

# Welcome to EECS 16A!

## Designing Information Devices and Systems I

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Fall 2021

Module 2  
Lecture 2  
Introduction to Circuit Analysis  
(Note 12)



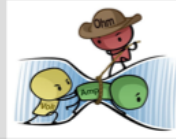
# Last Class (0203)...

## Electronic Materials

- Conductors
- Semiconductor
- Insulators

Quantities	Analytical Symbol	Units
Current	I	Amperes (A)
Voltage	V	Volts (V)
Resistance	R	Ohms ( $\Omega$ )

- I  $\Rightarrow$  flows through an element  
V  $\Rightarrow$  applied across an element  
R  $\Rightarrow$  opposition to current flow



\* **Charge**  $\Rightarrow$  Can be either positive or negative; basic element of electric flow. Unit: Coulomb [C]

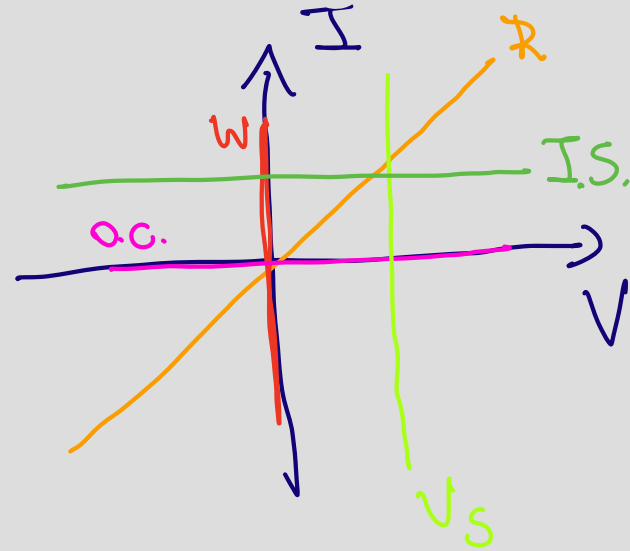
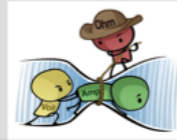
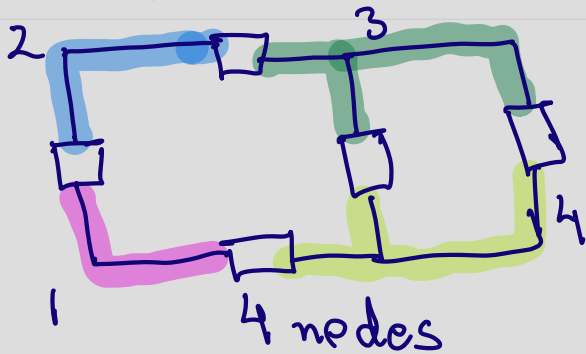
\* **Current**  $\Rightarrow$  Net amount of charge that passes through some cross-section area over a period of time.

$$[A] \rightarrow I = \frac{dQ}{dt} \frac{[C]}{[s]}$$

# Last Class (0203)...

Quantities	Analytical Symbol	Units
Current	$I$	Amperes (A)
Voltage	$V$	Volts (V)
Resistance	$R$	Ohms ( $\Omega$ )

$I \Rightarrow$  flows through an element  
 $V \Rightarrow$  applied across an element  
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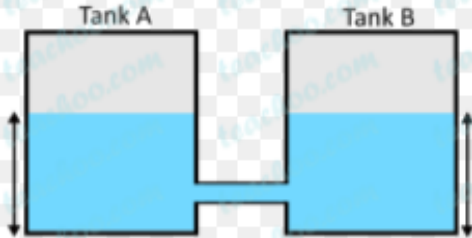


