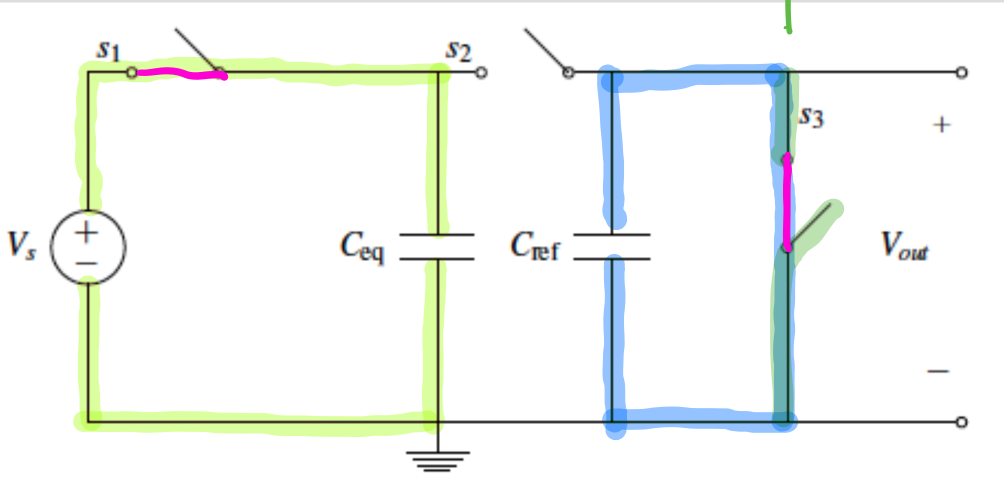
- There is a path for charge to move.
- Ceq can provide the energy needed for current.

Charge will split between Ceq and Cref

"charge sharing"

So close! But we don't know initial Cref.

Measuring Capacitance Models – Attempt #3 – known initial condition



Use S_3 to discharge C_{ref} so we know $C_{ref} = 0$

Phase 1: S_3 closed, S_1 closed, S_2 open

C_{req} discharges $V_{out} \to 0$

$q = C_{req} \cdot V_{out} = 0$ ✓

C_{eq} charges

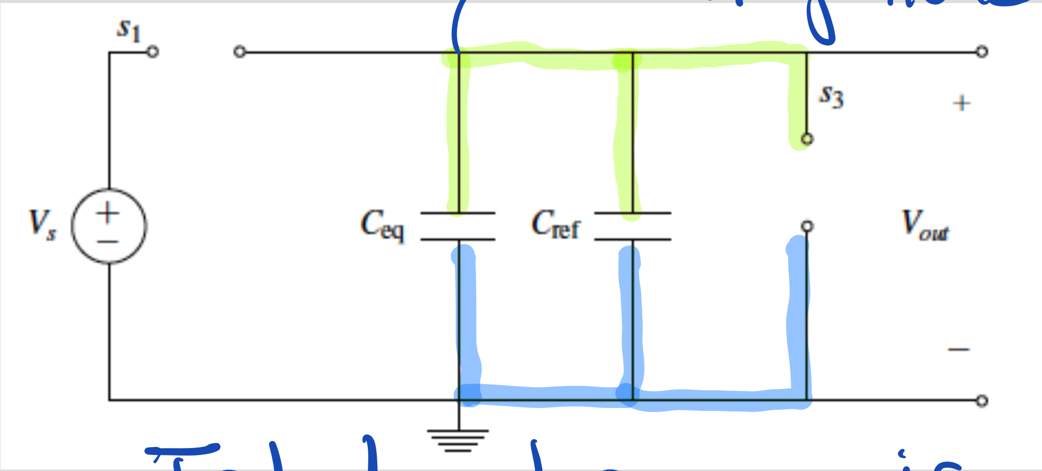
$q = C_{eq} \cdot V_s$

Phase 2: S_1 open, S_2 closed, S_3 open

C_{eq} – charged

Measuring Capacitance Models – Attempt #3 – known initial condition

floating node



Voltage across C_{eq} : V_{out}

Voltage across C_{ref} : V_{out}

charge in C_{eq}: $q_1 = C_{eq} \cdot V_{out}$

charge in C_{ref}: $q_2 = C_{ref} \cdot V_{out}$

Total charge is conserved!

