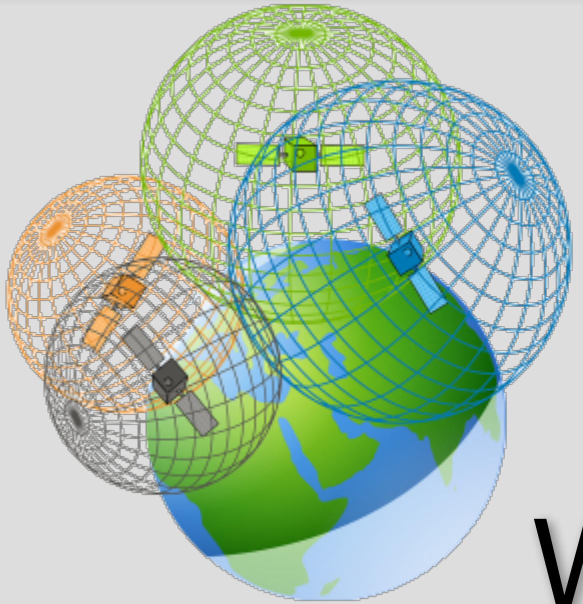
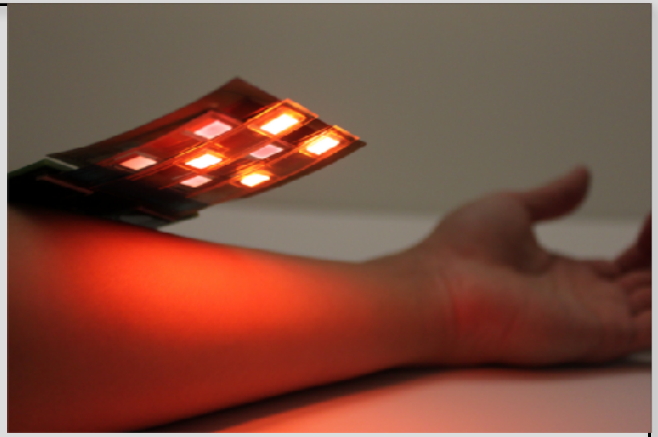




Welcome to EECS 16A!

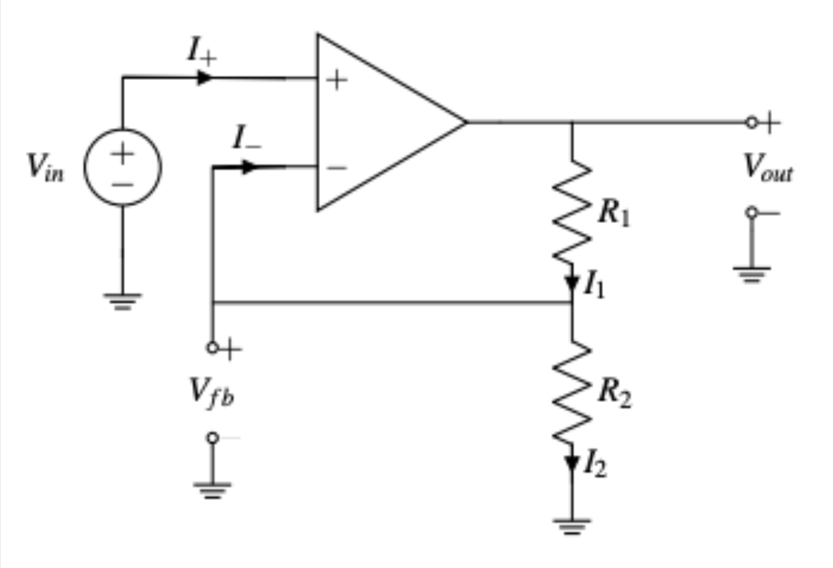
Designing Information Devices and Systems I

Ana Arias and Miki Lustig

2022

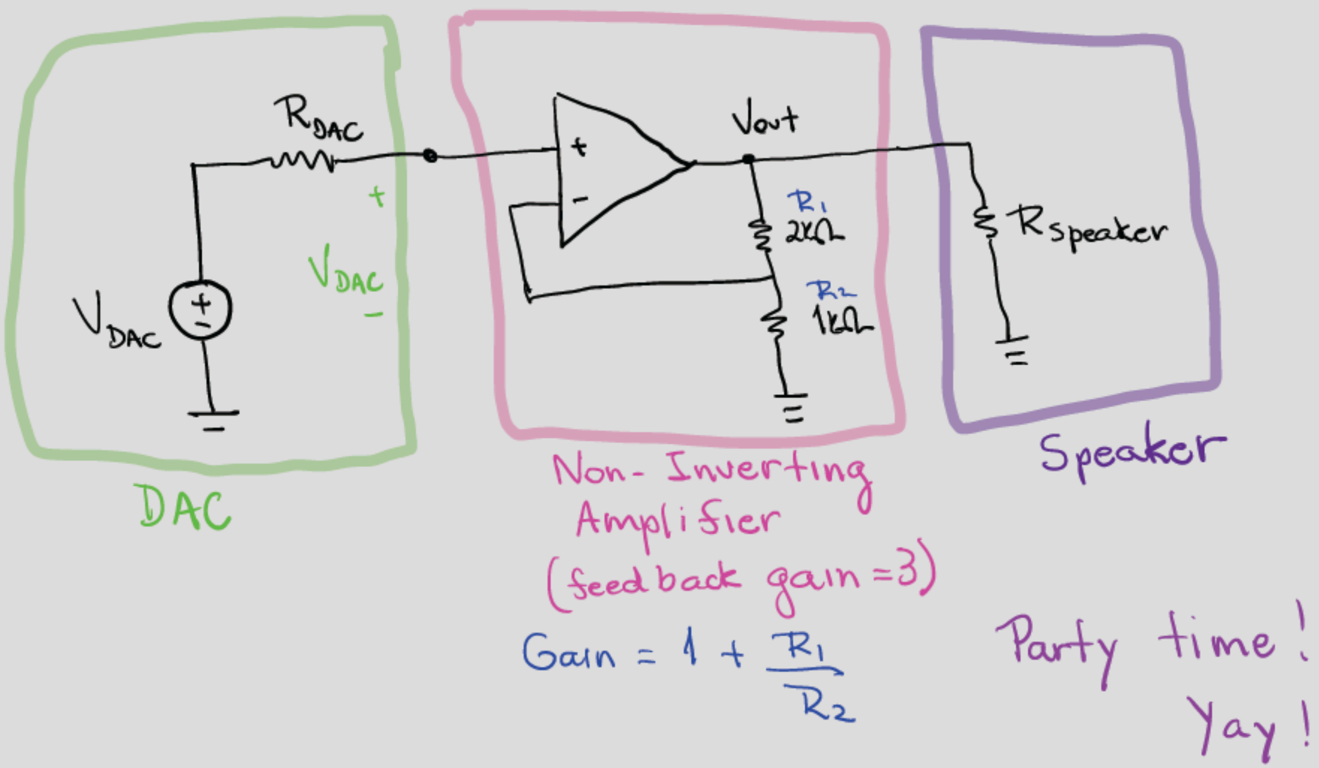
Lecture 10A
Op-amp circuit analysis

Last Lecture…



Toolbox

- Resistors
- Capacitors
- Open-circuits
- Voltage Dividers/Summers
- Op-Amps
- Thevenin and Norton Equivalence
- KCL/KVL
- Element Definitions
- DAC
- Negative Feedback
- Op-Amp in Negative Feedback
- "Golden Rules" for Op-Amps

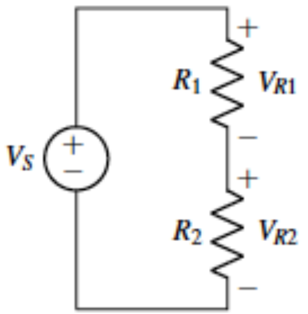


GR #1: $I_+ = 0, \quad I_- = 0$ no current into OpAmp

GR #2: in negative feedback: $U^+ = U^-$

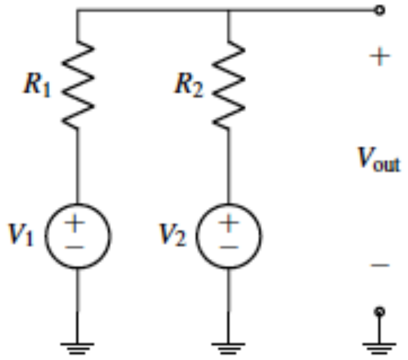
Today

Voltage Divider



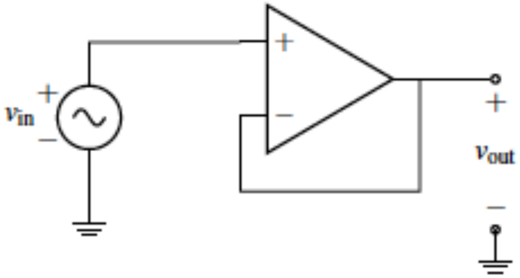
$$V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Voltage Summer



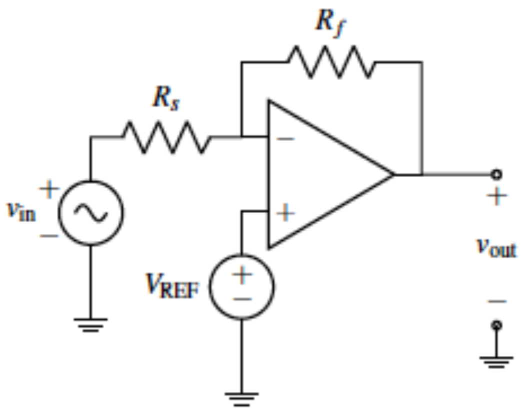
$$V_{\text{out}} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



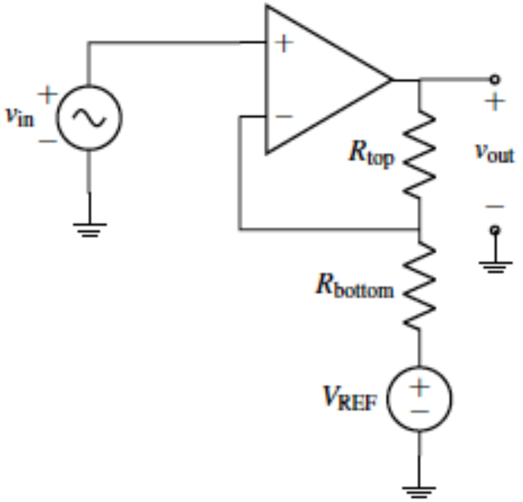
$$\frac{v_{\text{out}}}{v_{\text{in}}} = 1$$

Inverting Amplifier



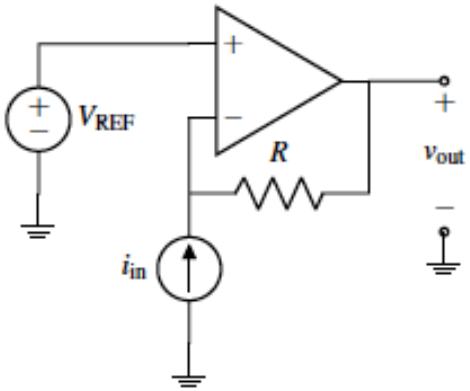
$$v_{\text{out}} = v_{\text{in}}\left(-\frac{R_f}{R_s}\right) + V_{\text{REF}}\left(\frac{R_f}{R_s}+1\right)$$

Non-inverting Amplifier



$$v_{\text{out}} = v_{\text{in}}\left(1+\frac{R_{\text{top}}}{R_{\text{bottom}}}\right) - V_{\text{REF}}\left(\frac{R_{\text{top}}}{R_{\text{bottom}}}\right)$$