Today

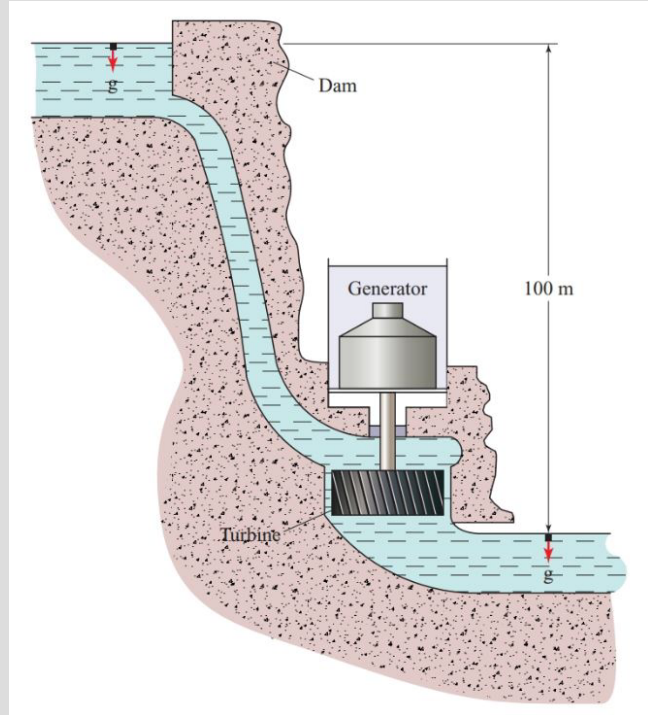
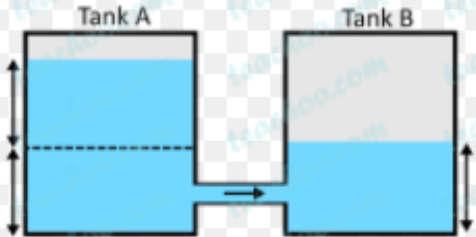
- KCL ; KVL  $\Rightarrow$  RULES
- Circuit analysis
- First operator!

# Potential Energy

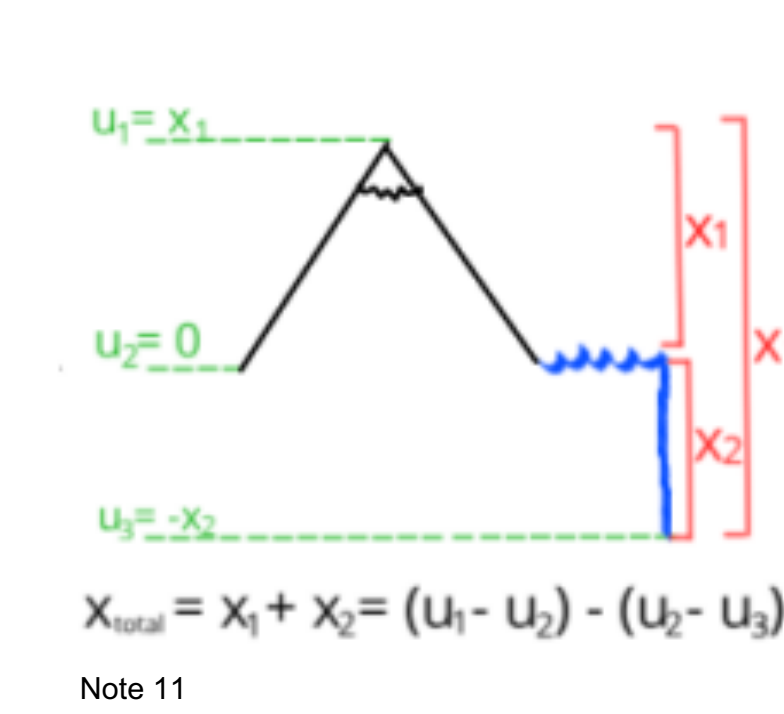
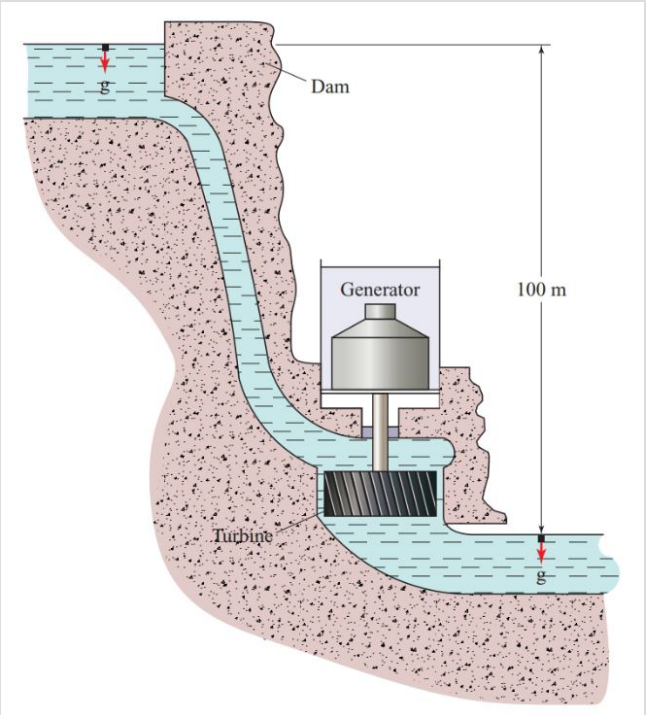
Water is exact the same level in both Tanks  
So, water will **not flow**