$$q(\text{phase }1) = q(\text{phase }2)$$

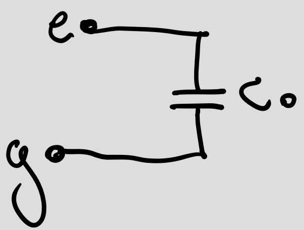
$$C_{eq} \cdot V_s = C_{eq} V_{out} + C_{ref} \cdot V_{out}$$

$$V_{out} = \frac{C_{eq} V_s}{C_{eq} + C_{ref}}$$

⇒ V_{out} changes when C_{eq} changes!!!

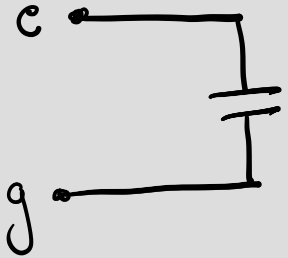
Effect of touch on total capacitance

when no touch:



$$\Rightarrow \quad V_{OUT} = \frac{C_0}{C_0 + C_{ref}} \cdot V_S$$

with touch:

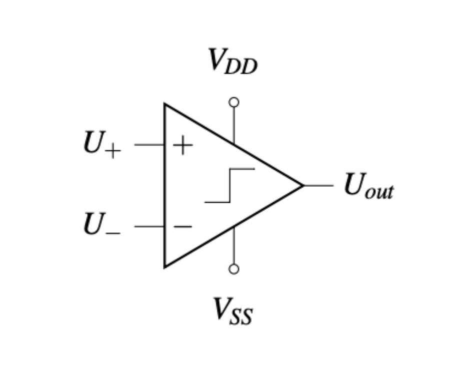


$$C_0 + \underbrace{\frac{C_1 C_2}{C_1 + C_2}}_{C_\Delta}$$

Extra Capacitance due to touch!

$$\Rightarrow \quad V_{OUT} = \frac{(C_0 + C_\Delta)}{C_0 + C_\Delta + C_{ref}} \cdot V_S$$

How can we go from voltage measurement to binary answer: touch or no touch?