Transresistance Amplifier



$$v_{\text{out}} = i_{\text{in}}(-R) + V_{\text{REF}}$$

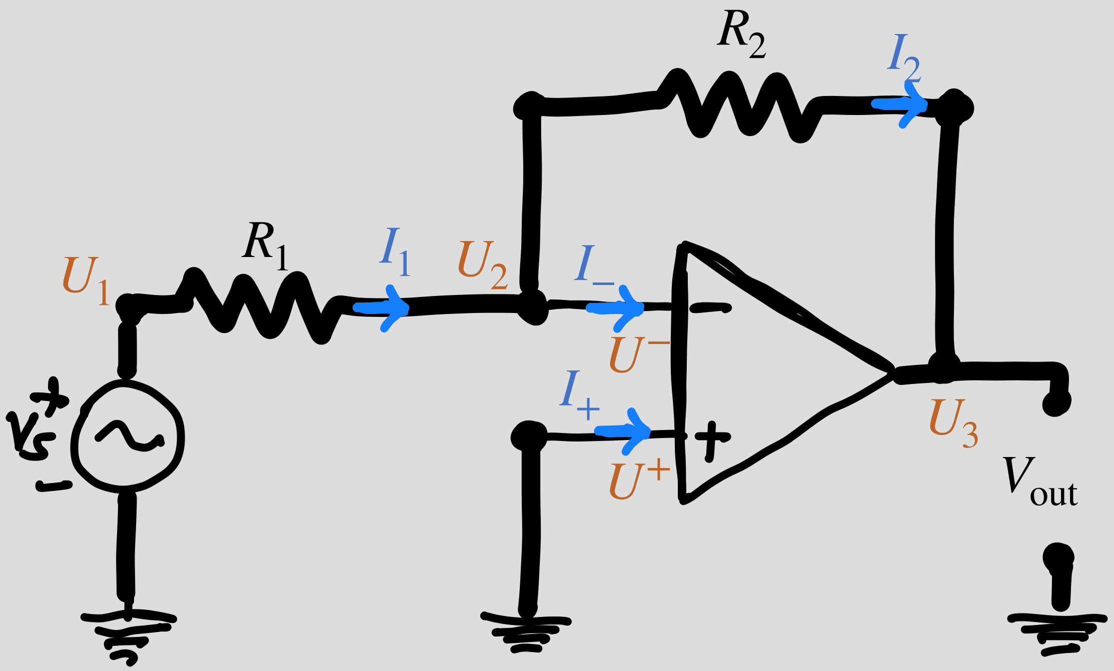
Checking for Negative Feedback

Step 1 – Zero out all independent sources

- replacing voltage sources with wires
- current sources with open circuits as in superposition

Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.

- if the $(U^+ - U^-)$ decreases, the output $A(U^+ - U^-)$ must also decrease. The circuit is in negative feedback
- if the $(U^+ - U^-)$ increases, the output $A(U^+ - U^-)$ must also increase. The circuit is in positive feedback



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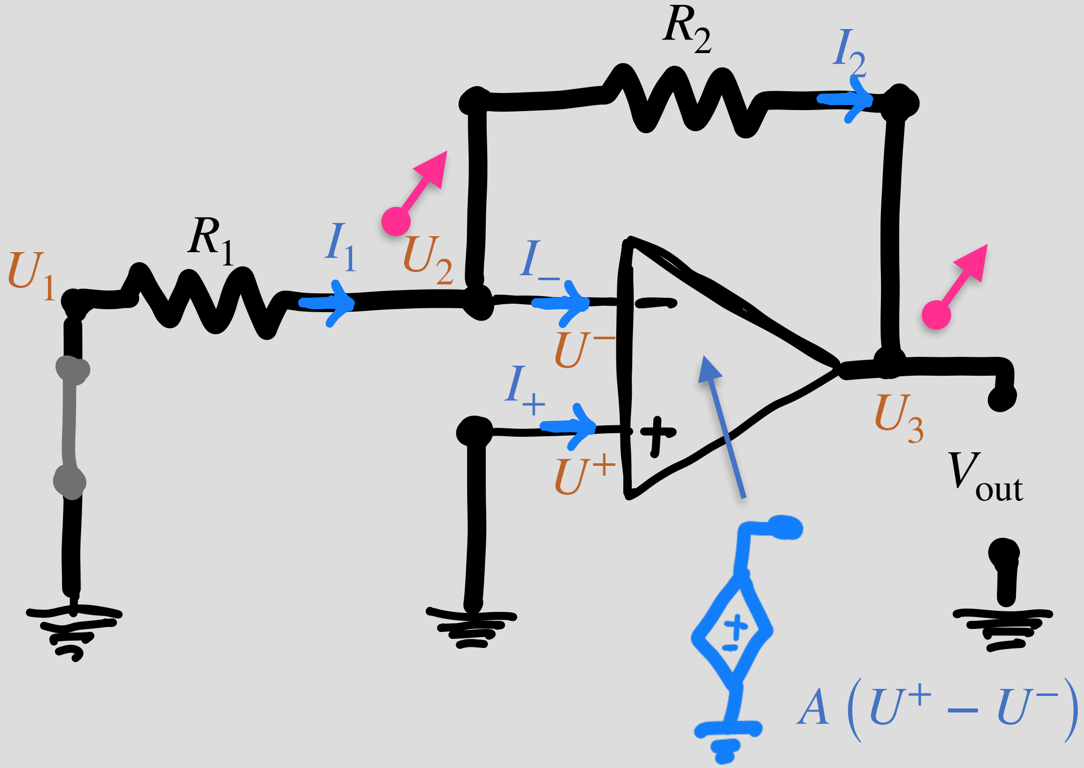
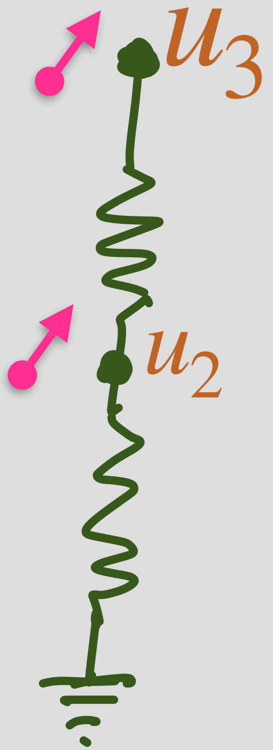
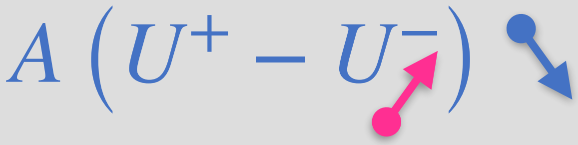
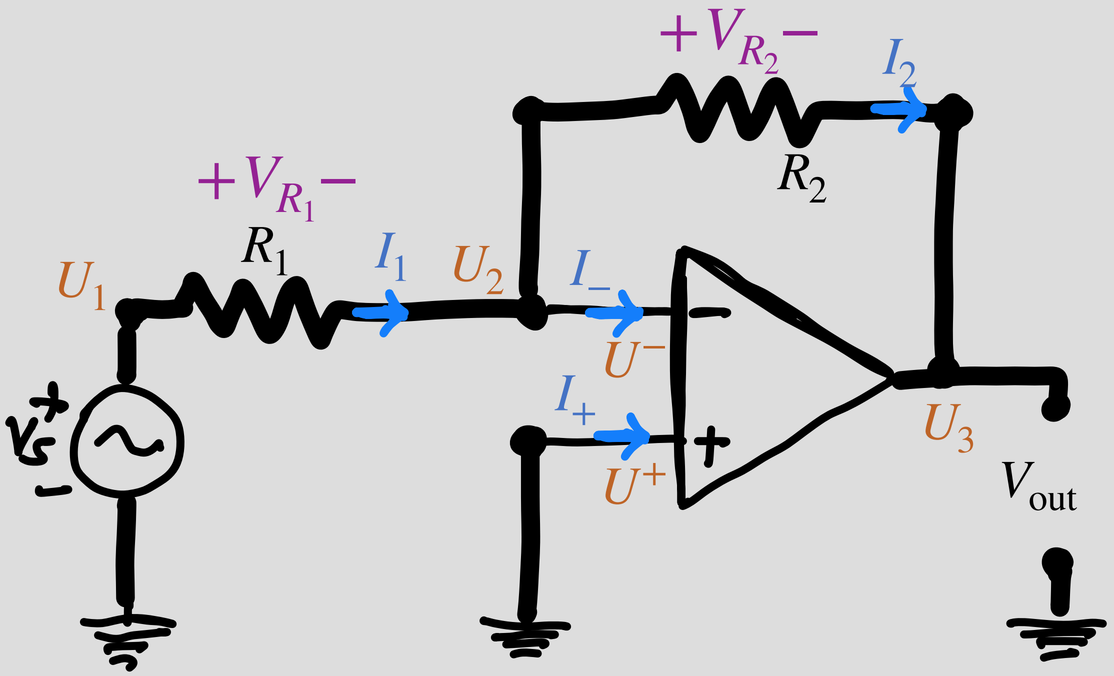






Negative feedback
Golden Rule #2 applies!



$$\text{NFB} \Rightarrow \text{Golden Rule \#2} \Rightarrow U^- = U^+$$

①

$U_1 = V_{in}$

$U_3 = V_{out}$

$U_2 = 0$

$$U_2 = U^- \Rightarrow \text{NFB} \Rightarrow U^- = U^+ = 0$$

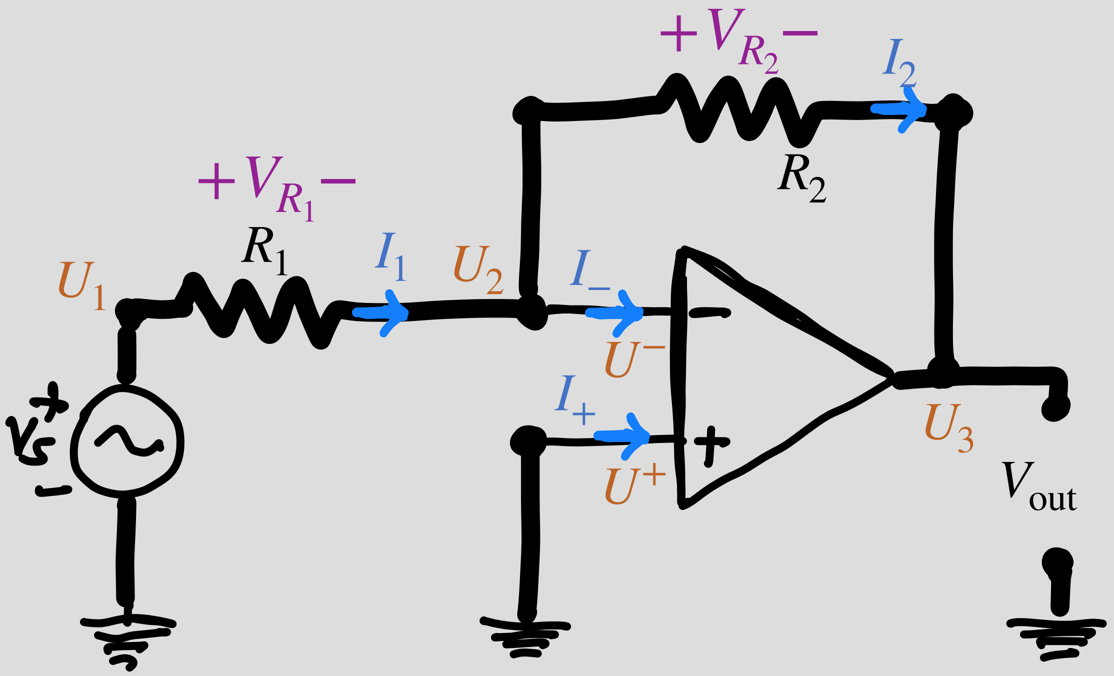
② Element Definitions:

$V_{R_1} = I_1 R_1$

$V_{R_2} = I_1 R_2$

$V_{R_1} = U_1 - \cancel{U_2} = V_{in}$

$V_{R_2} = \cancel{U_2} - U_3 = -V_{out}$



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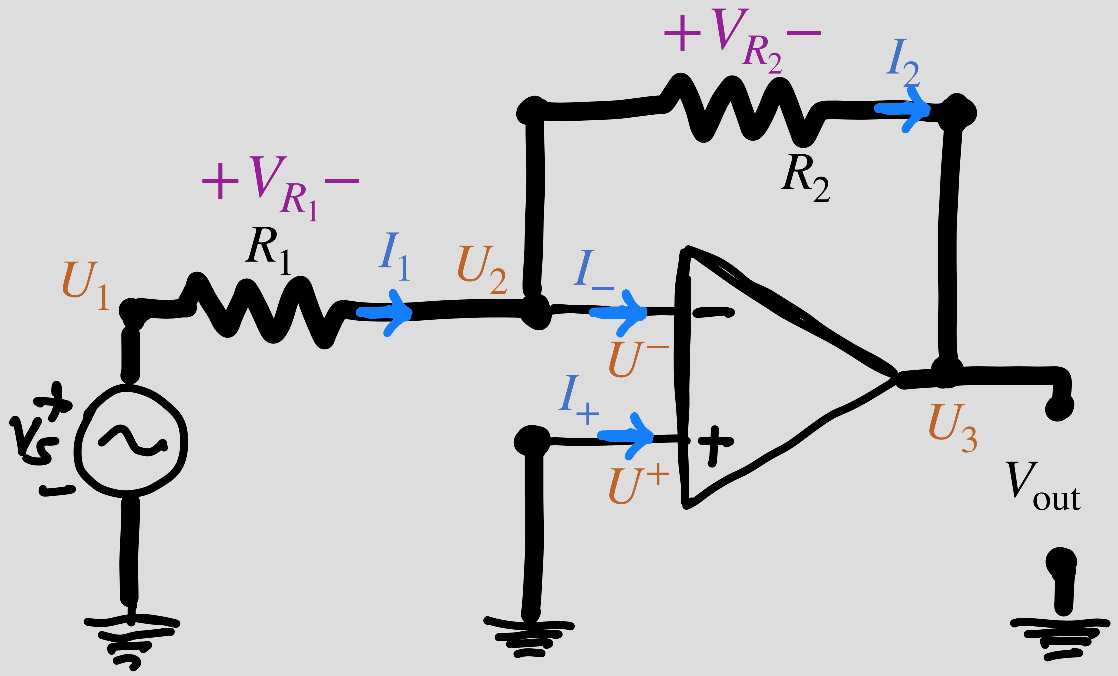
③ (KCL)

$I_1 = I_2 + \cancel{I_-}$ (GR#1)



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③ (KCL)

$I_1 = I_2 + \cancel{I_-}$ (GR#1)

$V_{in} = I_1 R_1$

$V_{out} = -I_2 R_2$

$I_2 = I_1$

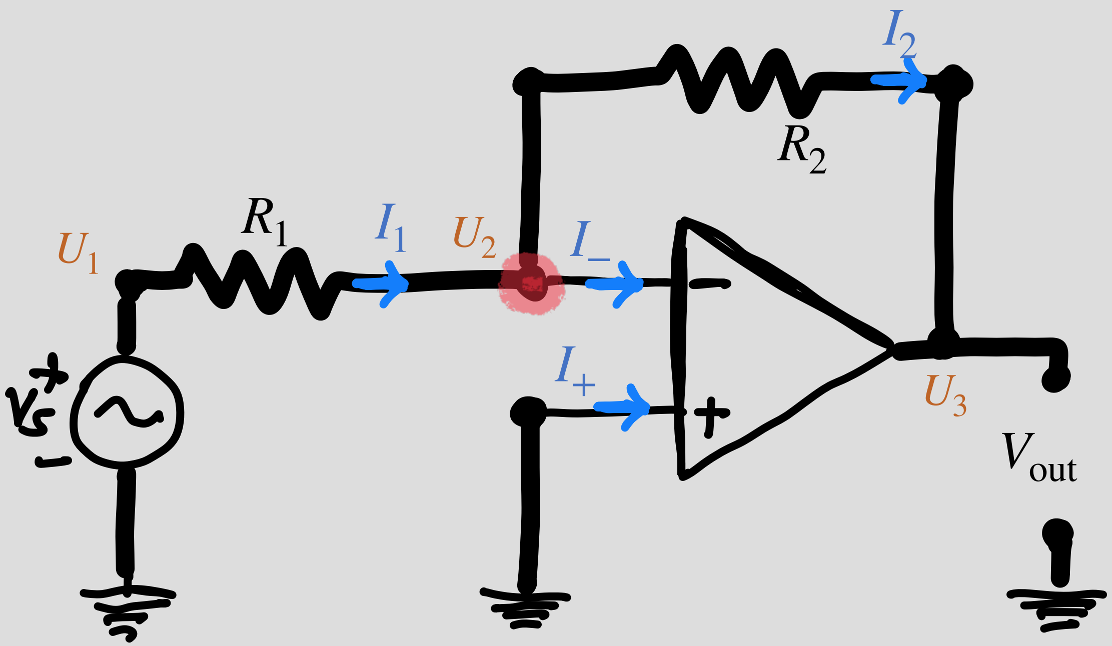
$$-\frac{V_{out}}{R_2} = \frac{V_{in}}{R_1}$$

$$V_{out} = -\frac{R_2 V_{in}}{R_1}$$

$$Av = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Inverting Amplifier!

A faster way…



NFB $\Rightarrow$ Golden Rule #2 $\Rightarrow U^- = U^+$

$$U^+ = 0 \;\Rightarrow\; U^- = 0 \;\Rightarrow\; U_2 = 0$$

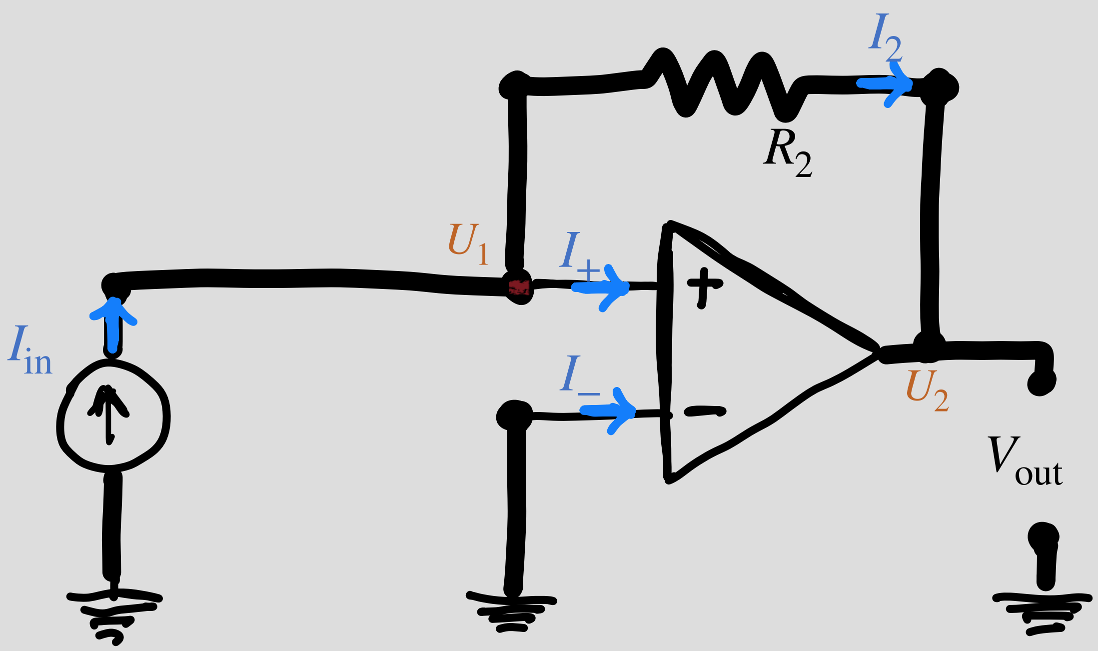
GR #1 + KCL $\quad I_1 = I_2 + \cancel{I_-}$

$$\frac{U_1 - \cancel{U_2}}{R_1} = \frac{\cancel{U_2} - U_3}{R_2}$$

$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{R_2}$$

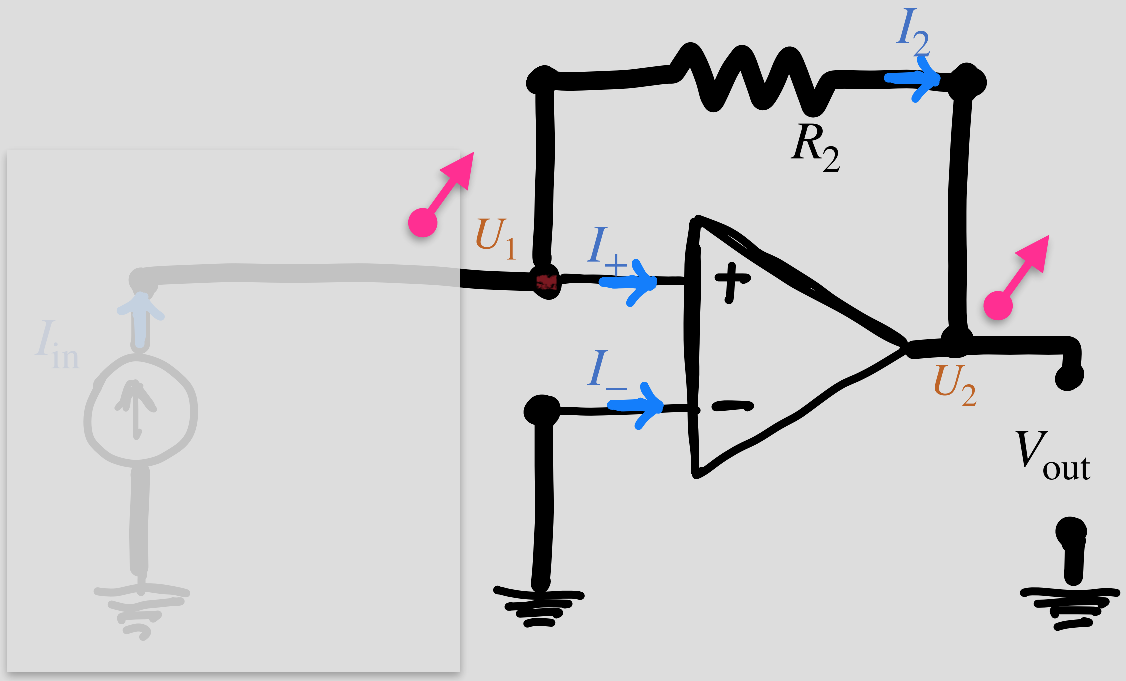
$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Example circuit 2 (trans-resistance amplifier)



① Zero-out independent sources

Example circuit 2 (trans-resistance amplifier)



① Zero-out independent sources

From GR #1: $I_+ = 0 \quad \Rightarrow I_2 = 0$

$\Rightarrow U_2 = U_1$

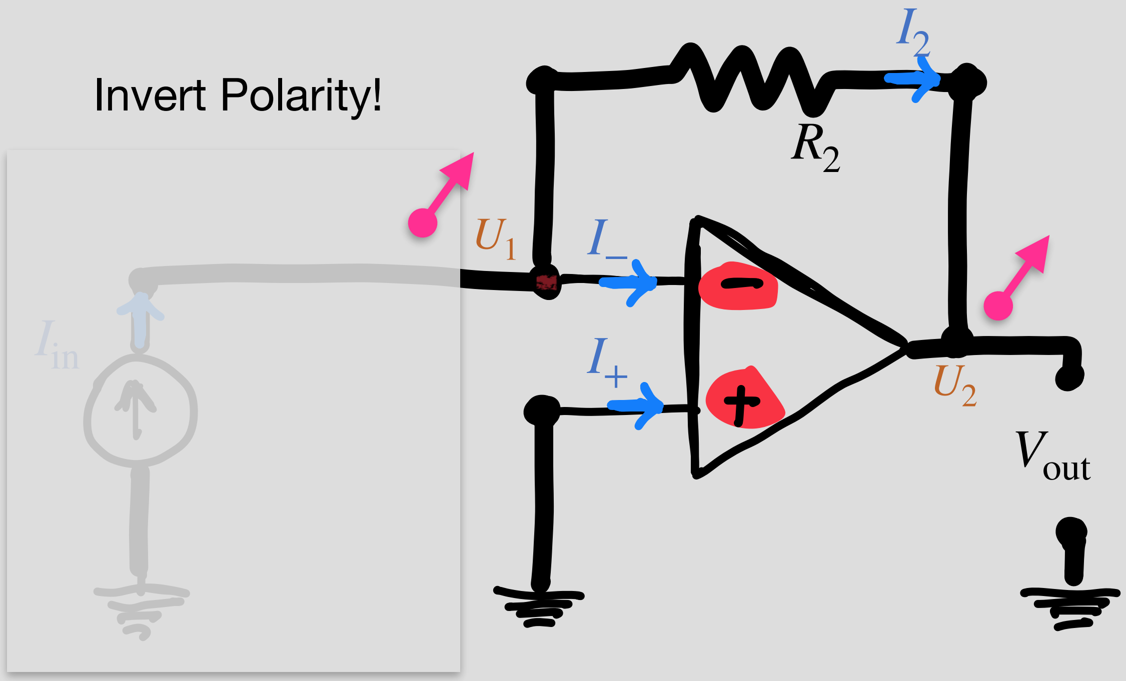
② Check for negative feedback

$$A\left(U^+ - U^-\right)$$

Increasing output, increases U^+, increases output

Not in Negative feedback 😭

Example circuit 2 (trans-resistance amplifier)