Since Tank A has higher water level,  
So, water **will flow** from Tank A to Tank B

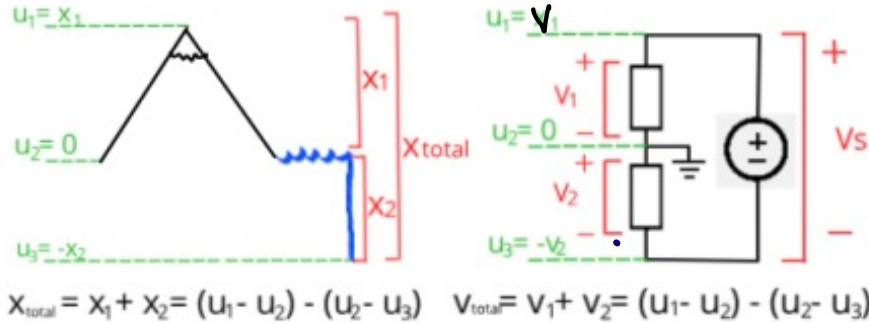


# Potential and Voltage



Note 11

# Electronic Devices depend on movement of charges



Note 11

We always need to define a reference for potentials.  
Ground = 0

$U_1, U_2, U_3$   
potentials

$$V_1 = U_1 - U_2$$

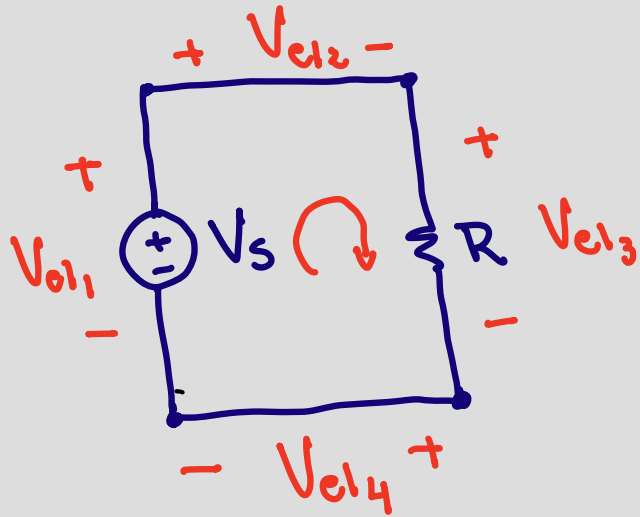
$$V_2 = U_2 - U_3$$

$$V_{total} = V_1 + V_2$$

$$V_{total} = V_S$$

# Rules for circuit analysis: Kirchoff's Voltage Law (KVL)

Sum of Voltages across the elements in a loop equal zero



$$V_{el1} - V_{el2} - V_{el3} - V_{el4} = 0$$

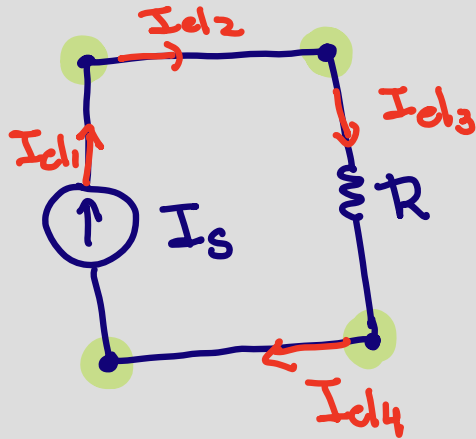
$$\begin{array}{l} + \quad \underline{\quad} V_{el2} \\ V_{el1} \quad \underline{\quad} V_{el3} \\ - \quad \underline{\quad} V_{el4} \end{array}$$

$$V_{el1} = V_s$$

$$V_{el3} = ?$$

# Rules for circuit analysis: Kirchoff's Current Law (KCL)

The current flowing into any junction must equal the current flowing out



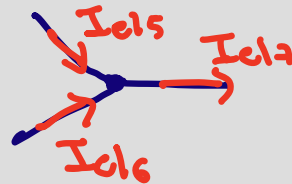
$$I_{cl1} = I_{cl2}$$

$$I_{cl2} = I_{cl3}$$

$$I_{cl3} = I_{cl4}$$

$$I_{cl4} = I_{cl1}$$

Example 2:

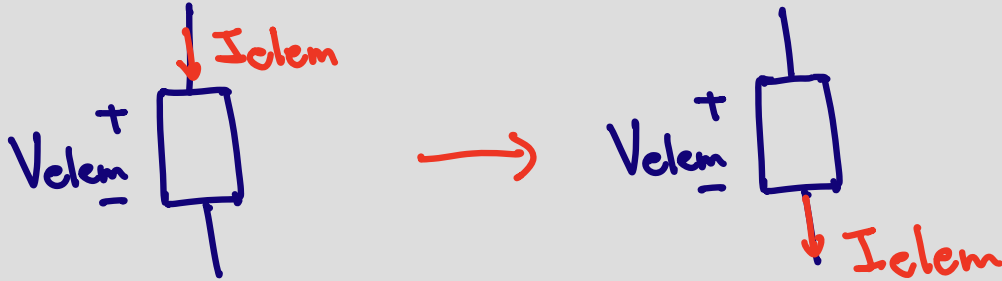


$$I_{cl5} + I_{cl6} = I_{cl7}$$



# Rules for circuit analysis: KCL within the element

The current flowing into any junction must equal the current flowing out



Same current!

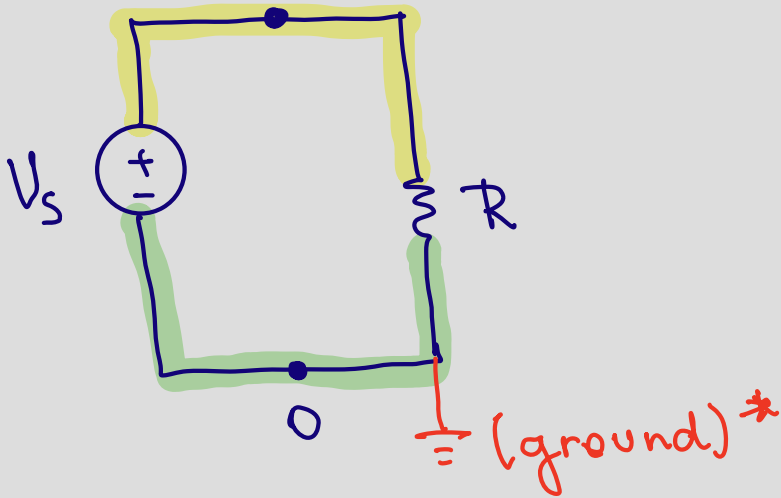
Both are allowed.

$I_{elem}$  goes into a  $\oplus$  or out of a  $\ominus$  terminal

\* Passive sign convention

# Circuit Analysis Algorithm : step 1

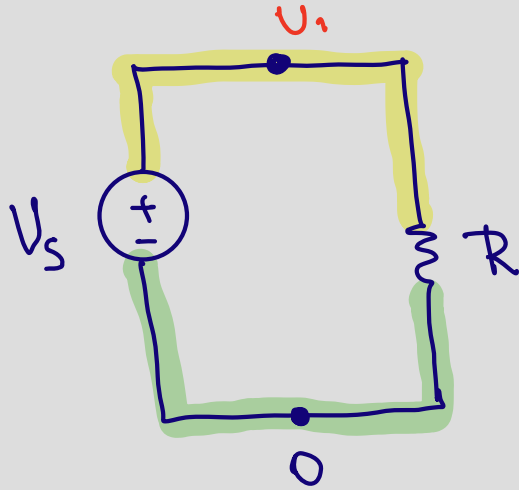
Pick a reference node and label it as 0 potential. All voltages measured relative to this node.



\* Tells you where the reference is.

# Circuit Analysis Algorithm : step 2

Label all remaining nodes as potentials  $U_i$   
[ $U_1 \dots U_{N-1}$ ]



$$U_1 - 0 = U_1 = V$$

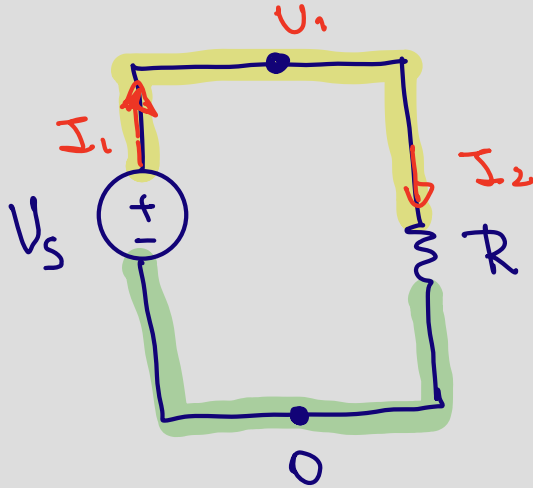
Voltage between  
node 1 and node "0".