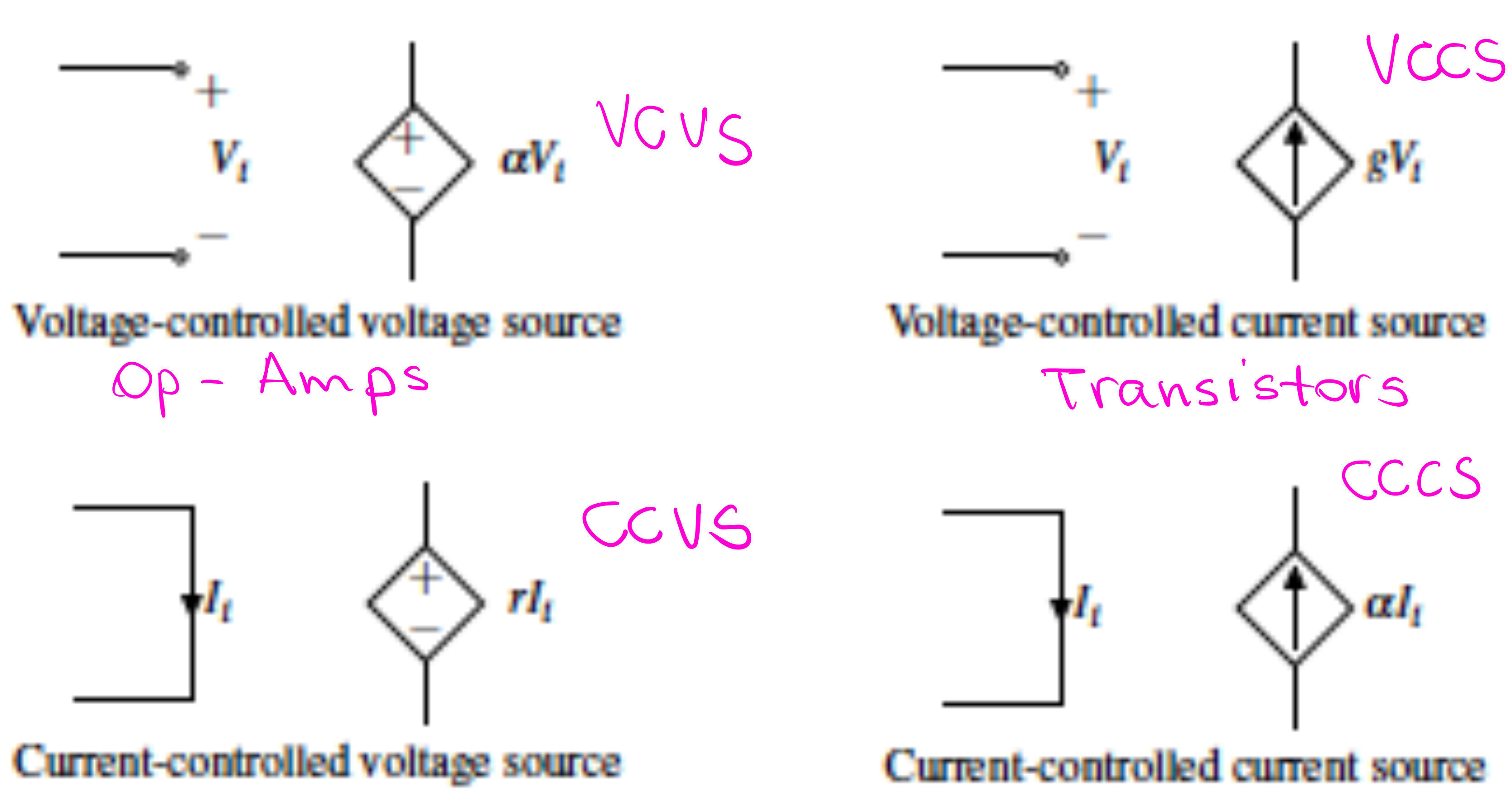
- We need to choose a Voltage that we call : Threshold Voltage (V_{th})

- Above V_{th} ∴ 1 (touch)
- Below V_{th} ∴ 0 (no-touch)

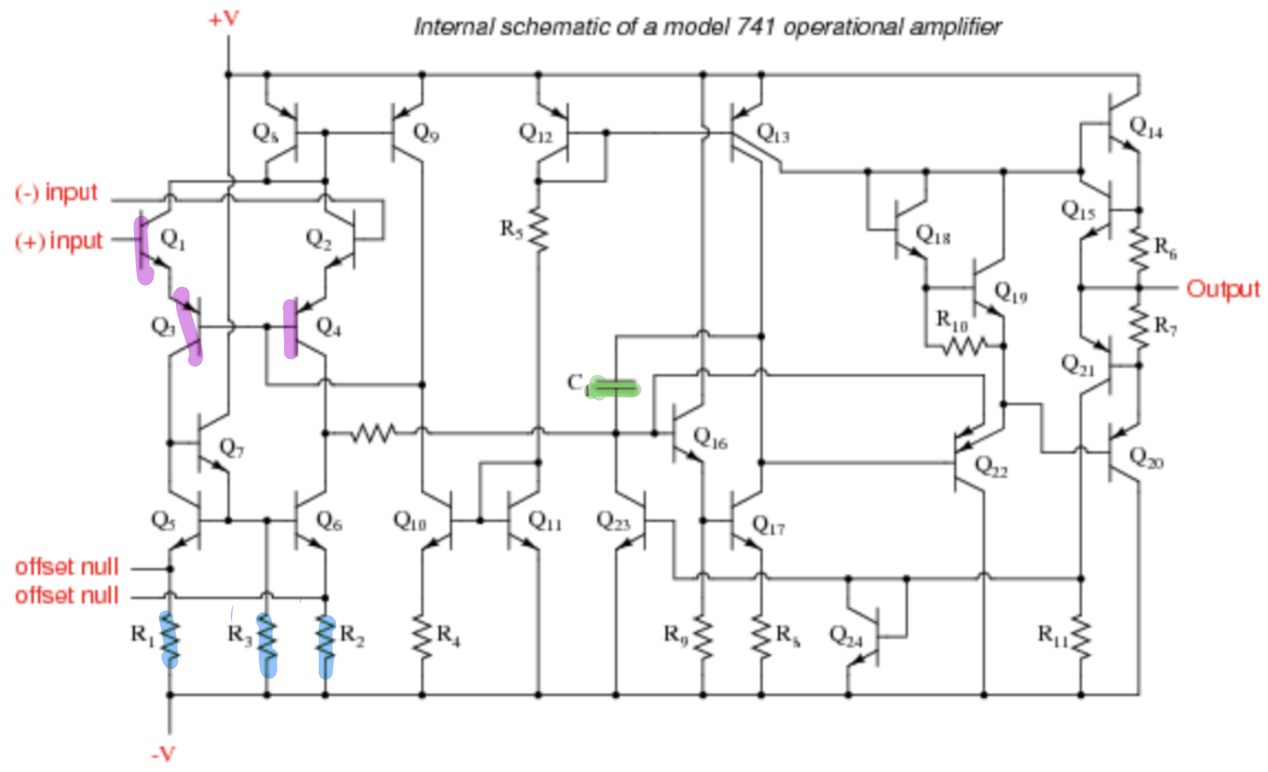
We need to compare Voltages to determine if 1 or 0

