Welcome to EECS 16A!
Designing Information Devices and Systems I

Ana Claudia Arias and Miki Lustig
Fall 2022

Module 2
Lecture 8
Capacitance Modeling and Comparators
(Note 16 and 17)
\( Q_{\text{elem}} = C \cdot V_{\text{elem}} \)

\[
[C] = [F] \cdot [V] \\
C, \text{ Farad}
\]

We know:
\[
I_{\text{elem}} = \frac{dQ}{dt}
\]

\[
I_{\text{elem}} = \frac{d}{dt} C \cdot V_{\text{elem}}
\]

\[
C = \text{constant over time}
\]
Simple Circuit 3

Looking for $U_i$ value when $V_c = \text{const. (steady-state)}$

$$I_c = C \frac{\text{d}V_c}{\text{d}t} = 0$$

KCL: $I_c^0 + I_R = 0$

$$I_R = 0$$

Ohm's law: $V_R = I_R R = 0$

Voltage Def: $U_i - 0 = V_R^0$

If current is zero $\Rightarrow$ OPEN-CIRCUIT

$U_i = 0$
Equivalent Circuits with Capacitors

* Capacitor-only circuits

Step 1: Find $V_{th}$ and $I_{no}$

Step 2: $C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}}$

only if

\[
(\text{match } \frac{dV_{elem}}{dt})
\]
Two Methods:

a) Apply $I_{test}$ and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure $I_{test}$

$$\text{\text{\text{Eq}}} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$
Example 1

\[ V_{C_1} = U_1, \quad V_{C_2} = U_1 \quad \text{and} \quad U_1 = V_{\text{test}} \]

\[ \frac{dU_1}{dt} = \frac{dV_{\text{test}}}{dt} \]

**Elem def:** \[ I_{C_1} = C_1 \frac{dV_1}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

**Elem def:** \[ I_{C_2} = C_2 \frac{dV_2}{dt} = C_2 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

**KCL:** \[ I_{\text{test}} = I_{C_1} + I_{C_2} = C_1 \frac{dV_{\text{test}}}{dt} + C_2 \frac{dV_{\text{test}}}{dt} \]
\[ I_{\text{test}} = (C_1 + C_2) \frac{dV_{\text{test}}}{dt} \]

\[ C_{\text{eq}} = \frac{I_{\text{test}}}{dV_{\text{test}}/dt} = C_1 + C_2 \]

\[ R_1 \quad \text{Series} \quad R_2 \quad \Rightarrow R_{\text{eq}} = R_1 + R_2 \]
**Example 2**: "Capacitors in series"

\[ I_{C_1} = I_{C_2} = I_{test} \]

**KCL**: \[ I_{C_1} = I_{C_2} = I_{test} \]

**Elements**: 
\[ I_{C_2} = C_2 \frac{dV_{C_2}}{dt} \]
\[ I_{C_1} = C_1 \frac{dV_{C_1}}{dt} \]

**Voltage Def**: 
\[ V_{C_2} = U_2 - 0 \]
\[ V_{C_1} = U_1 - U_2 \]
\[ V_{test} = U_1 - 0 \]

**For \( V_{C_2} \)**:
\[ I_{C_2} = C_2 \frac{dV_{C_2}}{dt} \]
\[ I_{test} = C_2 \frac{dU_2}{dt} = \frac{dU_2}{dt} = \frac{I_{test}}{C_2} \]

**For \( V_{C_1} \)**:
\[ I_{C_1} = C_1 \frac{dV_{C_1}}{dt} \]
\[ \frac{dV_1}{dt} = \frac{I_{C_1}}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1} \]
\[ \frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1} \]
\[ \frac{dU_1}{dt} = \frac{dV_{test}}{dt} = \frac{I_{test}}{C_1} \left( \frac{1}{C_2} + \frac{1}{C_1} \right) \]

\[ C_{eq} = \frac{I_{test}}{dV_{test}/dt} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_{1||C_2} \]

\[ C_{eq} = C_{1||C_2} \text{ (|| - parallel mathematical operator)} \]
Example 3

\[ C_{eq} = \frac{C_1}{C_1 + C_2 + C_3} \]

\[ \Rightarrow C_{eq} = C_2 + C_3 \]

\[ C_{eq} = \frac{C_1}{C_1 + C_{eq_1}} \]
Capacitive Touchscreen – Model without touch

\[ C_0 = \varepsilon \cdot \frac{A}{\lambda} \]
Capacitive Touchscreen – Model with touch

When there is a touch, it makes a capacitor!

Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture
We only have access to nodes e and g, not f.

Redraw to focus on terminals (nodes) e and g.

Equivalent capacitance for $C_1$ in series with $C_2$.

When no touch:

When with touch:

Extra capacitance due to touch!

$C_0 + \frac{C_1 C_2}{C_1 + C_2}$

$C_0 \frac{C_1 C_2}{C_1 + C_2}$

$\Rightarrow$ Equiv. capacitance for $C_0$ in parallel to $\frac{C_1 C_2}{C_1 + C_2}$.
2D View – How do we measure Capacitance?

We want to measure capacitance here.

We will try ideas to get to a final model.

Problem: We don’t have a capacitance meter!
Measuring Capacitance Models – Attempt #1

If there is touch: \( V_c = V_s \)
If there is no touch: \( V_c = V_s \)

\( V_{out} \) does not change!