# Circuit Analysis Algorithm : step 3

Label all branch currents with  $I_m$

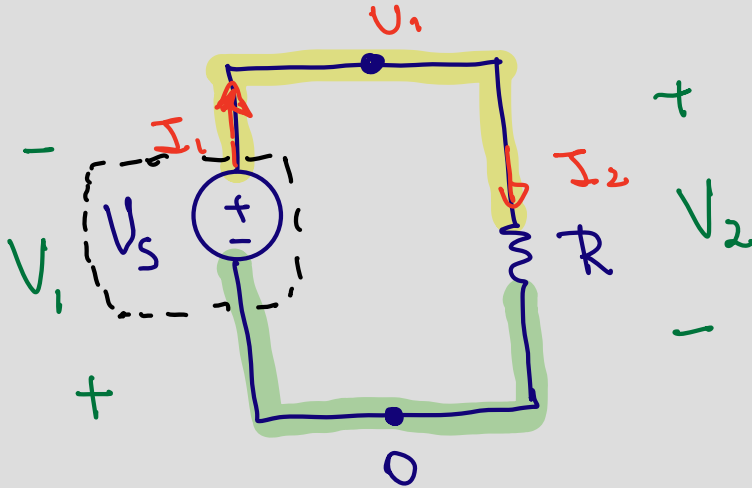
Arbitrarily pick directions of  $I_m$

[ $I_1 \dots I_k$ ]



# Circuit Analysis Algorithm : step 4

Add signs + and - element voltages to each element following the passive sign convention