How can we go from voltage measurement to binary answer: touch or no touch?

- New tools are needed – new circuit elements



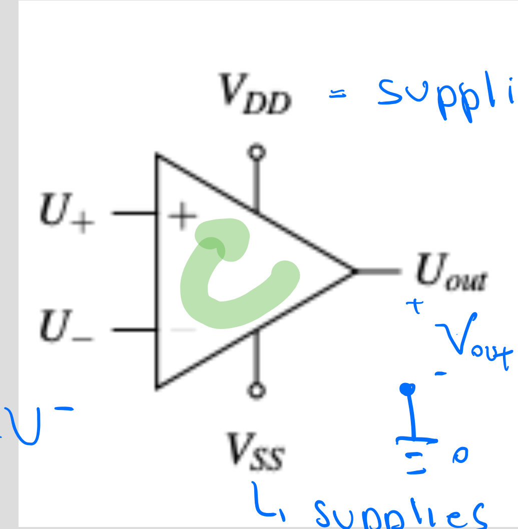
An example of an Op-amp circuit diagram



Schematic diagram of a model 741 op-amp.

Operational Amplifier

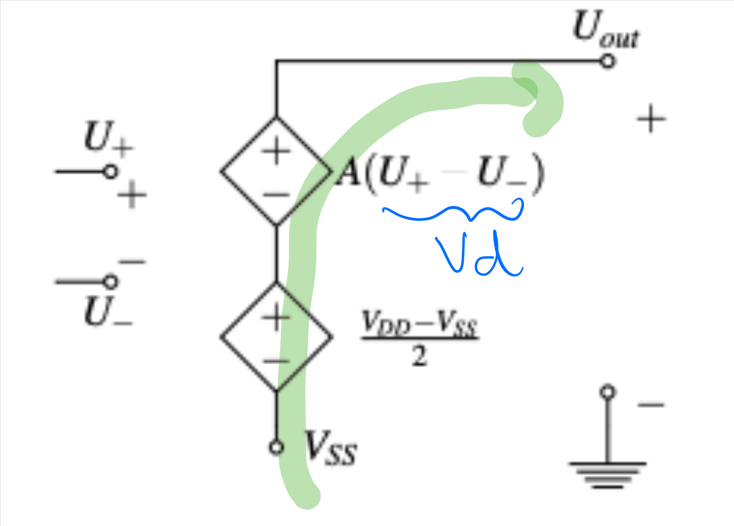
An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.

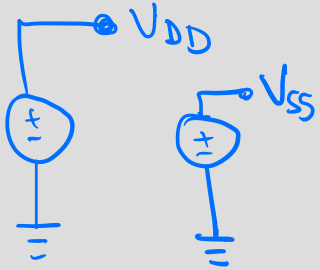


$V_d = U^+ - U^-$

An op-amp has two input terminals marked (+) and (−) with potentials U+ and U−, two power supply terminals called VDD and VSS, and one output terminal with potential Uout.