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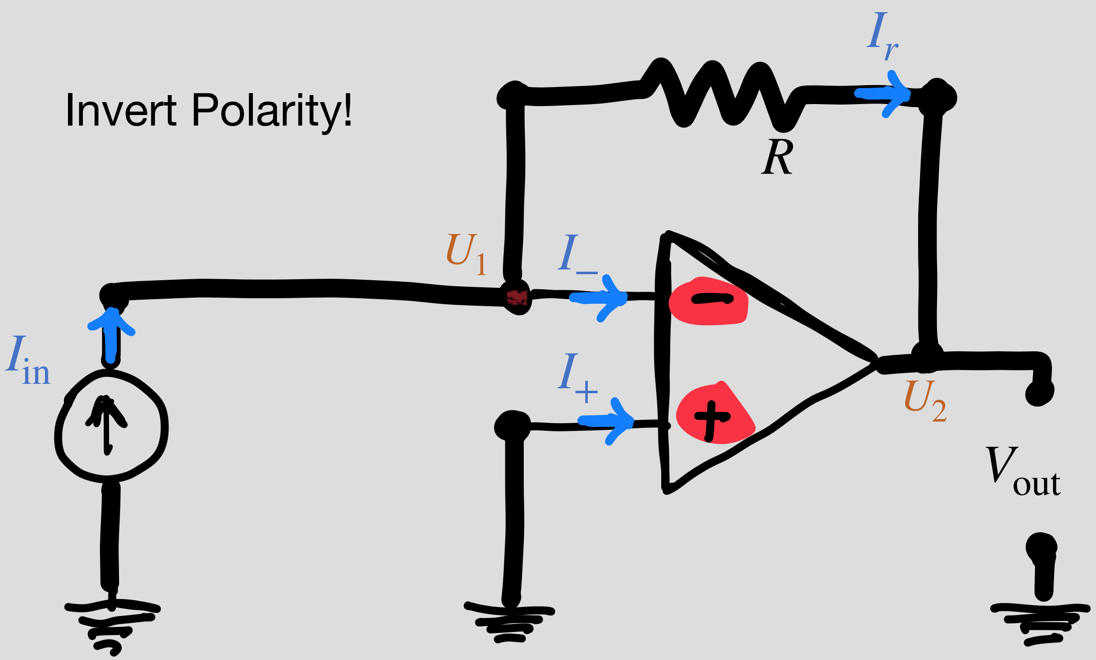
Increasing output, increases U^-, decreases output

in Negative feedback 😍

Example circuit 2 (trans-resistance amplifier)

Invert Polarity!



NFB $\Rightarrow$ Golden Rule #2 $\Rightarrow U^- = U^+$

$$U^+ = 0 \quad \Rightarrow U^- = 0 \quad \Rightarrow U_1 = 0$$

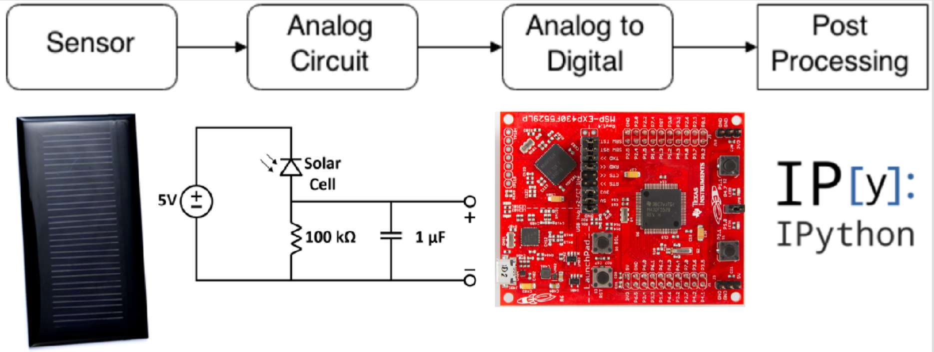
Golden Rule #1 & KCL $\quad I_{in} = I_r + \cancel{I_-}$

$$I_{in} = \frac{\cancel{U_1} - U_2}{R_1}$$

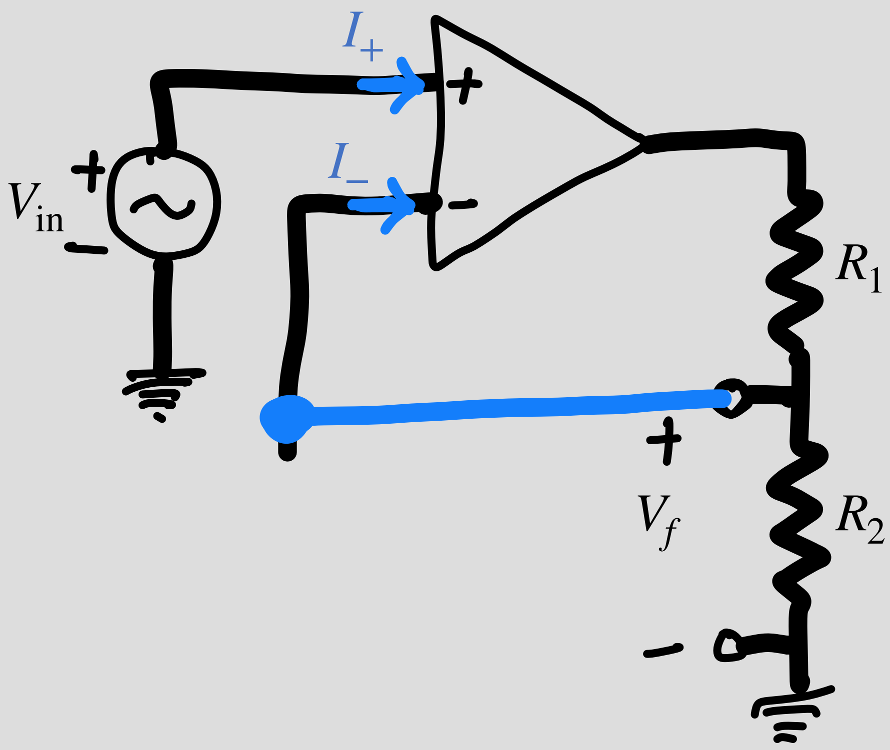
$$V_{out} = -R_1 I_{in}$$

$$\boxed{\frac{V_{out}}{I_{in}} = -R}$$

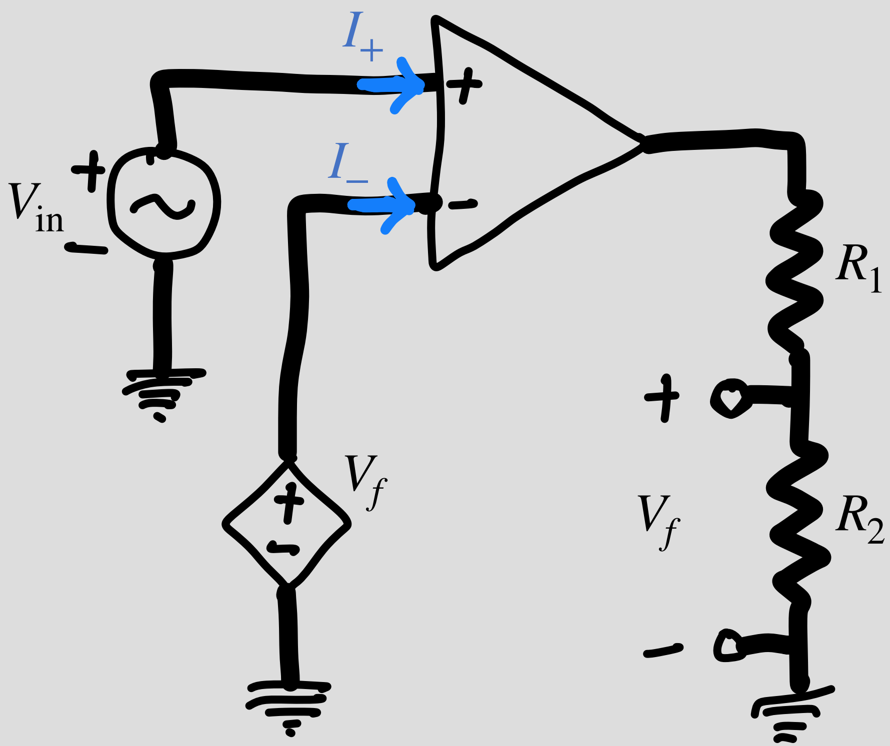
Input current, output is voltage!