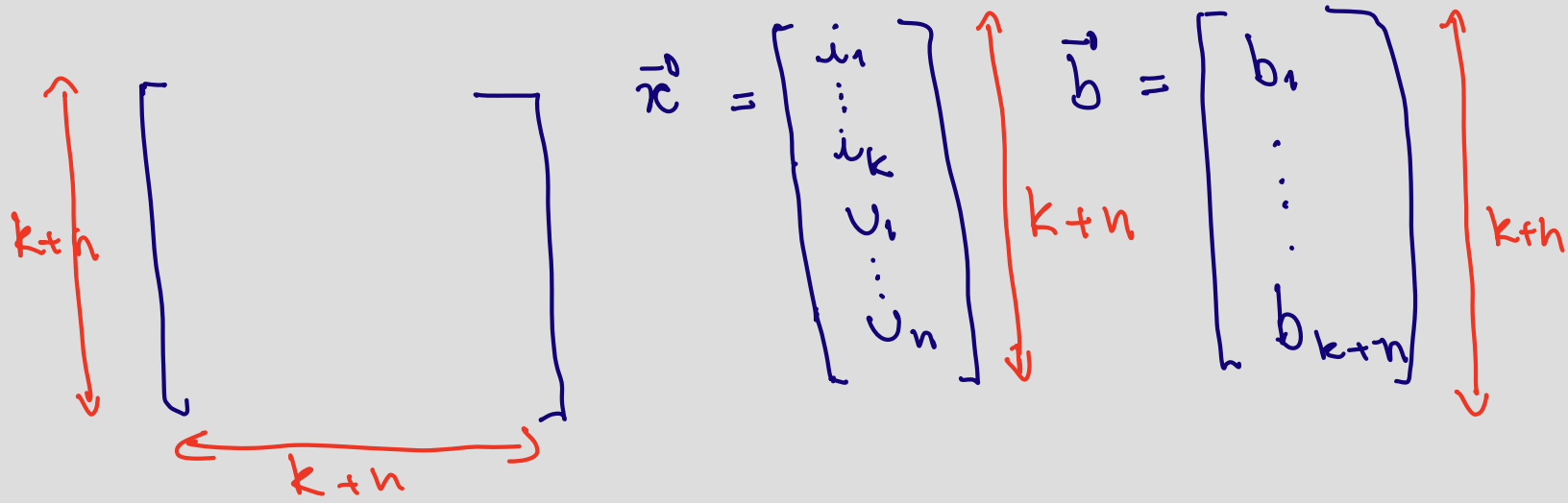
# Circuit Analysis Algorithm : step 5

$$A \vec{x} = \vec{b}$$

$\vec{x} \therefore$  unknowns

$\vec{b} \therefore$  knowns / constants

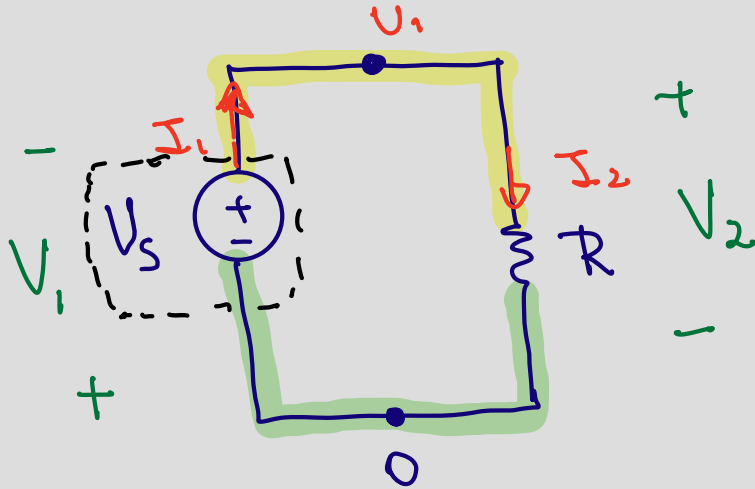
$A \therefore$  knowns / constants



# Circuit Analysis Algorithm : step 6

Use KCL to fill as many rows of A as possible (linear independence) # Nodes

$$-1 = N-1$$



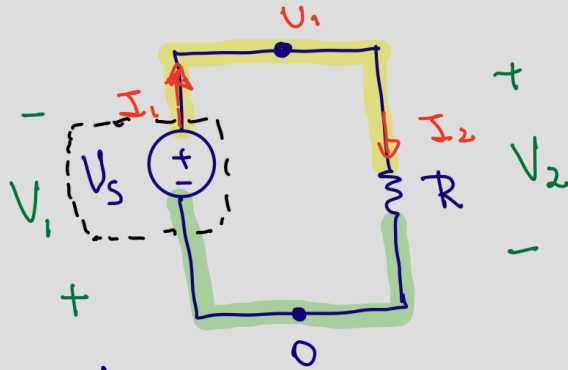
$$I_1 = I_2$$

$$I_1 - I_2 = 0$$

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ U_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

# Circuit Analysis Algorithm : step 7

Use current-voltage relationships for each element to fill the rest of the A matrix



Voltage Element =  $-V_s$

$$Vel_1 = 0 - U_1$$

$$U_1 = V_1 = V_s$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & R & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ U_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \end{bmatrix}$$

$$U_1 = V_s$$

$$I_1 = V_s / R$$

$$I_2 = V_s / R$$

Resistor

$$Vel_2 = I_2 \cdot R$$

$$Vel_2 = U_1 - 0 = U_1$$

$$U_1 = I_2 \cdot R$$

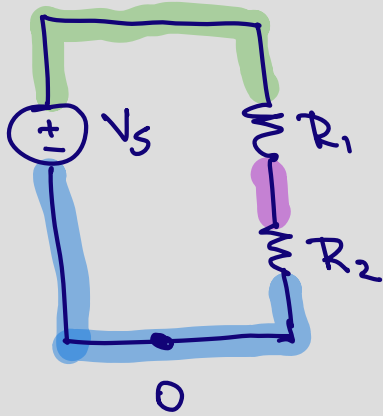
$$I_2 \cdot R - U_1 = 0$$



# Node Voltage Analysis – Voltage Divider

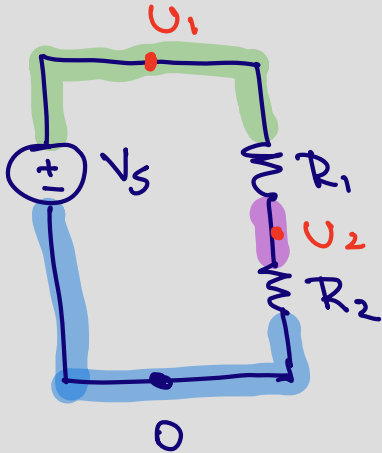
(Operator) → make circuit analysis faster

✓ Step 1 – Pick a node and label it as ground



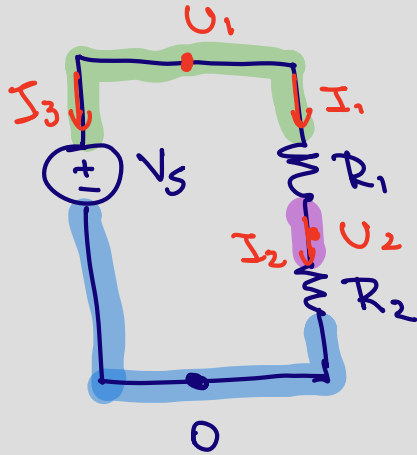
# Node Voltage Analysis – Voltage Divider

Step 2 – Label all remaining nodes as some potential  $U_i$ .