Model





$$V_{out} = V_{SS} + \frac{V_{DD}-V_{SS}}{2} + A \cdot V_d$$

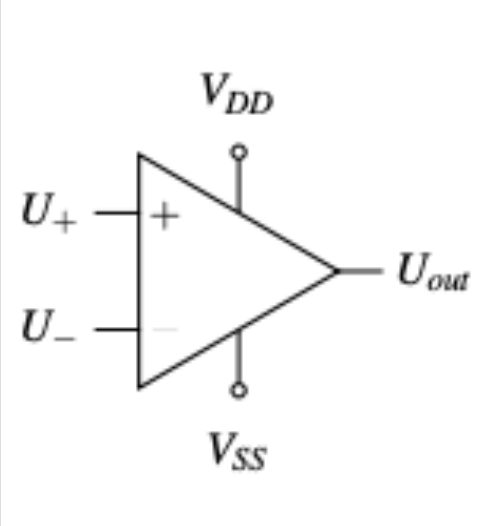
when

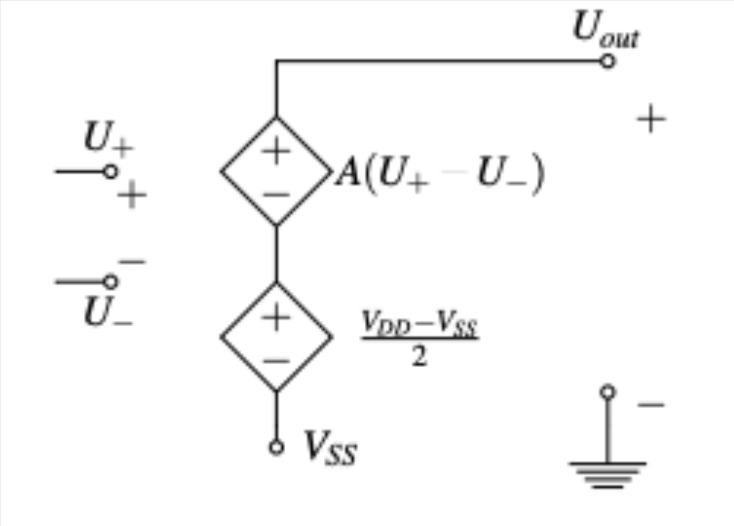
$$V_{SS} \leq \frac{V_{DD}-V_{SS}}{2} + A V_d \leq V_{DD}$$

Operational Amplifier

$\frac{V_{DD} - V_{SS}}{2} + A V_d$ V^*

An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.





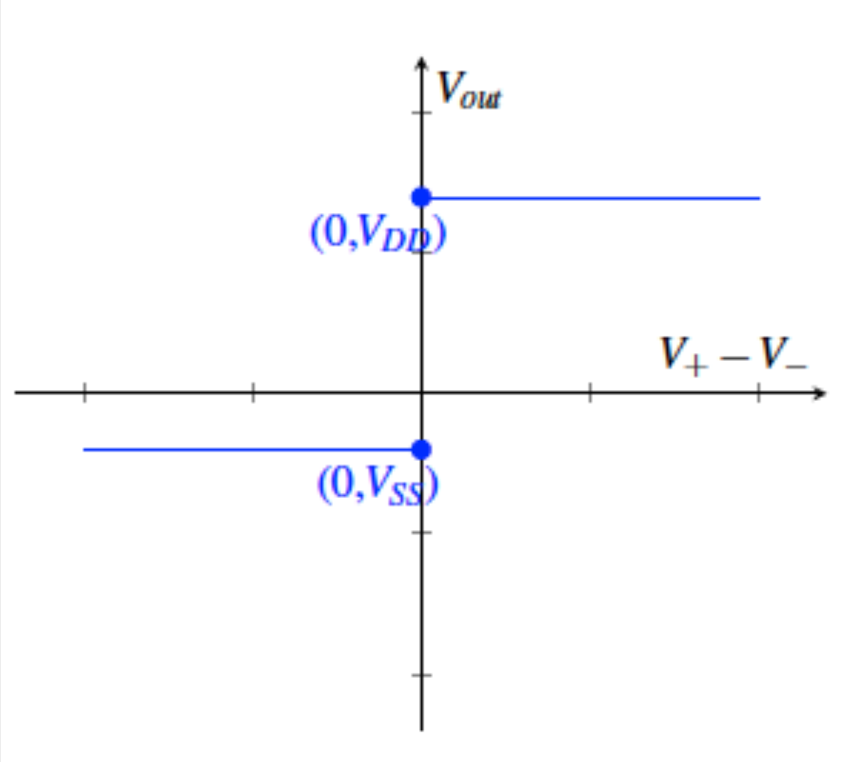
An op-amp has two input terminals marked (+) and (−) with potentials U+ and U−, two power supply terminals called VDD and VSS, and one output terminal with potential Uout.