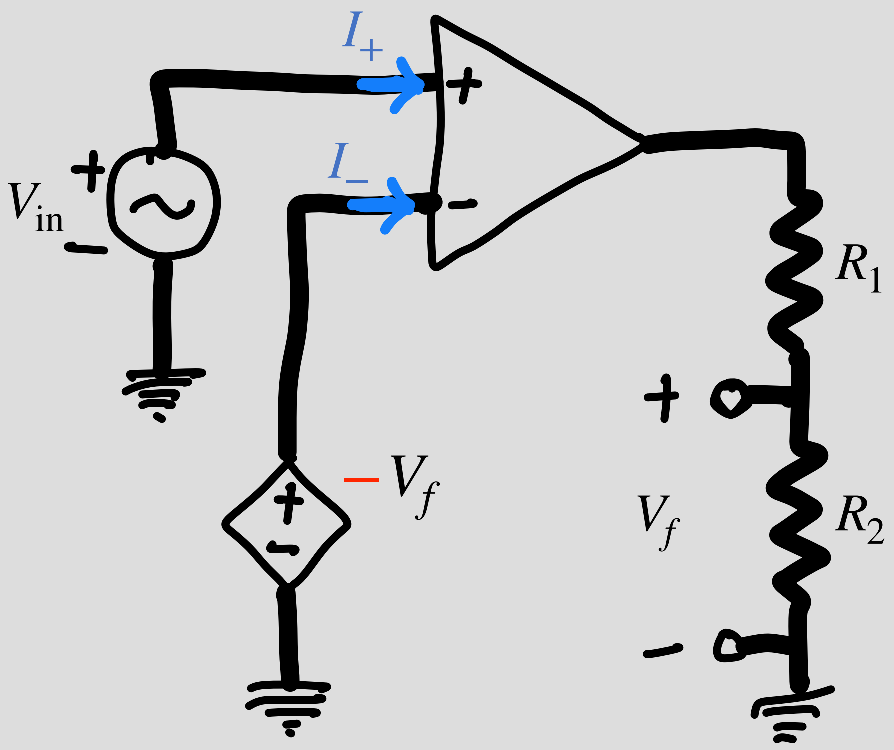
Example circuit 3 -



Example circuit 3 -

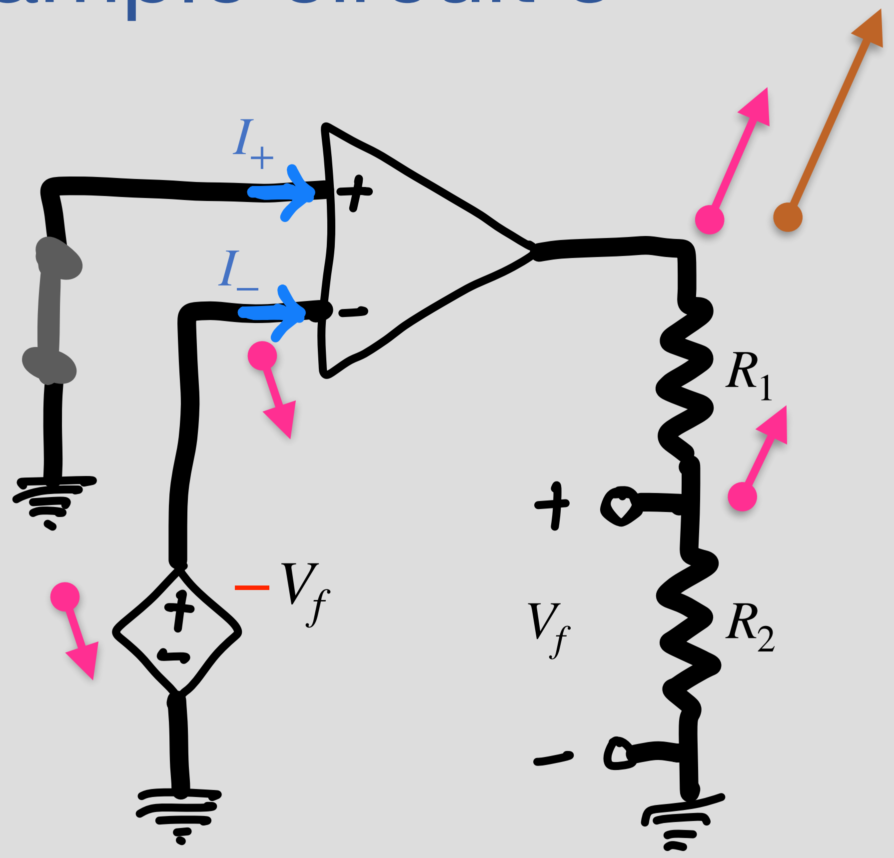


Example circuit 3 -



1. Zero-out independent sources

Example circuit 3 -



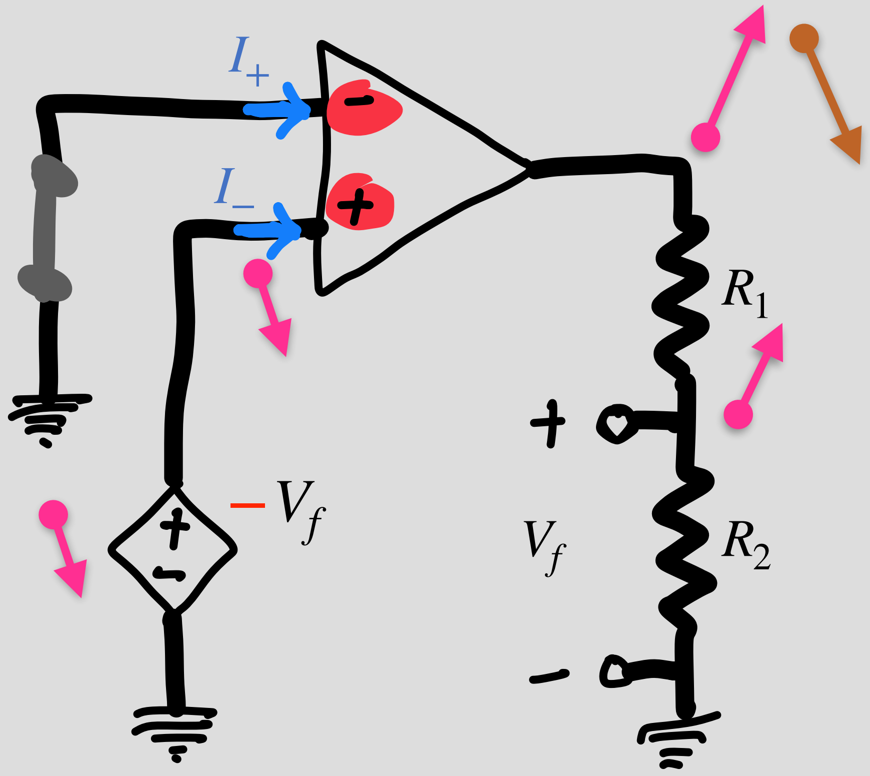
1. Zero-out independent sources
2. Check for negative feedback

$$A\left(U^+ - U^-\right)$$

Increasing output, decreases U^-, increases output

Not in Negative feedback 😭

Example circuit 3 -



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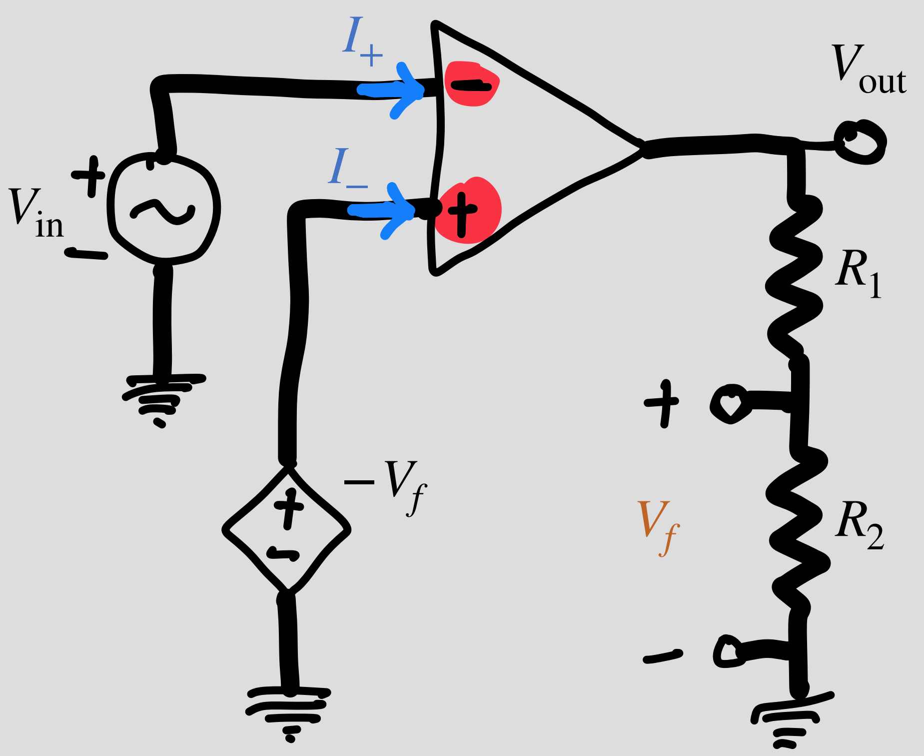


NFB $\Rightarrow$ Golden Rule #2 $\Rightarrow U^- = U^+$

$$\Rightarrow V_{in} = -V_f$$

Voltage divider:

$$V_f = \frac{R_2}{R_1 + R2} V_{out}$$

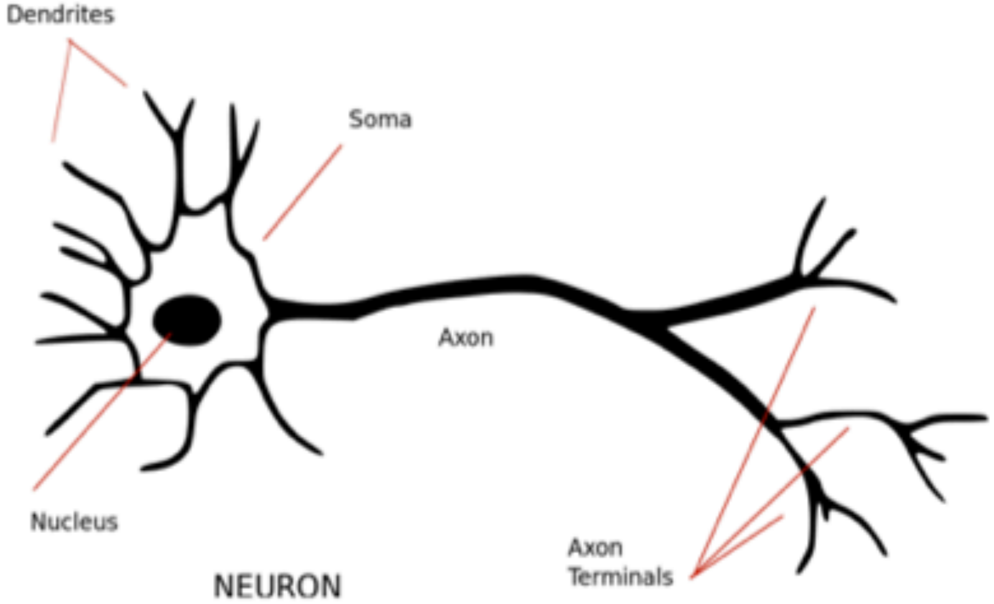
$$V_{in} = -\frac{R_2}{R_1 + R2} V_{out}$$

$$Av = \frac{V_{out}}{V_{in}} = -\frac{R_1 + R2}{R_2} = -\left(1 + \frac{R_1}{R_2}\right)$$

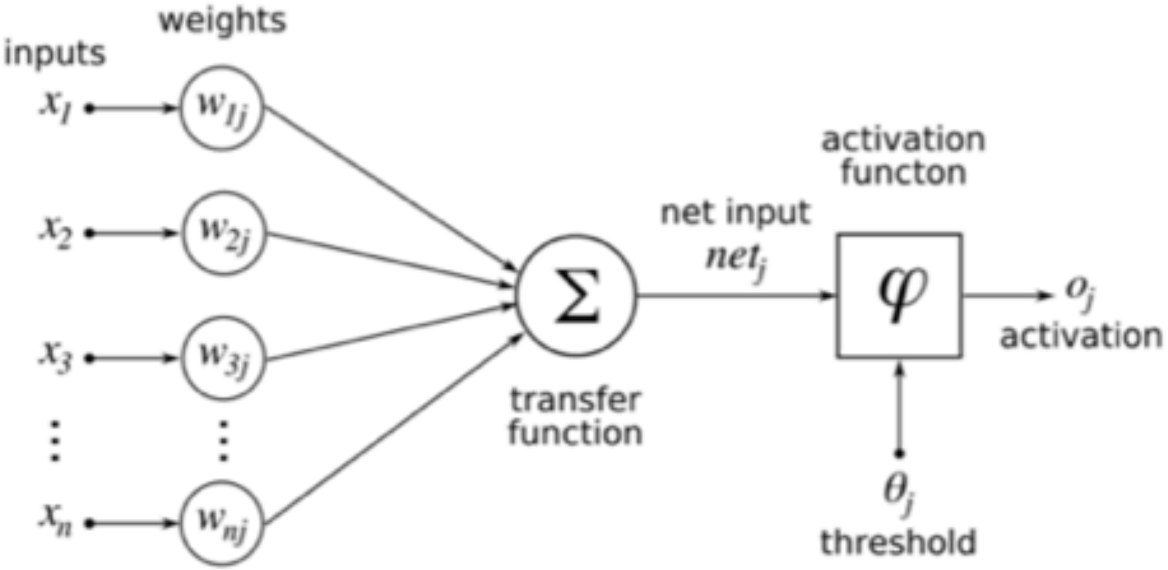
Artificial Neuron

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$$\begin{bmatrix} w_{1j} & w_{2j} & \cdots w_{nj} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \sum_{i=1}^{n} w_{ij} x_i$$