# Node Voltage Analysis – Voltage Divider

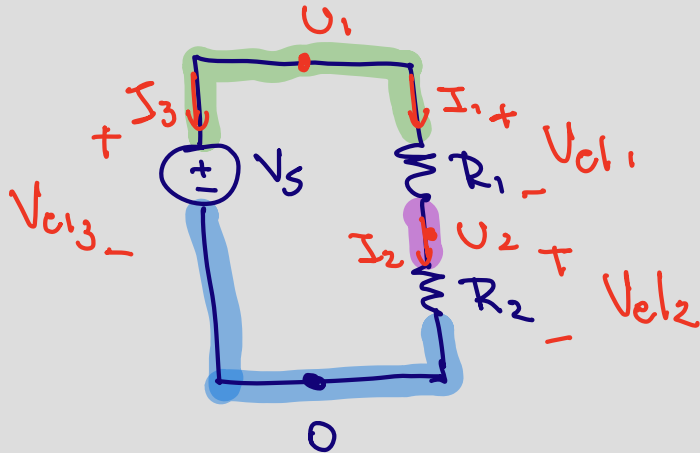
Step 3 – Label the current through every non-wire element in the circuit with  $I_n$ . ✓



# Node Voltage Analysis – Voltage Divider

Requires some thinking

Step 4 – Add +/- labels on each non-wire element, following the passive sign convention.



\* Important!

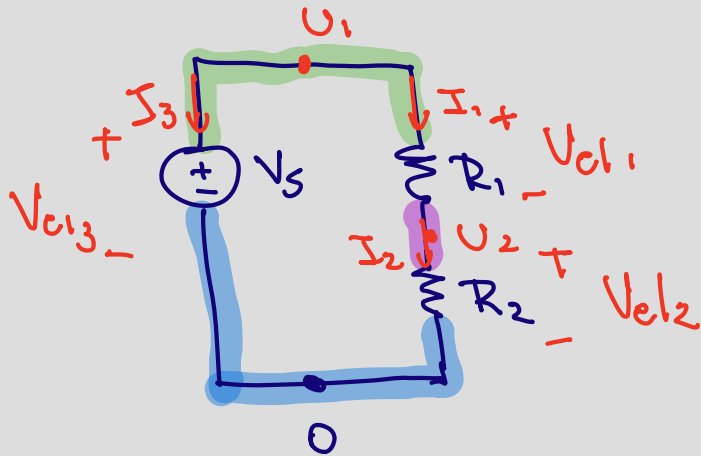
**Passive sign convention**: the current enters at the positive terminal and exits at the negative terminal.

# Node Voltage Analysis – Voltage Divider

$$A \vec{x} = \vec{b}$$

Step 5 – Set up the relationship  $A$  consists of the unknown currents and potentials.

Identify the unknowns.

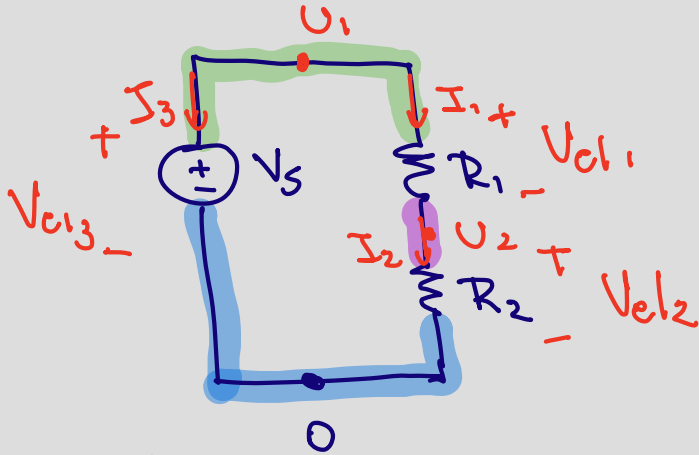


$$\vec{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}$$

**Passive sign convention:** the current enters at the positive terminal and exits are the negative terminal.

# Node Voltage Analysis – Voltage Divider

Step 6 – Use KCL to fill in as many linearly independent rows in  $\mathbf{A}$  and  $\vec{\mathbf{b}}$



$$\text{For } U_1 \Rightarrow 0 = I_1 + I_3 \quad (1)$$

$$\text{For } U_2 \Rightarrow I_1 = I_2$$

$$I_1 - I_2 = 0 \quad (2)$$

$$\vec{\mathbf{a}} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}$$

**KCL**: the current flowing into a junction must equal the current flowing out of that junction.

# Node Voltage Analysis – Voltage Divider

Step 7 – Use the IV relationships of each of the non-wire elements to fill in the remaining rows of A and the remaining rows of A are