$V_{out} = V_{DD}$ if $V^* > V_{DD}$

$V_{out} = V_{SS}$ if $V^* < V_{SS}$

Can be used to compare Voltage

Comparator – optimized for binary output

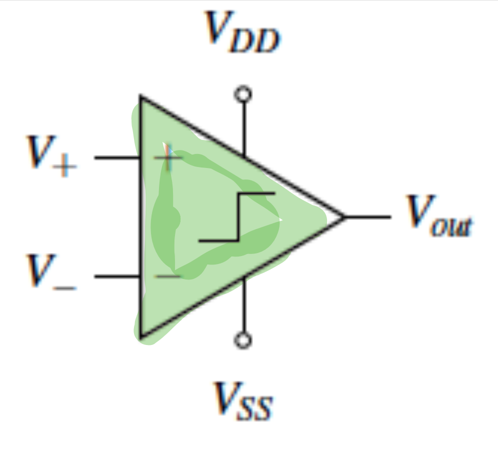


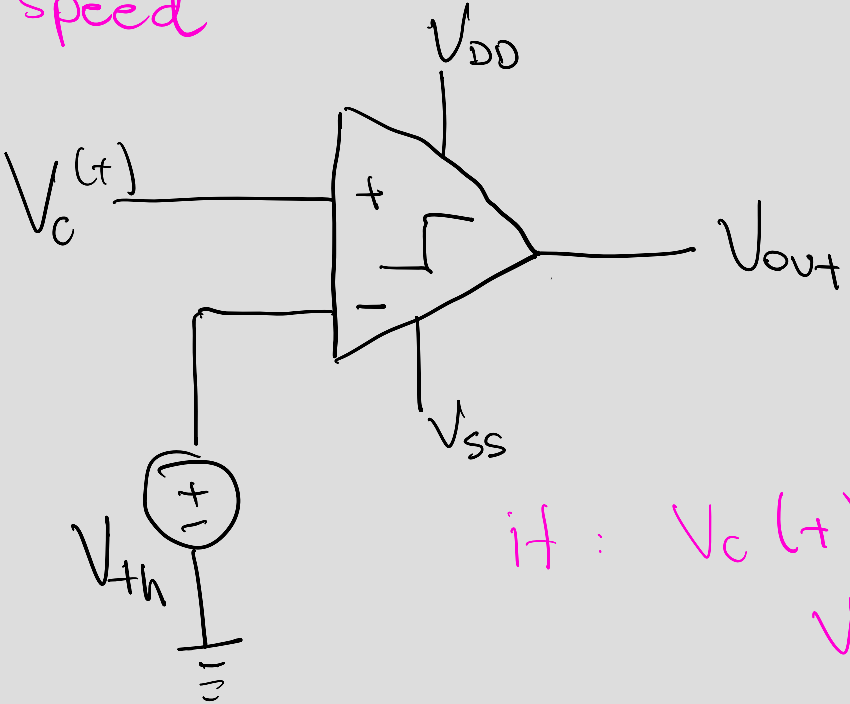
V_{DD} can be much higher than V_{SS}

∴ it amplifies the signal.