A biological Neuron



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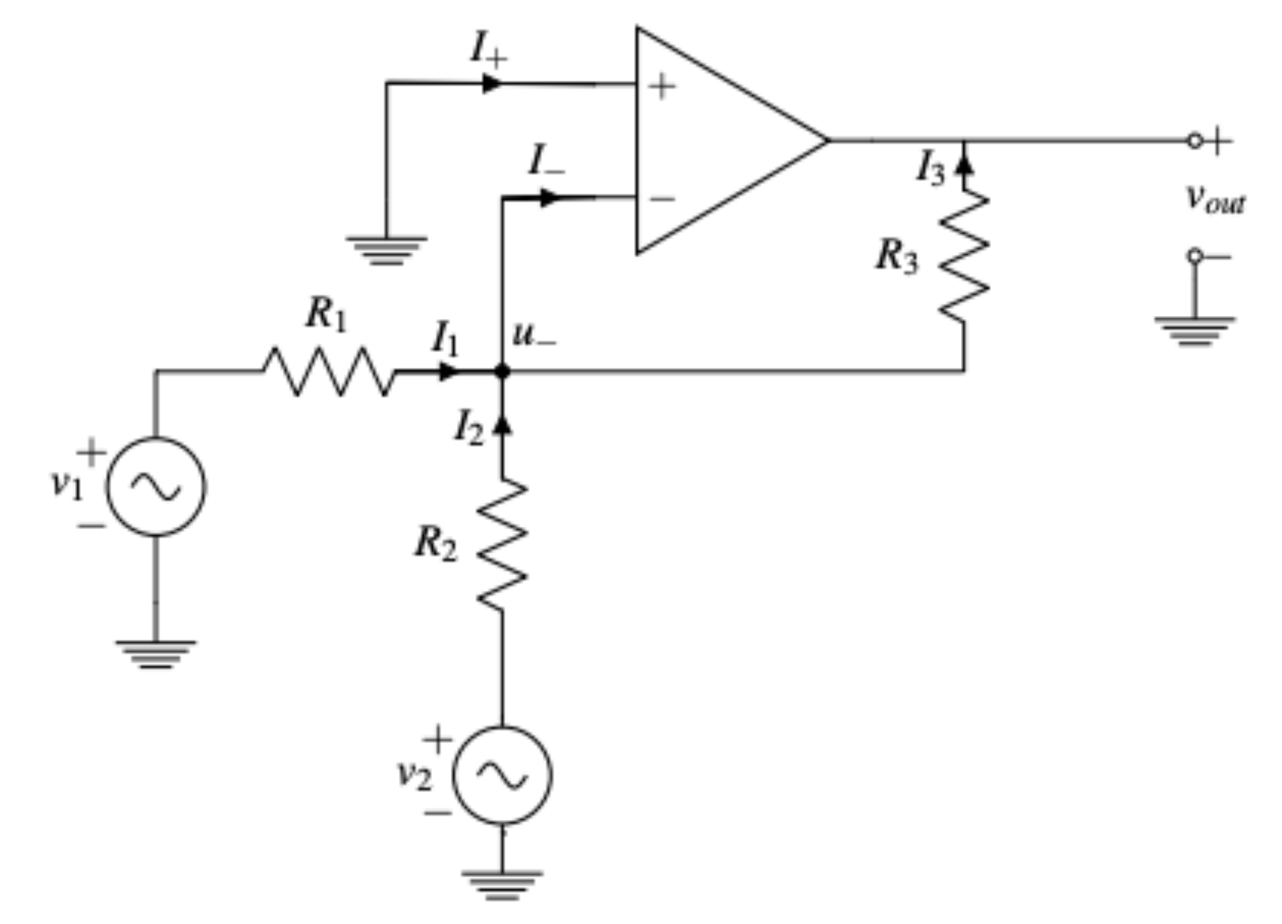

(1) Zero-out independent sources

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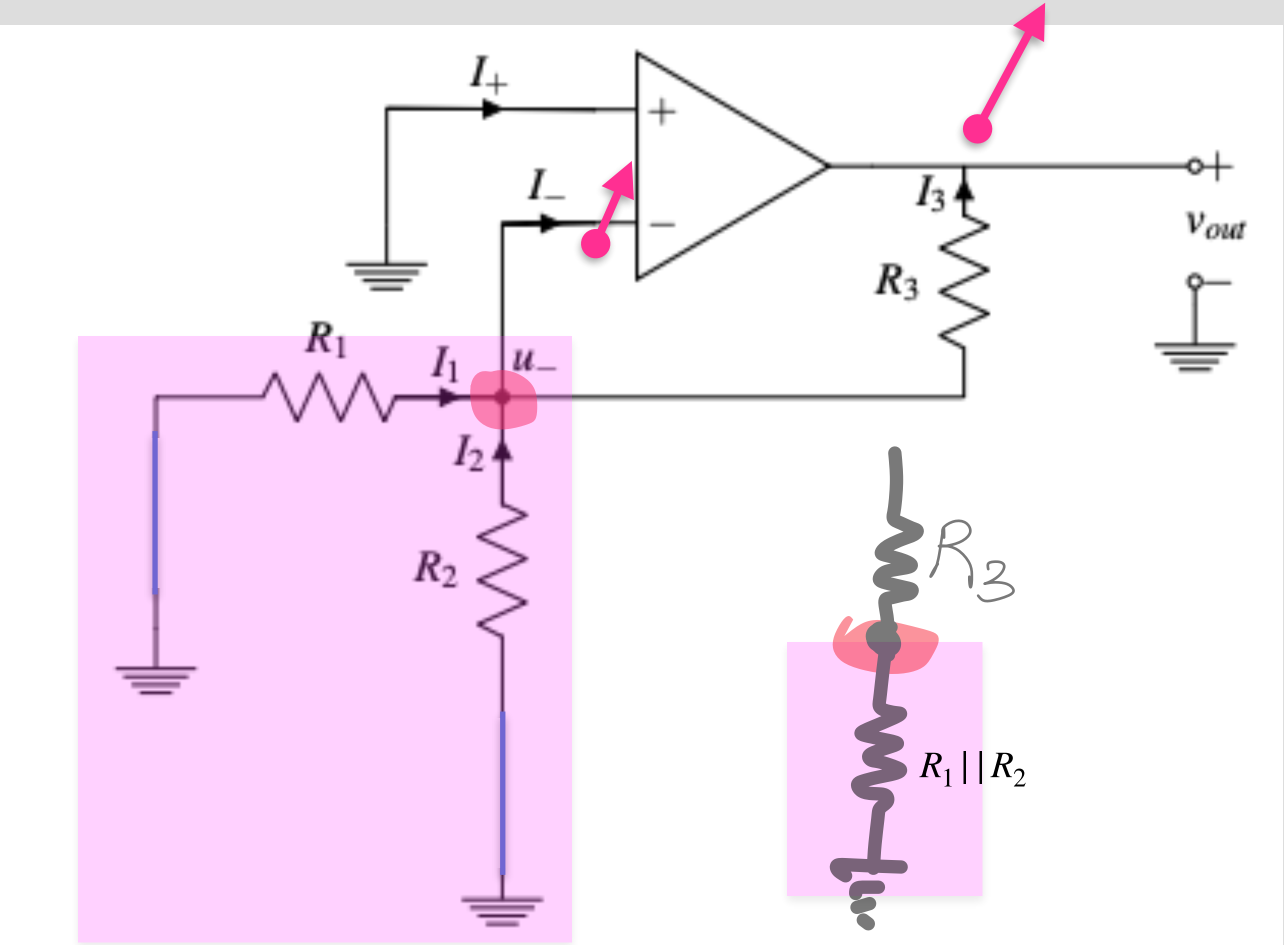

① Zero-out independent sources

② Check for negative feedback

$$A\left(U^+ - U^-\right)$$

Increasing output, increases U^-, decreases output

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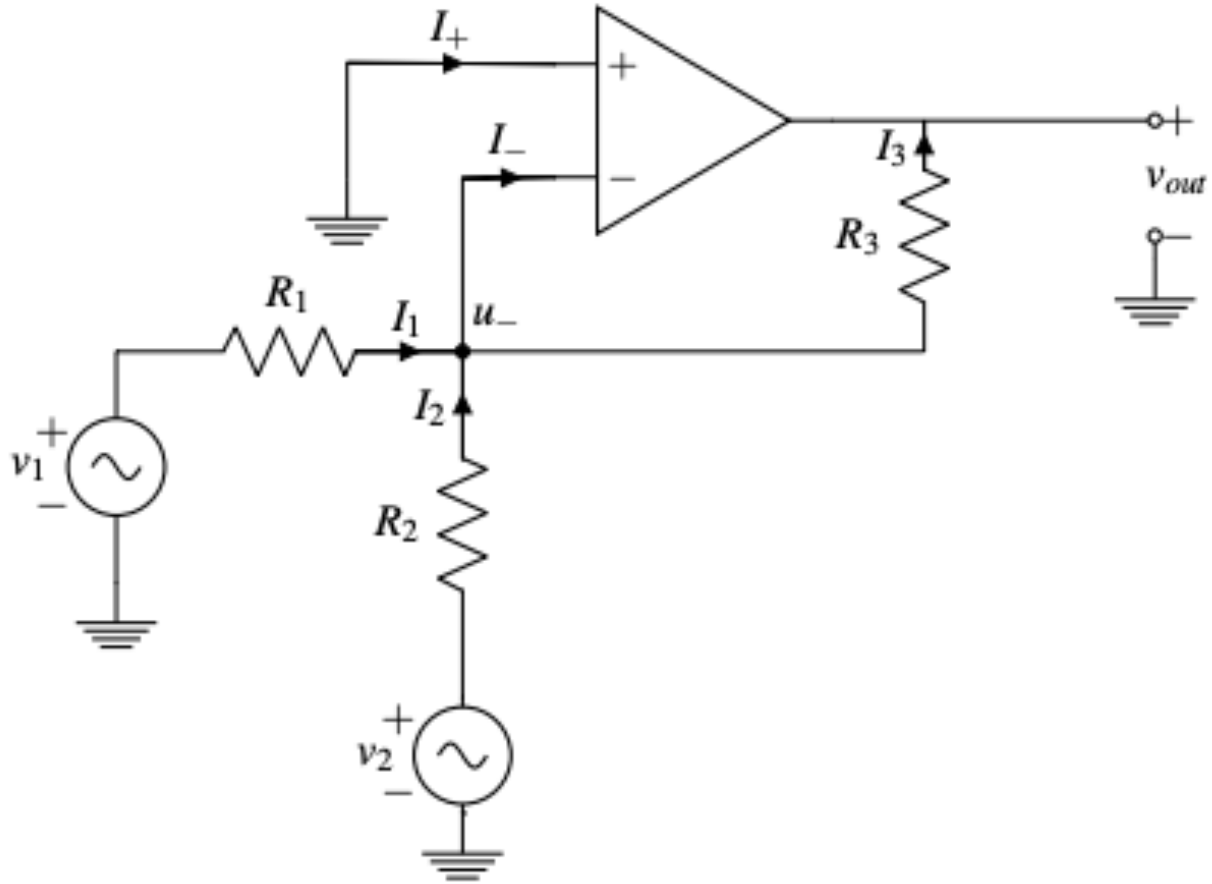


$\text{NFB} \Rightarrow$ Golden Rule #2 $\Rightarrow U^- = U^+$

$U^+ = 0 \Rightarrow U^- = 0$

KCL: $I_1 + I_2 = I_3 + \cancel{I_-}$

$$\frac{\cancel{U^-} - V_1}{R_1} + \frac{\cancel{U^-} - V_2}{R_2} = \frac{V_{out} - \cancel{U^-}}{R_3}$$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3}$$

$$V_{out} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 \cdots - \frac{R_3}{R_i}V_1 \cdots$$