Bad idea! \( \times \)

Assume starts out discharged:
\( V_{out}(t=0) = 0 \)

\[ I_s = C_{eq} \frac{dV_{out}(t)}{dt} \Rightarrow V_{out}(t) = \int_{0}^{t} \frac{I_s}{C_{eq}} dt \]

\( V_{out} = \frac{I_s}{C_{eq}} \times \) \( \Rightarrow \) \( C_{eq} = \frac{I_s}{dV_{out}(t)} \)

Very hard to make current sources! \( \times \)
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

1st. Close both switches
We want to charge $C_{ref}$ and measure $V_{out}$ as $C_{ref}$ discharges.
If both closed – nothing happens. ! Attempt #1

Phase 1: Close $S_1$; Open $S_2$

$C_{eq}$ is charging
$q = C_{eq} \cdot V_s$ accumulates on capacitor plates.
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Phase 2: close $S_2$, open $S_1$

- There is a path for charge to move.
- $C_{eq}$ can provide the energy needed for current.

Charge will split between $C_{eq}$ and $C_{ref}$.

“charge sharing”

So close! But we don’t know initial $C_{ref}$. 
Measuring Capacitance Models – Attempt #3 – known initial condition

Phase 1: $S_3$ closed, $S_1$ closed, $S_2$ open
- $C_{eq}$ discharges $V_{out} \to 0$
- $q = C_{eq} \cdot V_{out} = 0$

Phase 2: $S_1$ open, $S_2$ closed, $S_3$ open
- $C_{eq}$ charges
- $q = C_{eq} \cdot V_s$

Use $S_3$ to discharge $C_{ref}$ so we know $C_{ref} = 0$
Measuring Capacitance Models – Attempt #3 – known initial condition

Total charge is conserved!

\[ q_1 \text{ (phase 1)} = q_2 \text{ (phase 2)} \]

\[ C_{\text{eq}} \cdot V_s = C_{\text{eq}} \cdot V_{\text{out}} + C_{\text{ref}} \cdot V_{\text{out}} \]

\[ V_{\text{out}} = \frac{C_{\text{eq}} \cdot V_s}{C_{\text{eq}} + C_{\text{ref}}} \]

Voltage across \( C_{\text{eq}} \): \( V_{\text{out}} \)

Voltage across \( C_{\text{ref}} \): \( V_{\text{out}} \)

Charge in \( C_{\text{eq}} \): \( q_1 = C_{\text{eq}} \cdot V_{\text{out}} \)

Charge in \( C_{\text{ref}} \): \( q_2 = C_{\text{ref}} \cdot V_{\text{out}} \)

\( \Rightarrow \) \( V_{\text{out}} \) changes when \( C_{\text{eq}} \) changes!!!
Effect of touch on total capacitance

when no touch:

\[ e \quad \frac{1}{C_0} \]

with touch:

\[ e \quad \frac{1}{C_0 + \frac{C_1 C_2}{C_1 + C_2}} \]

Extra Capacitance due to touch!

\[ \Rightarrow \quad V_{\text{out}} = \frac{C_0}{C_0 + C_{\text{res}}} \cdot V_S \]

\[ \Rightarrow \quad V_{\text{out}} = \frac{(C_0 + C_\Delta)}{C_0 + C_\Delta + C_{\text{res}}} \cdot V_S \]
How can we go from voltage measurement to binary answer: touch or no touch?

- We need to choose a Voltage that we call: Threshold Voltage ($V_{th}$)

- Above $V_{th}$: 1 (touch)
- Below $V_{th}$: 0 (no-touch)

We need to compare voltages to determine if 1 or 0.
How can we go from voltage measurement to binary answer: touch or no touch?

- New tools are needed – new circuit elements
An example of an Op-amp circuit diagram

Schematic diagram of a model 741 op-amp.
Operational Amplifier

An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.

An op-amp has two input terminals marked (+) and (−) with potentials $U_+$ and $U_-$, two power supply terminals called $V_{DD}$ and $V_{SS}$, and one output terminal with potential $U_{out}$.

$V_d = U_+ - U_-$

$V_{out} = V_{SS} + \frac{V_{DD} - V_{SS}}{2} + AV_d$

when $V_{SS} \leq \frac{V_{DD} - V_{SS} + AV_d}{2} \leq V_{DD}$
Operational Amplifier

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$$V_{DD} - V_{SS} + A V_{in} \frac{V_{DD} - V_{SS}}{2}$$

$V_{out} = V_{DD}$ if $V^* > V_{DD}$

$V_{out} = V_{SS}$ if $V^* \leq V_{SS}$

Can be used to compare Voltage
Comparator – optimized for binary output

\[ V_{DD} \text{ can be much higher than } V_{SS} \]

It amplifies the signal.
Comparator – optimized for binary output

Also optimized for speed

\[ \text{if: } V_C(+) > V_{th} \]
\[ V_{out} = V_{DD} \]

\[ \text{if: } V_C(+) \leq V_{th} \]
\[ V_{out} = V_{SS} \]
Back to our Capacitive Touchscreen

$C_{eq} \Rightarrow C_0 + C_A - \text{touch}$

$C_0 - \text{no touch}$

$V_{DD} - \text{touch}$

$\text{no touch } V_{SS}$

Should be halfway between $V_{touch}$ and $V_{no-touch}$