Comparator – optimized for binary output

Also optimized for speed





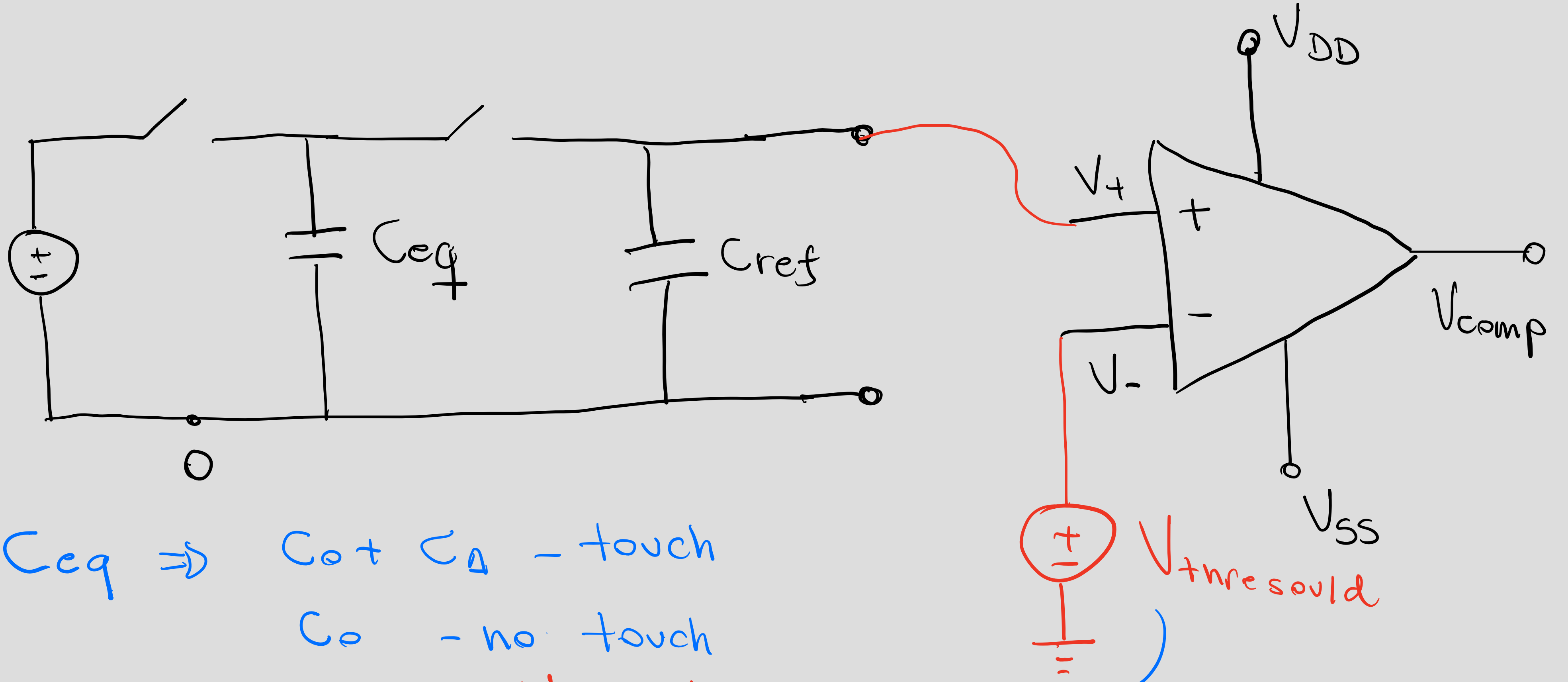
if: $V_c(t) > V_{th}$

$V_{out} = V_{DD}$

if: $V_c(t) \leq V_{Th}$

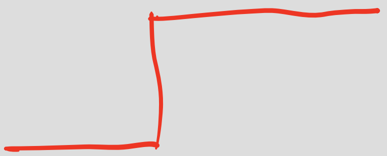
$V_{out} = V_{SS}$

Back to our Capacitive Touchscreen



$C_{eq} \Rightarrow C_0 + C_\Delta$ - touch

C_0 - no touch



Should be half way between V_{touch} and $V_{notouch}$