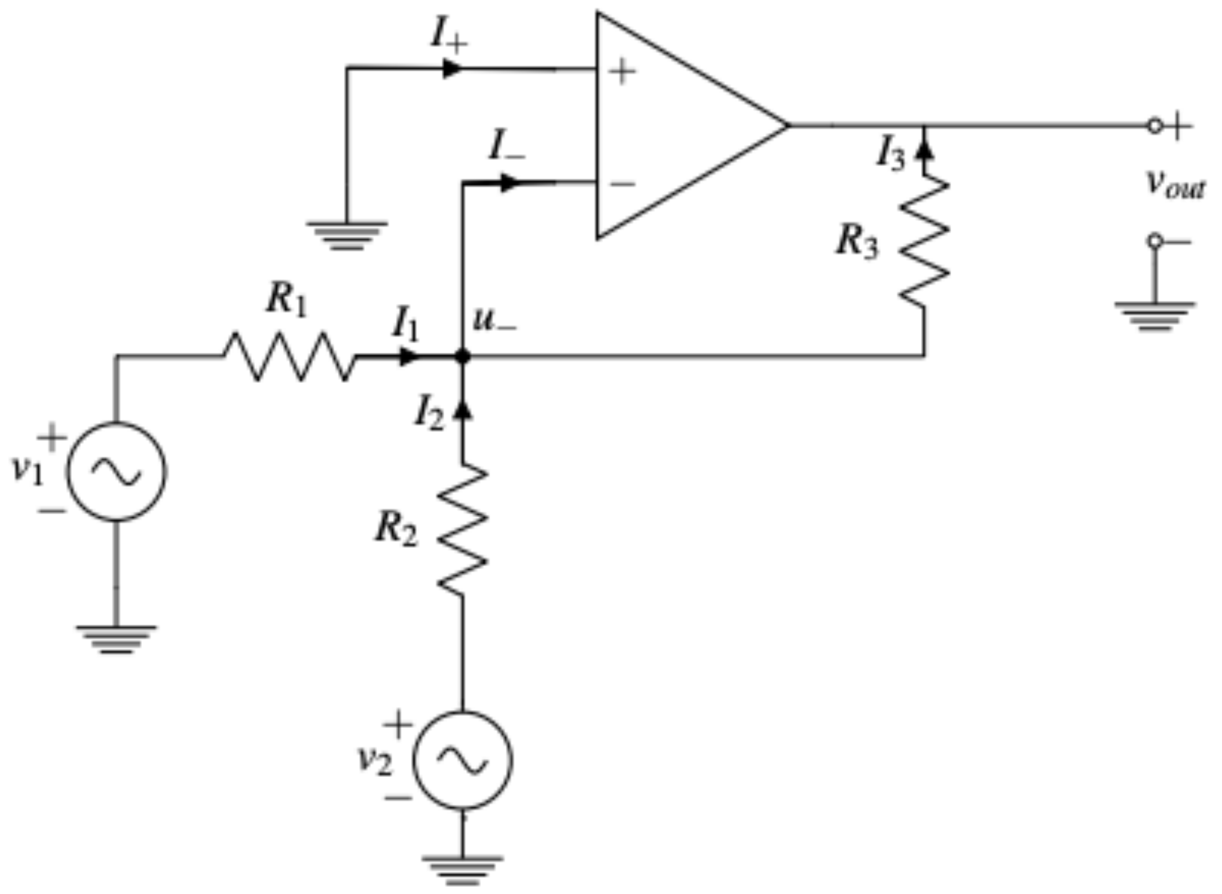
Artificial Neuron

$$V_{\text{out}} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 \cdots -\frac{R_3}{R_i}V_1 \cdots$$

w_1 w_2 w_i

Q: All weights are negative. How can we change sign?

A: Add an inverting amp circuit?



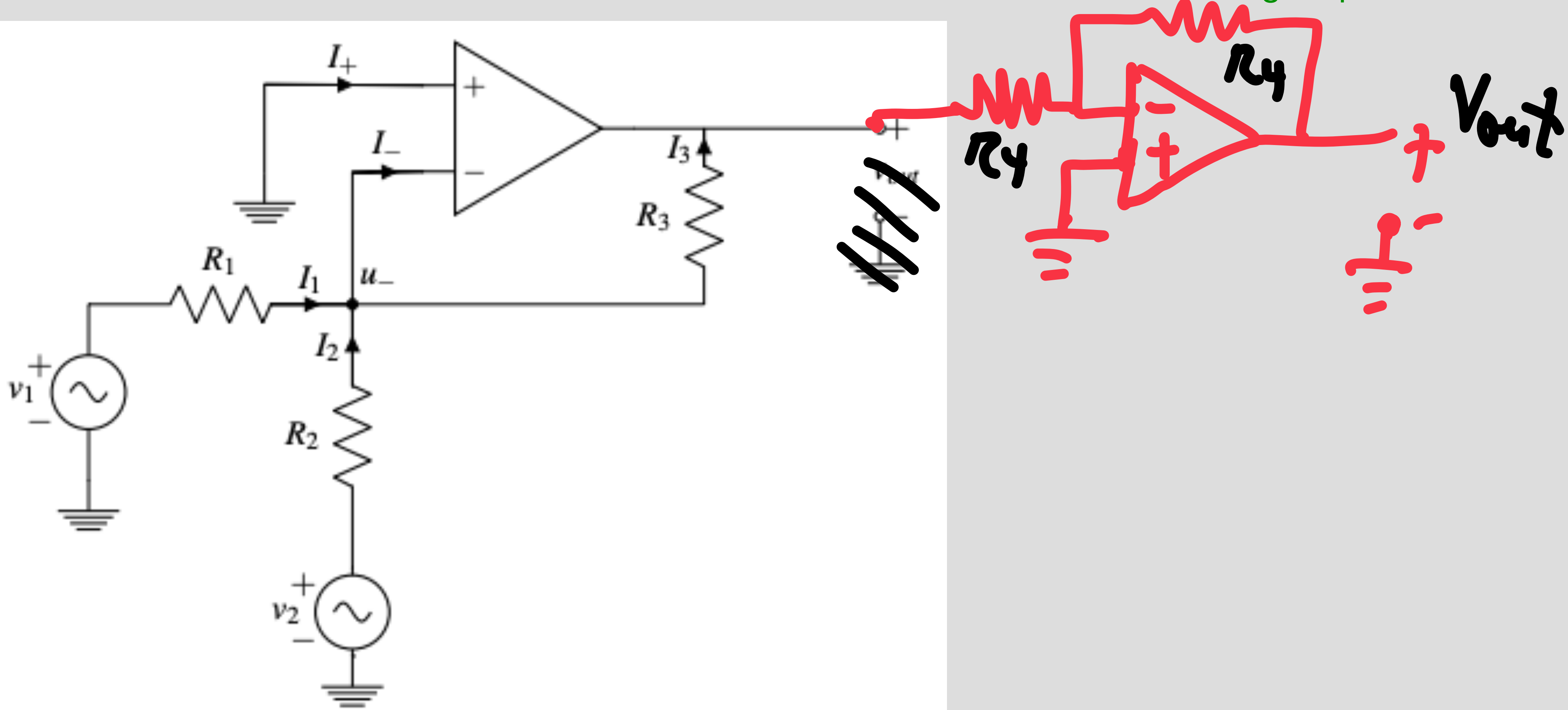
Artificial Neuron

$$V_{out} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 \cdots - \frac{R_3}{R_i}V_1 \cdots$$

$\omega_1 \quad \omega_2 \quad \omega_i$

Q: All weights are negative. How can we change sign?

Q: Can we inverting amp circuit?



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ω_1 ω_2 ω_i

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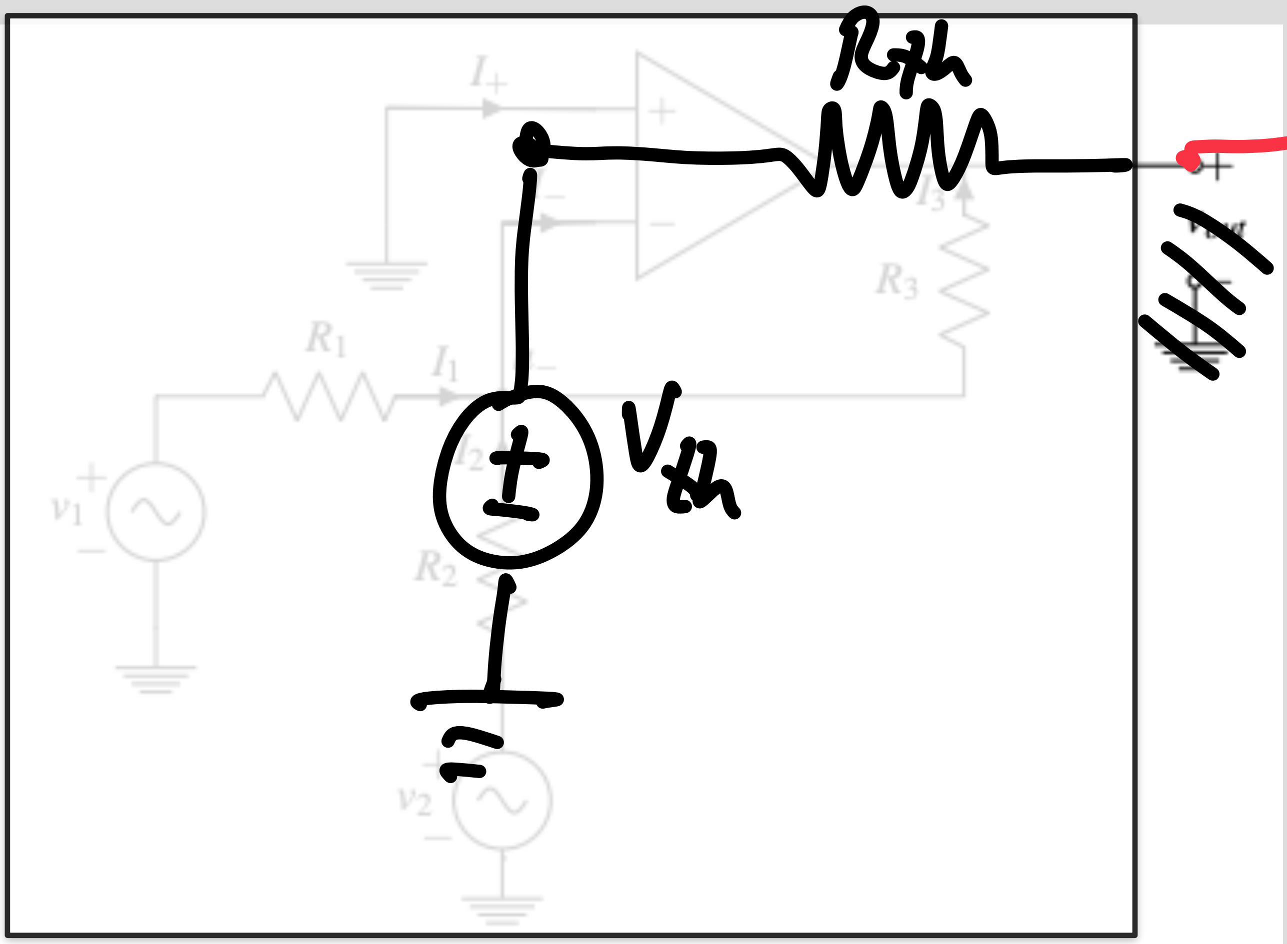

A: Not always…. But perhaps here is OK.

Q: What’s the requirement on Rth?

A: Rth = 0?

