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EECS 16A    Designing Information Devices and Systems I    Homework 3  
 Spring 2022

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**This homework is due February 11th, 2022, at 23:59.**

**Self-grades are due February 18th, 2022, at 23:59.**

**Submission Format**

Your homework submission should consist of a single PDF file that contains all of your answers (any hand-written answers should be scanned).

- 1. Reading Assignment** For this homework, please read Note 3 and 11A. Note 3 overview of span and an introduction to thinking about and writing proofs. Note 11A will give you an introduction to circuits.

Please answer the following questions:

- What does span of a set of vectors mean?
- How can you check if a particular vector is in the span of a set of vectors?
- Given that  $\vec{b} \in \text{span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$  and  $\vec{a}_1, \vec{a}_2, \vec{a}_3$  are column vectors of  $\mathbf{A}$ , which *one* of the following statements does not make sense:
  - $\vec{b}$  is in the span of matrix  $\mathbf{A}$
  - $\vec{b}$  is in the range of  $\mathbf{A}$
  - $\vec{b}$  is in the column space of  $\mathbf{A}$

**2. Vectors in the Span**

**Learning Goal:** Practice determining whether a vector is in the span of a set of vectors.

Determine whether a vector  $\vec{v}$  is in the span of the given set of vectors. If it is in the span of given set, write  $\vec{v}$  as a linear combination of given set of vectors (you will need to find the scalar coefficients in the linear combination).

(a)  $\vec{v} = \begin{bmatrix} -10 \\ 4 \end{bmatrix}$  and  $\left\{ \begin{bmatrix} -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$

(b)  $\vec{v} = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$  and  $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \\ -1 \end{bmatrix} \right\}$

(c)  $\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$  and  $\left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \right\}$

(d)  $\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$  and  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 2 \\ 0 \end{bmatrix} \right\}$

### 3. Span Proofs

**Learning Objectives:** This is an opportunity to practice your proof development skills. Refer to problem 3 in Discussion 2B for a similar proof with span.

- (a) Given some set of vectors  $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ , show the following:

$$\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} = \text{span}\{\vec{v}_1 + \vec{v}_2, \vec{v}_2, \dots, \vec{v}_n\}$$

In other words, we can replace one vector with the sum of itself and another vector and not change their span.

In order to show this, you have to prove the two following statements:

- If a vector  $\vec{q}$  belongs in  $\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ , then it must also belong in  $\text{span}\{\vec{v}_1 + \vec{v}_2, \vec{v}_2, \dots, \vec{v}_n\}$ .
- If a vector  $\vec{r}$  belongs in  $\text{span}\{\vec{v}_1 + \vec{v}_2, \vec{v}_2, \dots, \vec{v}_n\}$ , then it must also belong in  $\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ .

In summary, you have to prove the problem statement from both directions.

- (b) Consider the span of the set  $(\vec{v}_1, \dots, \vec{v}_n, \vec{u})$ . Suppose  $\vec{u}$  is in the span of  $\{\vec{v}_1, \dots, \vec{v}_n\}$ . Then, show that any vector  $\vec{r}$  in  $\text{span}\{\vec{v}_1, \dots, \vec{v}_n, \vec{u}\}$  is in  $\text{span}\{\vec{v}_1, \dots, \vec{v}_n\}$ .

### 4. Basic Circuit Components

**Learning Objectives:** Review basics of circuit components and current-voltage relationships

- (a) Fill in the units for the following quantities:

Quantity	Symbol	Units
Voltage	V	
Current	I	
Resistance	R	

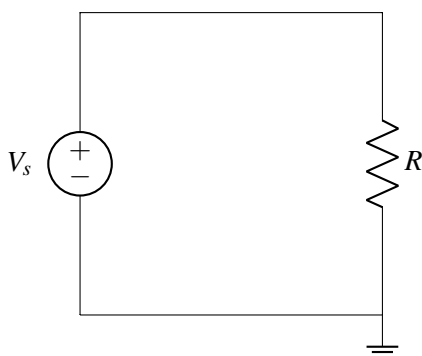
- (b) What is the voltage across a **short circuit (wire)**? What is the current going through it? Draw the symbol and sketch the IV relationship.
- (c) What is the voltage across an **open circuit**? What is the current going through it? Draw the symbol and sketch the IV relationship.
- (d) What is the relationship between voltage and current for a **resistor**? Draw the symbol and IV relationship.
- (e) What is the voltage across a **voltage source**  $V_s$ ? What is the current going through it? Draw the symbol and sketch the IV relationship.
- (f) What is the voltage across a **current source**  $I_s$ ? What is the current going through it? Draw the symbol and sketch the IV relationship.

### 5. Ohm's Law

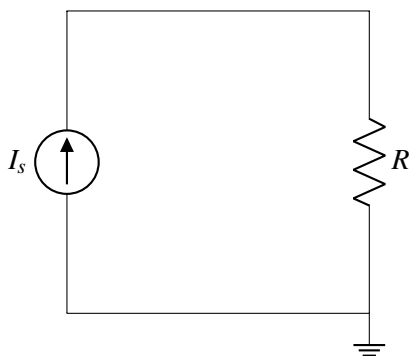
**Learning Objectives:** Practice implementing Ohm's Law in basic circuits.

- (a) Take the circuit diagram below.

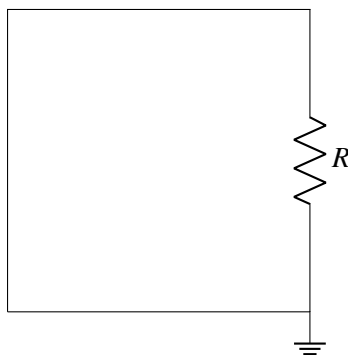
$$V_s = 5 \text{ V and } R = 10 \Omega.$$



- i. How many nodes are in this circuit?
  - ii. What is the potential at ground?
  - iii. What is  $V_R$ , the voltage across  $R$ ?
  - iv. What is  $I_R$ , the current through  $R$ ?
- (b) Now switch the voltage source with a current source.  
 $I_s = 0.002 \text{ A} = 2 \text{ mA}$  and  $R = 10,000,000 \text{ } \Omega = 10 \text{ M}\Omega$ .



- i. How many nodes are in this circuit?
  - ii. What is the potential at ground?
  - iii. What is  $I_R$ , the current through  $R$ ?
  - iv. What is  $V_R$ , the voltage across  $R$ ?
- (c) Now short the circuit.



- i. How many nodes are in this circuit?
- ii. What is  $V_R$ , the voltage across  $R$ ?

iii. What is  $I_R$ , the current through  $R$ ?

## 6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.