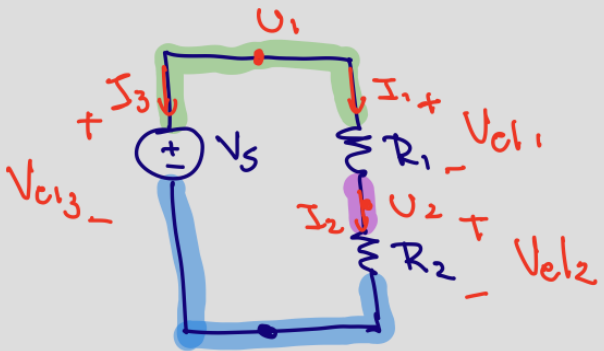
$= 0$  (1)  
 $= 0$  (2)  
 $\dots$

## Voltage Def.

$$V_{el1} = U_1 - U_2$$

$$V_{el2} = U_2 - 0 = U_2$$

$$V_{el3} = U_1 - 0 = U_1$$



Element IV

$$V_{el1} = R_1 I_1$$

$$V_{el2} = R_2 I_2$$

$$V_{el3} = V_s$$

Substitution:

$$El_1 \therefore U_1 - U_2 = R_1 I_1 \Rightarrow R_1 I_1 - U_1 + U_2 = 0 \quad (3)$$

$$El_2 \therefore U_2 = R_2 I_2 \Rightarrow R_2 I_2 - U_2 = 0 \quad (4)$$

$$El_3 \therefore U_1 = V_s \quad (5)$$

# Node Voltage Analysis – Voltage Divider

Step 8 – Solve the system of equations to determine values of unknown variables.

$$\begin{aligned} I_1 + I_3 &= 0 \quad (1) \\ -I_1 + I_2 &= 0 \quad (2) \\ R_1 I_1 - U_1 + U_2 &= 0 \quad (3) \\ R_2 I_2 - U_2 &= 0 \quad (4) \\ U_1 &= V_s \quad (5) \end{aligned}$$

$$\overset{A}{\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ R_1 & 0 & 0 & -1 & 1 \\ 0 & R_2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}} \overset{x}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ U_1 \\ U_2 \end{bmatrix}} = \overset{b}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \end{bmatrix}}$$

$$I_1 = \frac{V_s}{R_1 + R_2}, \quad I_2 = \frac{V_s}{R_1 + R_2}, \quad I_3 = -\frac{V_s}{R_1 + R_2}$$

$$U_1 = V_s$$

$$U_2 = \frac{R_2}{R_1 + R_2} \cdot V_s$$

$\hookrightarrow \alpha < 1$

$\alpha$  is an operator



# Electrical Circuit Analysis Algorithm (tool)

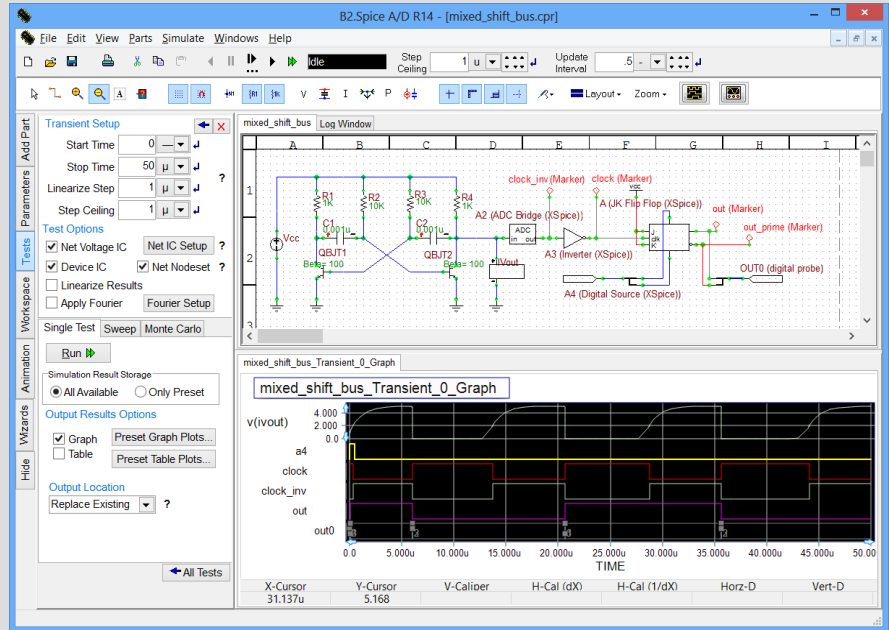
SPICE (Simulation Program with Integrated Circuit Emphasis): started as a student project at Berkeley!

Now the basis for open-source electronic circuit simulation, to design and model device characteristics and check circuit boards

**cadence**<sup>®</sup>  
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Prof. Alberto L. Sangiovanni-Vincentelli



# Electrical Circuit Analysis Algorithm (tool)