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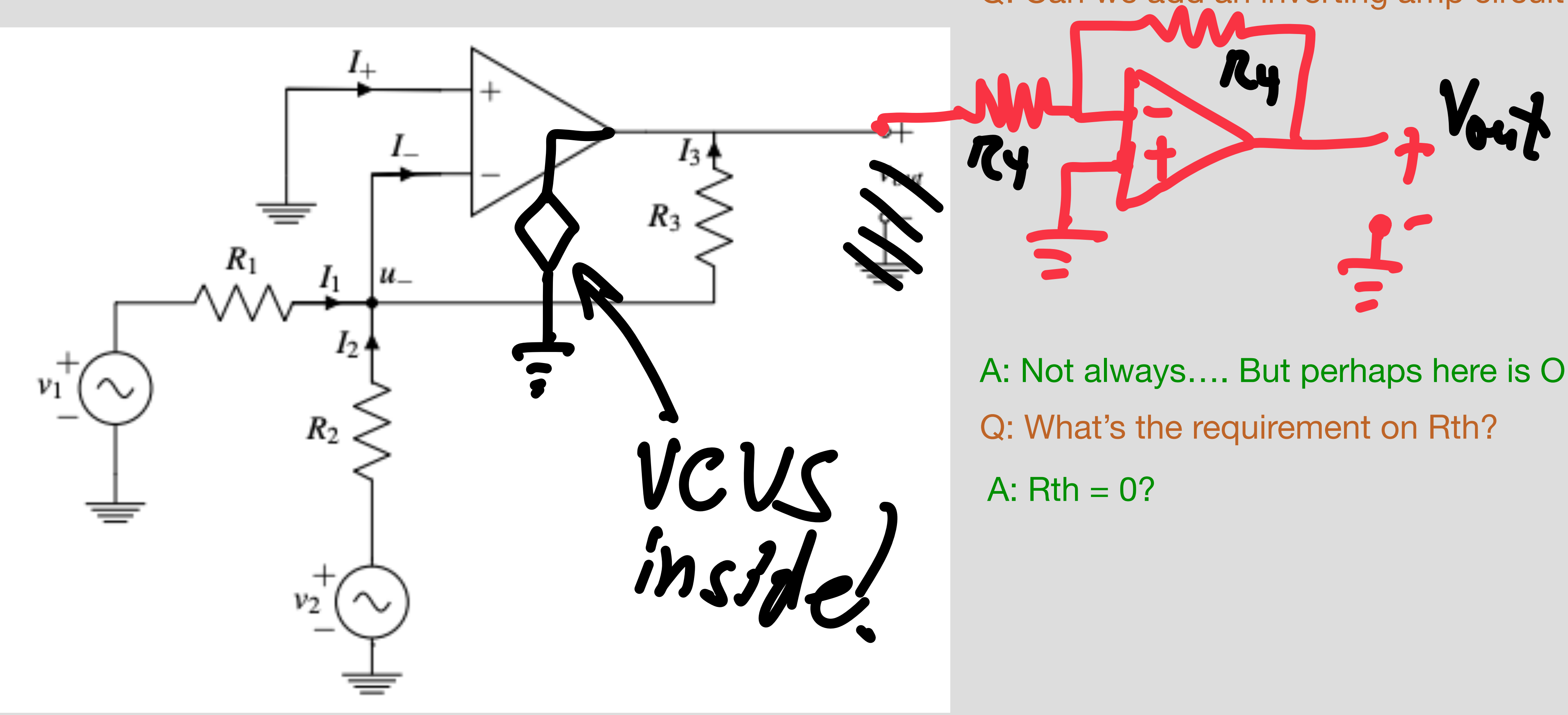

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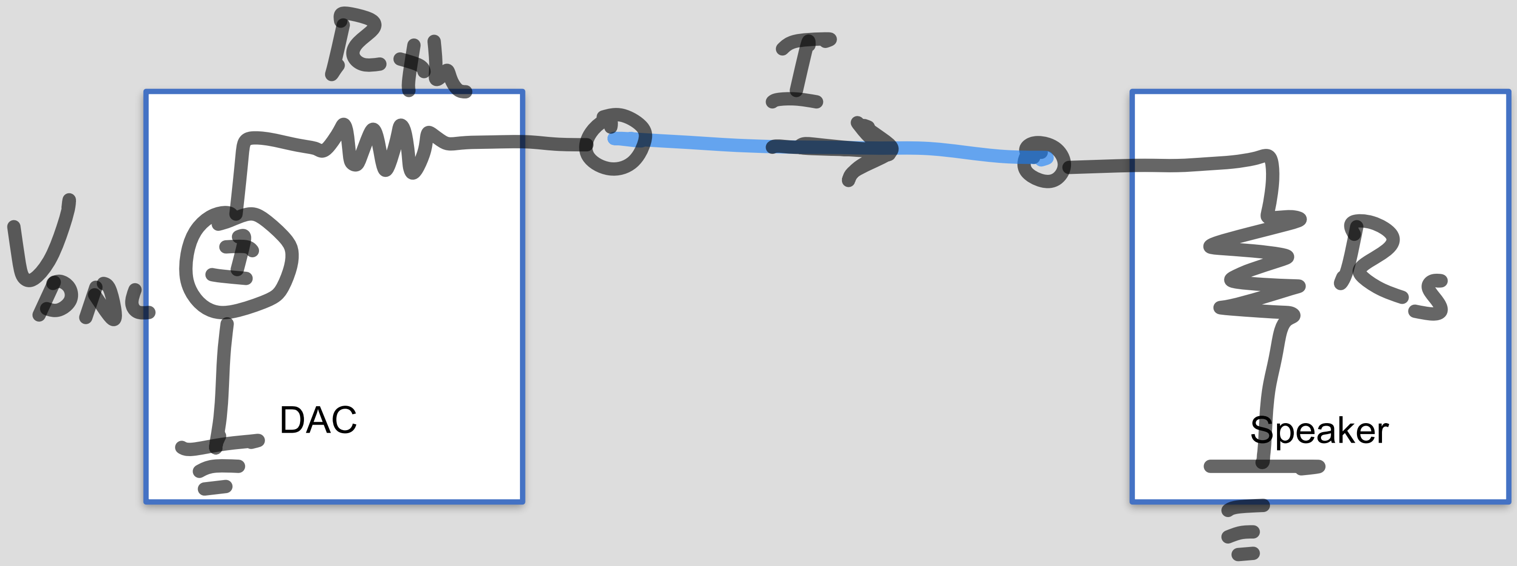
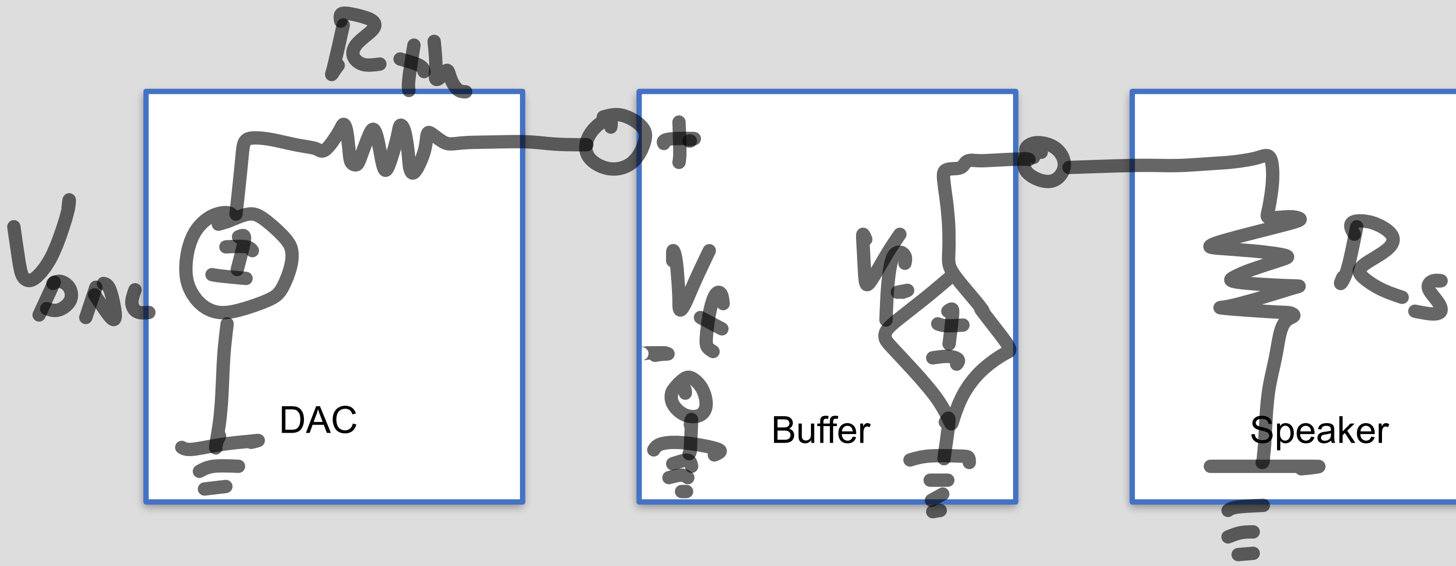
Unity Gain Buffer

- Safely cascading circuit modules



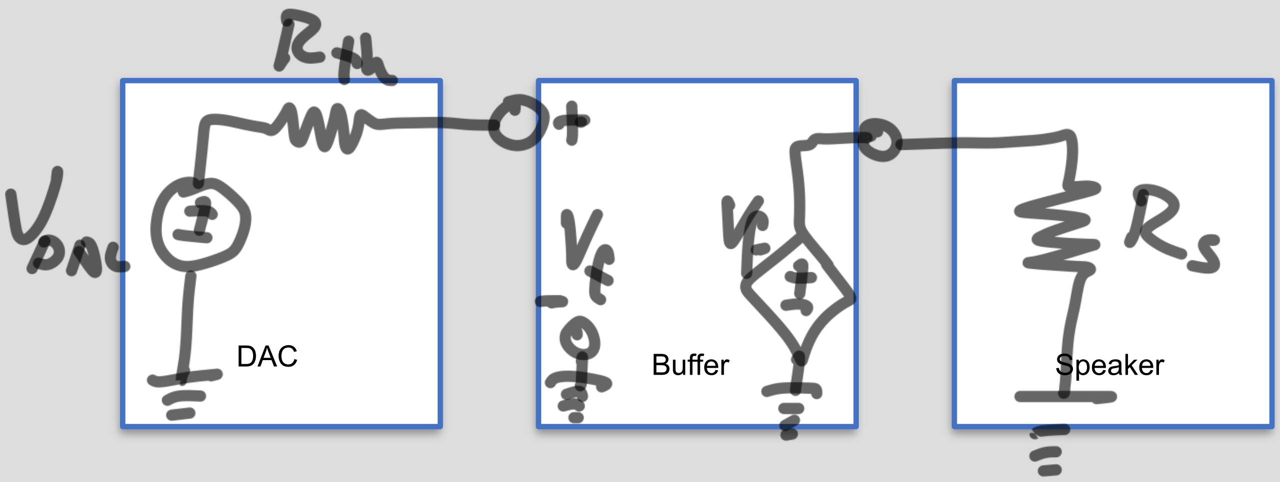
Unity Gain Buffer

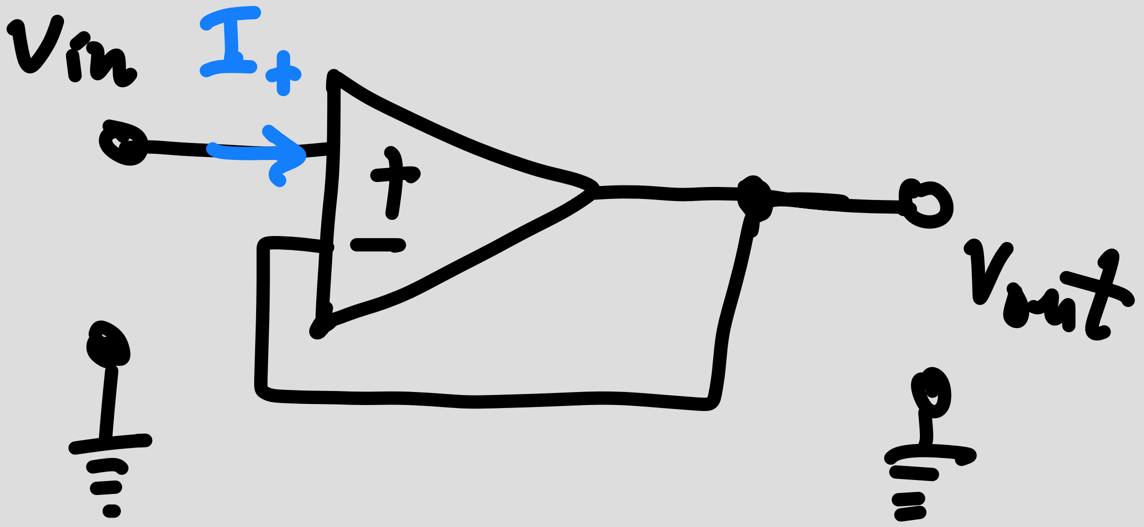
- Safely cascading circuit modules



Unity Gain Buffer

- Safely cascading circuit modules





$U^{+} = V_{in}$

$U^{-} = V_{out}$

NFB $\Rightarrow$ Golden Rule #2 $\Rightarrow U^{-} = U^{+}$

$V_{in} = V_{out}$