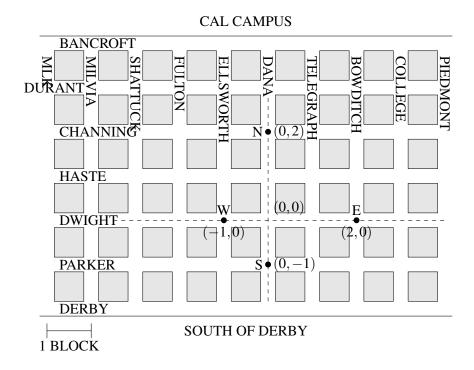
$\begin{array}{ccc} \text{EECS 16A} & \text{Designing Information Devices and Systems I} \\ \text{Spring 2022} & \text{Discussion 12B} \end{array}$

1. Search and Rescue Dogs

Berkeley's Puppy Shelter needs your help! While Mr. Mochi Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the shelter have a collar that sends a Bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of five city blocks. Can you help the shelter locate their lost puppy?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map). Mr. Mochi Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Mochi Muffin?' it is sufficient to answer with his intersection or 'between these two intersections.'



(a) You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	5
\mathbf{W}	$\sqrt{20}$
E	$\sqrt{5}$
S	$\sqrt{10}$

On the map provided above, identify where Mr. Mochi Muffin is!

(b) Can you set this up as a system of equations? Are these equations linear? If not, can these equations be linearized? If you can linearize these equations, write down a simplified form of your set of equations.

Hint: Set (0,0) to be Dwight and Dana.

Hint 2: Distance = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Hint 3: You don't need all 4 equations. You have two unknowns, *x* and *y*. You know from lecture that you need at least three circles to uniquely find a point on a 2D plane. How can you use the third circle/equation to get two equations and two unknowns?

Note: Remember to check for consistency for all nonlinear equations after finding the coordinates.

(c) Suppose Mr. Mochi Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

Sensor	Distance
N	5
\mathbf{W}	$\sqrt{20}$
E	Out of Range
S	Out of Range

Can you find Mr. Mochi Muffin? With a system of linear equations? Other methods? If so, on the map provided above, identify where Mr. Mochi Muffin is!

2. Mechanical Projection

In \mathbb{R}^n , the vector valued projection of vector \vec{b} onto vector \vec{a} is defined as:

$$\operatorname{proj}_{ec{a}}\left(ec{b}
ight) = rac{\left\langle ec{a}, ec{b}
ight
angle}{\left\| ec{a}
ight\|^2} ec{a}.$$

Recall $\|\vec{a}\|^2 = \langle \vec{a}, \vec{a} \rangle$.

- (a) Project $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ onto $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ that is, onto the *y*-axis. Graph these two vectors and the projection.
- (b) Project $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ onto $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Graph these two vectors and the projection.
- (c) Project $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Graph these two vectors and the projection.

